How to print values with uncertainties?

Weekly Group Meeting

Patrick Ecker | 23.08.2022

Problem

We often determine values which have a uncertainty assigned. For example:

$$\Gamma_{h'} = 0.000\,634\,56\,\mathrm{GeV}/c^2 \quad \text{with} \quad \sigma = 0.000\,423\,456\,\mathrm{GeV}/c^2$$

Why should we round this?

- Rounding increases the readability of the results
- Decimals smaller than the uncertainty do not give any additional useful information

But how do we decide how many decimals should be printed and how do we round?

Solution

Luckily the German Industry Norm **DIN 1333** gives us the answer!

How to define the number of decimals to take into account?

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- From the left take the position of the **first non-zero decimal**

$$\sigma_1 = 0.0054321 \qquad \sigma_2 = 0.00012345$$

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- If this value is ≥ 3 round to this precision, else round to the decimal to the right

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$$\sigma_1 = 0.0054321 \qquad \sigma_2 = 0.00012345$$

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$$\sigma_1 = 0.005 \qquad \sigma_2 = 0.00012$$

Rounding Position: $r_1 = 0.001 \qquad r_2 = 0.00001$

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$$\sigma_1 = 0.005 \qquad \sigma_2 = 0.00012$$

$$\text{Rounding Position:} \quad r_1 = 0.001 \qquad r_2 = 0.00001$$

- **Why is there a differentation?**

Why do we round to different decimals?

Rounding Deviation

$$\epsilon = \frac{|m_{true} - m_{round}|}{m_{round}}$$

The maximal deviation is half the rounding position value, e.g.

$$x = 0.1 \qquad \delta m_{max} = 0.05$$
$$\rightarrow \epsilon_{max} = \frac{0.05}{0.1} = 0.5$$
$$x = 0.7 \qquad \delta m_{max} = 0.05$$
$$\rightarrow \epsilon_{max} = \frac{0.05}{0.7} = 0.07$$

Value	$\epsilon_{max,1}(\%)$	$\epsilon_{max,2}(\%)$
0.1	50	5
0.2	25	2.5
0.3	17	1.7
0.4	13	1.3
0.5	10	1.0
0.6	8	0.8
0.7	7	0.7
0.8	6	0.6
0.9	6	0.6

How to round? $\rightarrow$ Positive numbers

Rounding

Add half of the rounding position and remove all numbers that are located behind the rounding position afterwards

$$0.0054321 + 0.0005 = 0.0059321 \rightarrow 0.005$$
$$0.0056321 + 0.0005 = 0.0061321 \rightarrow 0.006$$

Rounding down

Remove the decimals after the rounding position without any previous addition

Rounding up

Add the rounding position and remove all numbers that are located behind the rounding position afterwards

$$0.0054321 + 0.001 = 0.0064321 \rightarrow 0.006$$
$$0.0056321 + 0.001 = 0.0066321 \rightarrow 0.006$$

How to round? $\rightarrow$ Negative numbers

Rounding
Round the absolute value of the number according to the explanation for positive numbers and set the minus sign afterwards

Rounding down
Add the rounding position to the absolute value and leave out the numbers after the rounding position afterwards. Then add the minus sign.

$$-(0.0054321 + 0.001) = -0.0064321 \rightarrow -0.006$$
$$-(0.0056321 + 0.001) = -0.0066321 \rightarrow -0.006$$

Rounding up
Leave out every number that is located behind the rounding position without any previous addition

How to print a value with given uncertainty

$$\Gamma_{h'} = 0.000\,634\,56\,\mathrm{GeV}/c^2 \quad \text{with} \quad \sigma = 0.000\,423\,456\,\mathrm{GeV}/c^2$$

How to print a value with given uncertainty

$$\Gamma_{h'} = 0.000\,634\,56\,\mathrm{GeV/c^2} \quad \text{with} \quad \sigma = 0.000\,423\,456\,\mathrm{GeV/c^2}$$

- Get the significant digit of the uncertainty σ

$$\Gamma_{h'} = 0.000\,634\,56\,\mathrm{GeV/c^2} \quad \text{with} \quad \sigma = 0.000\textcolor{red}{4}23456\,\mathrm{GeV/c^2}$$

How to print a value with given uncertainty

$$\Gamma_{h'} = 0.000\,634\,56\,\mathrm{GeV/c^2} \quad \text{with} \quad \sigma = 0.000\,423\,456\,\mathrm{GeV/c^2}$$

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$$\Gamma_{h'} = 0.000\,634\,56\,\mathrm{GeV/c^2} \quad \text{with} \quad \sigma = 0.000\mathbf{4}23456\,\mathrm{GeV/c^2}$$

- Round **up** the uncertainty at this position

$$\Gamma_{h'} = 0.000\,634\,56\,\mathrm{GeV/c^2} \quad \text{with} \quad \sigma = 0.000\mathbf{5}\,\mathrm{GeV/c^2}$$

How to print a value with given uncertainty

$$\Gamma_{h'} = 0.000\,634\,56\,\mathrm{GeV}/c^2 \quad \text{with} \quad \sigma = 0.000\,423\,456\,\mathrm{GeV}/c^2$$

- Get the significant digit of the uncertainty σ

$$\Gamma_{h'} = 0.000\,634\,56\,\mathrm{GeV}/c^2 \quad \text{with} \quad \sigma = 0.000{\color{red}4}23456\,\mathrm{GeV}/c^2$$

- Round **up** the uncertainty at this position

$$\Gamma_{h'} = 0.000\,634\,56\,\mathrm{GeV}/c^2 \quad \text{with} \quad \sigma = 0.000{\color{red}5}\,\mathrm{GeV}/c^2$$

- Round the value to the **same precision** (independent of the value at this position!)

$$\Gamma_{h'} = 0.000{\color{red}6}\,\mathrm{GeV}/c^2 \quad \text{with} \quad \sigma = 0.000{\color{red}5}\,\mathrm{GeV}/c^2$$

How to print a value with given uncertainty

$$\Gamma_{h'} = 0.000\,634\,56\,\mathrm{GeV/c^2} \quad \text{with} \quad \sigma = 0.000\,423\,456\,\mathrm{GeV/c^2}$$

- Get the significant digit of the uncertainty σ

$$\Gamma_{h'} = 0.000\,634\,56\,\mathrm{GeV/c^2} \quad \text{with} \quad \sigma = 0.000423456\,\mathrm{GeV/c^2}$$

- Round **up** the uncertainty at this position

$$\Gamma_{h'} = 0.000\,634\,56\,\mathrm{GeV/c^2} \quad \text{with} \quad \sigma = 0.0005\,\mathrm{GeV/c^2}$$

- Round the value to the **same precision** (independent of the value at this position!)

$$\Gamma_{h'} = 0.0006\,\mathrm{GeV/c^2} \quad \text{with} \quad \sigma = 0.0005\,\mathrm{GeV/c^2}$$

- Print the result

$$\Gamma_{h'} = (6 \pm 5) \times 10^{-4}\,\mathrm{GeV/c^2}$$

Some Comments

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- Do not fill the rounded digits with zeros $\rightarrow$ The comma must not be further to the right than directly next to the rounding position

$$1234 \approx 1.2 \times 10^3 \quad \text{or} \quad 1234 \approx 12 \times 10^2 \quad \text{but not} \quad 1234 \approx 1200$$

- While rounding improves the readability of a value you should keep in mind that with every rounding you loose information
- Use the **rounded values only to print** them and keep the exact value for your calculation. Easy example:

$$\frac{1}{3} \cdot \frac{1}{3} \cdot \frac{1}{3} = \frac{1}{27} \approx 0.03704$$
$$0.3 \cdot 0.3 \cdot 0.3 = 0.027$$

  Which makes a huge difference
- I have written python code which follows the presented rules $\rightarrow$ Will put it on github

Backup