



NNLO interpolation grids for jet production at the LHC

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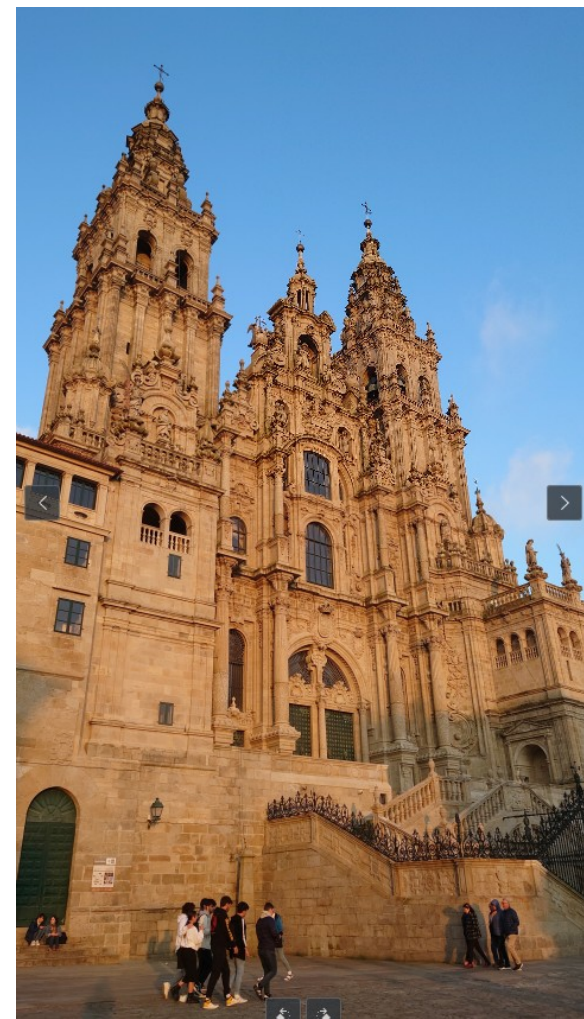




Outline



- Interpolation grids at NNLO for DIS published
- Interpolation grids for numerous jet datasets at LHC computed
- Inclusive jets with examples for
 - ➔ Grid closure
 - ➔ Scale dependence
 - ➔ PDF uncertainties
 - ➔ NNLO K factors
- Dijets with application example
 - ➔ PDF (+ α_s) fit
- Outlook





DIS interpolation grids

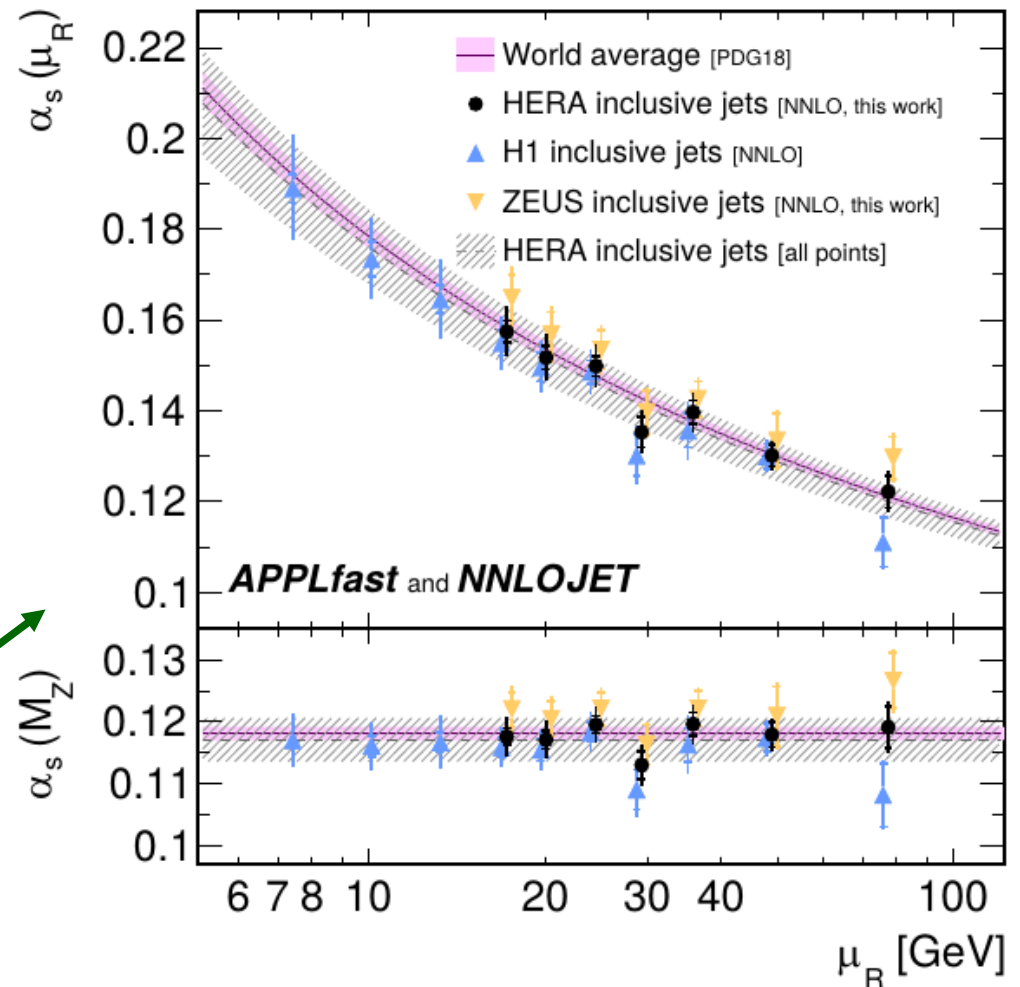


Successfully used for jets at NNLO in DIS:

- ➡ **H1**, EPJC 77 (2017) 791; Err. EPJC 81 (2021) 738.
- ➡ **APPLfast**, EPJC 79 (2019) 845; Err. EPJC 81 (2021) 957.
- ➡ **HERAPDF2.0Jets**, EPJC 82 (2022) 243.

See also yesterday's talk by Katarzyna Wichmann

Example:
Running of α_s from inclusive jets



Interpolation grids available on:
<https://ploughshare.web.cern.ch>

Ploughshare



Ingredients



• Theory:



- ➔ **NNLOJET:** T. Gehrmann et al., RADCOR2017 PoS (2018) 074, arXiv:1801.06415.
- ➔ **Inclusive jets:** J. Currie et al, PRL 118 (2017) 072002; JHEP 10 (2018) 155.
- ➔ **Dijets:** J. Currie et al., PRL 119 (2017) 152001; A. Gehrmann-de Ridder et al., PRL 123 (2019) 102001.

• Tools:



- ➔ **APPLfast interface:** D. Britzger et al., EPJC 79 (2019) 845, arXiv:1906.05303.
- ➔ **fastNLO:** D. Britzger et al., Proc. DIS2012 (2012) 217, arXiv:1208.3641.
- ➔ **APPLgrid:** T. Carli et al., EPJC 66 (2010) 503, arXiv:0911.2985.
- ➔ **xfitter:** S. Alekhin et al., EPJC 75 (2015) 304, arXiv:1410.4412.

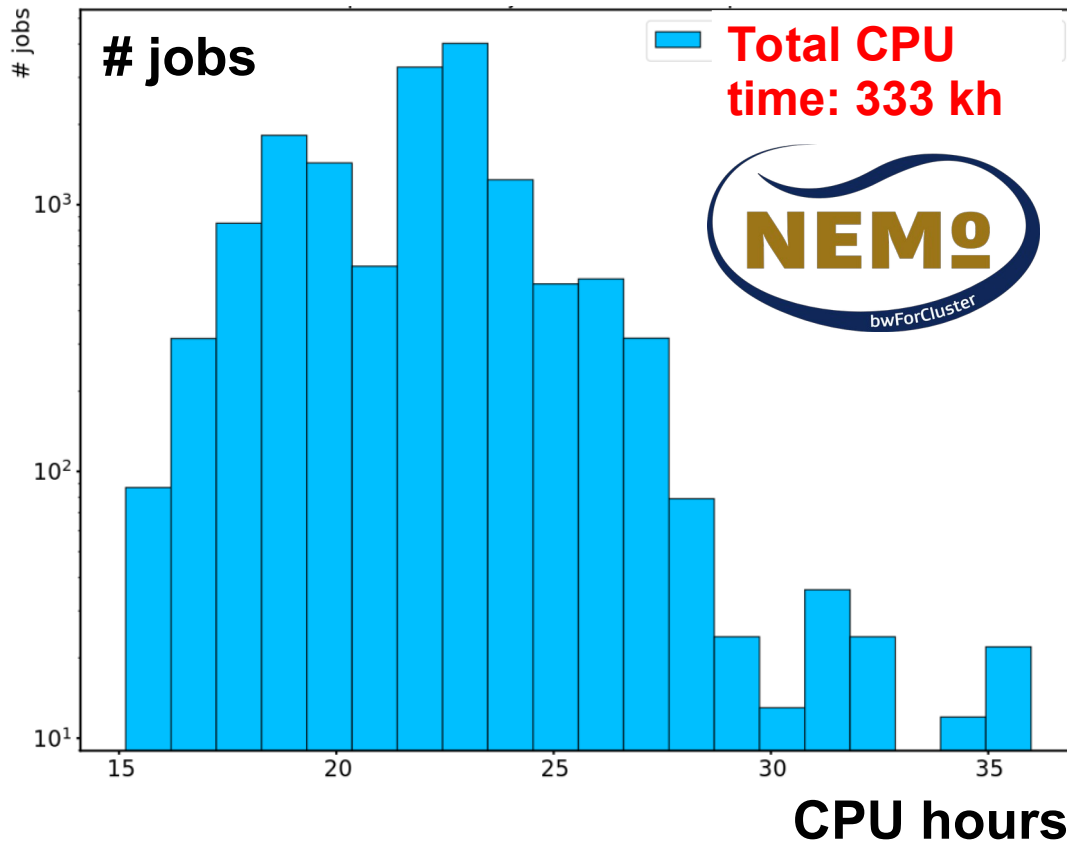




The investment



Typical total runtime of a grid production



Total # of ~24h jobs: $O(15000)$

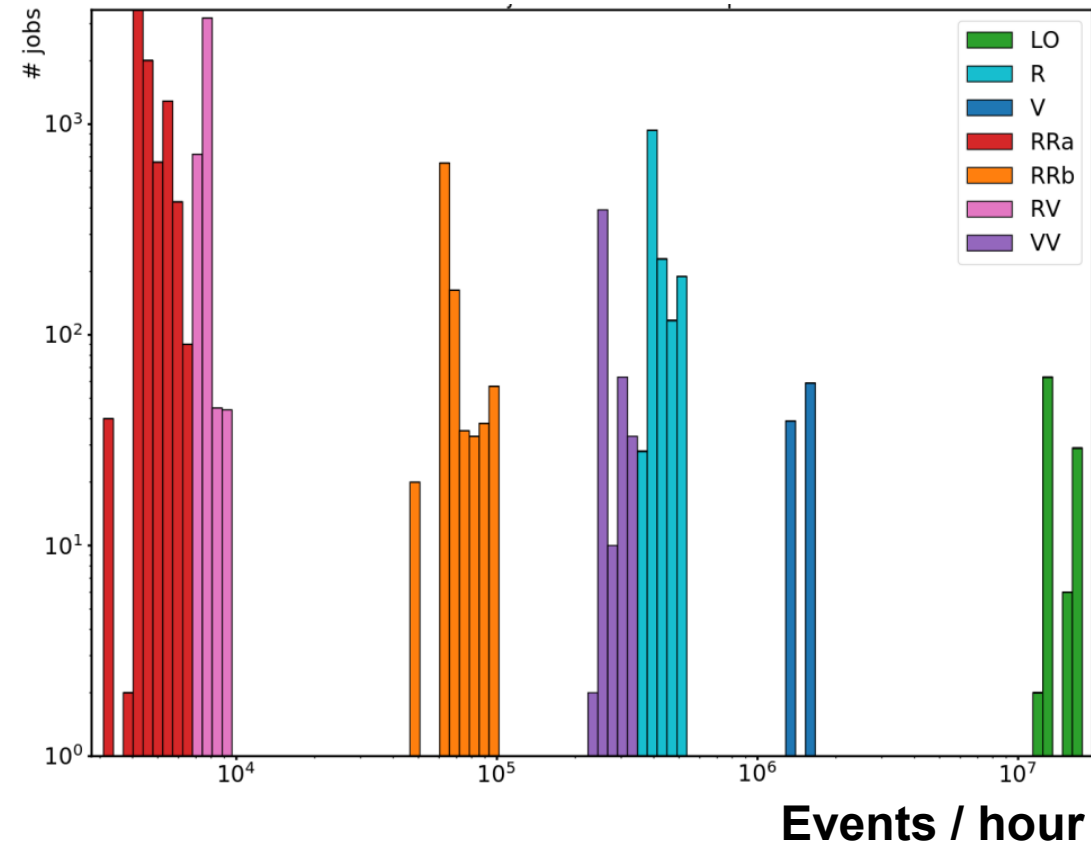
“Events” / hour

RRa, RV:
most expensive

LO, NLO, VV:
easy

$O(10^3 - 10^4)$

$O(10^5 - 10^7)$





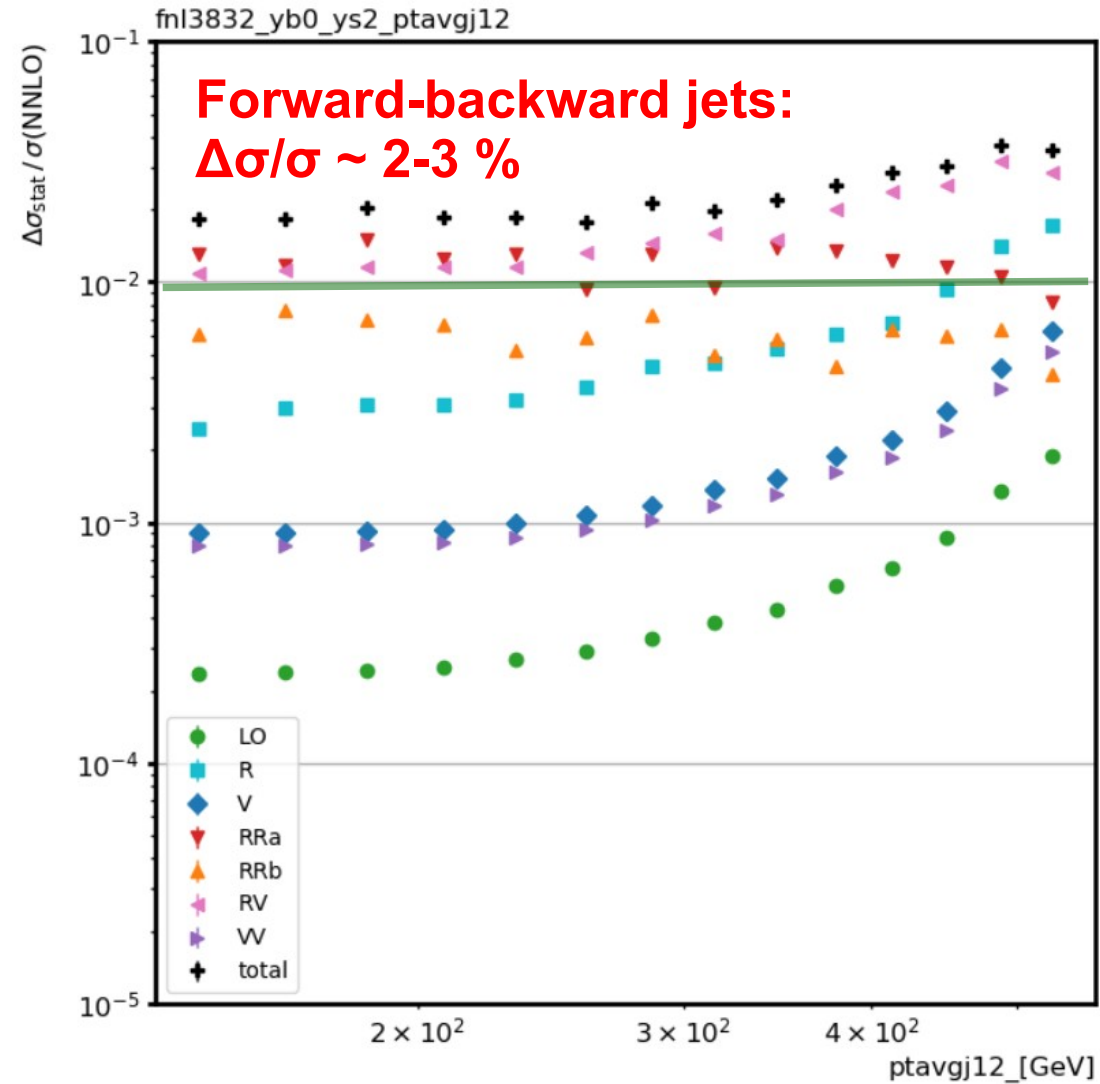
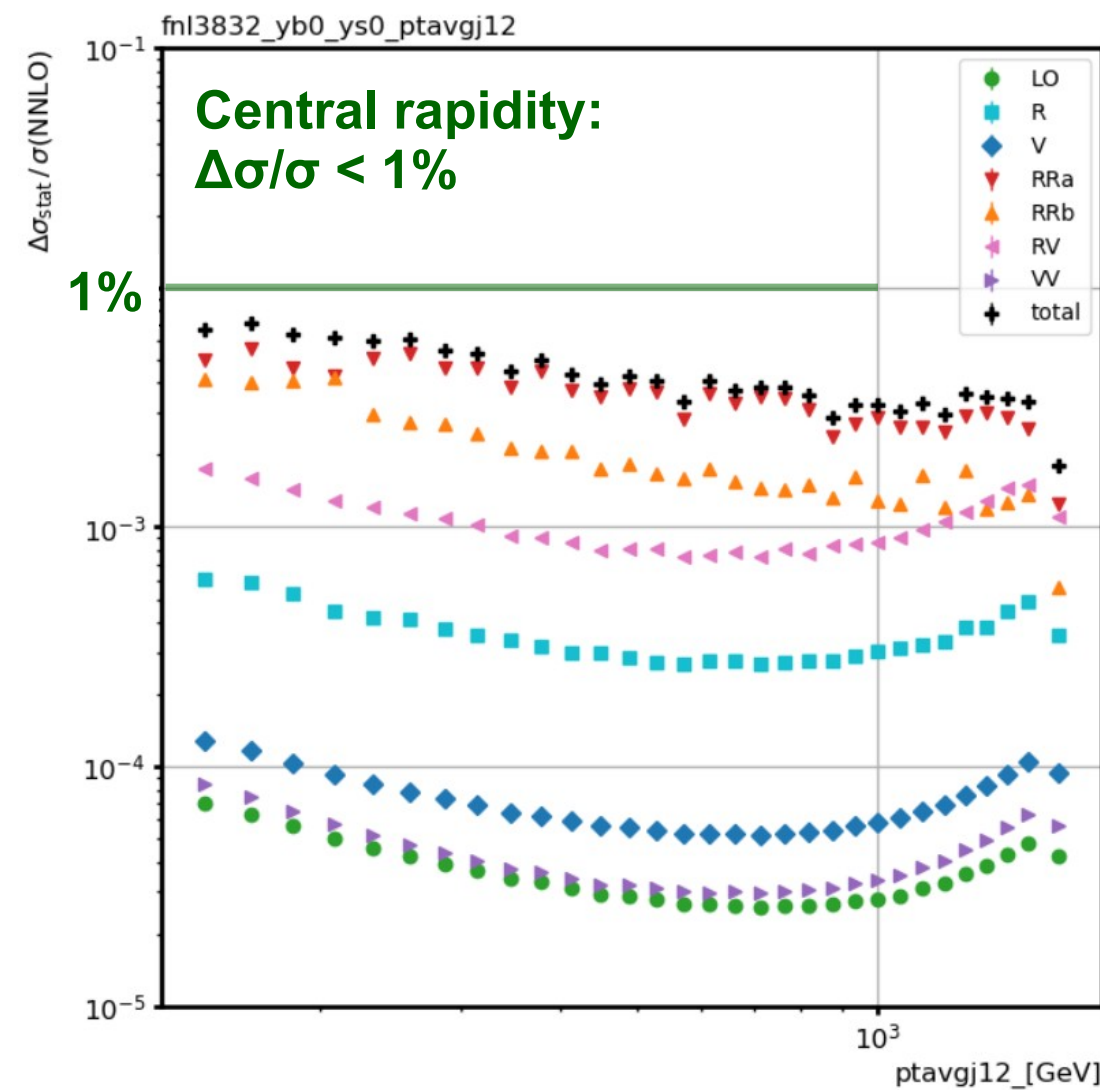
Return on investment



Relative numerical uncertainty of NNLO 3D dijet cross section

Dominated by RRa and RV channels

Numerical uncertainty provided inside grids





Inclusive jet datasets



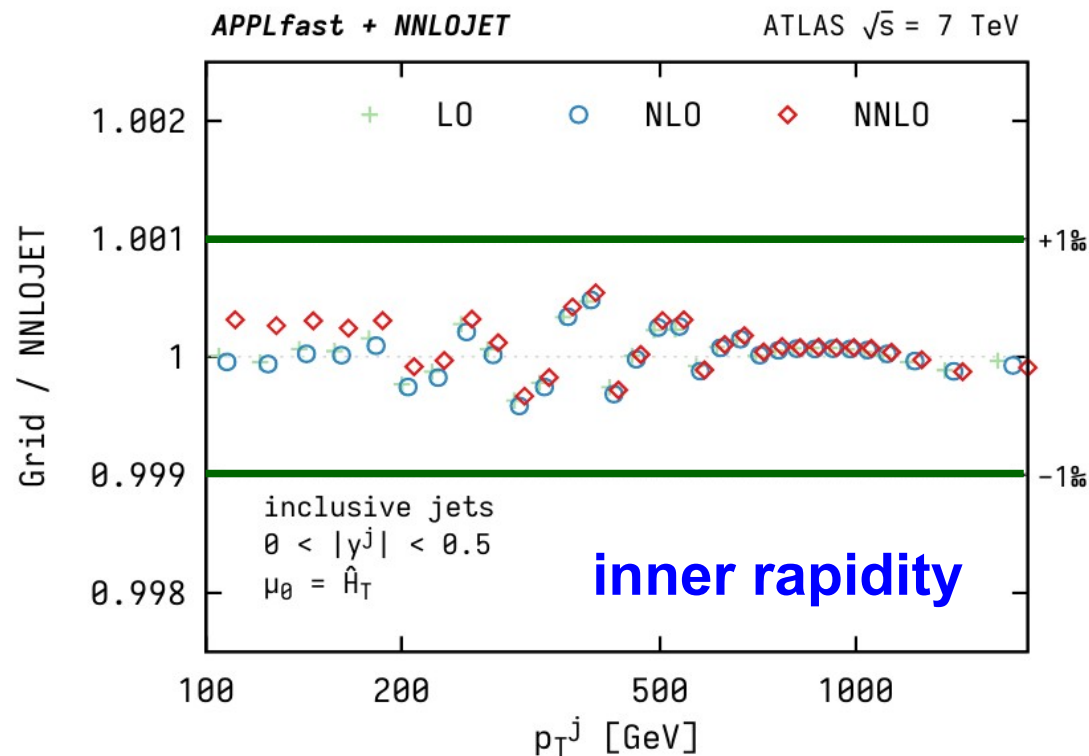
- Seven inclusive jet datasets from ATLAS & CMS, 2D in p_T and y
 - Four centre-of-mass energies, three jet radii R
 - Two central scales for $\mu_{R/F}$

**Sample plots
in this talk**

Data	\sqrt{s} [TeV]	\mathcal{L} [fb $^{-1}$]	no. of points	anti- k_T R	kinematic range [GeV]	fiducial cuts	$\mu_{R/F}$ -choice
CMS [29]	2.76	0.00543	81	0.7	$p_T^{\text{jet}} \in [74, 592]$	$ y < 3.0$	$p_T^{\text{jet}}, \hat{H}_T$
ATLAS [27]	7.0	4.5	2×140	0.4, 0.6	$p_T^{\text{jet}} \in [100, 1992]$	$ y < 3.0$	$p_T^{\text{jet}}, \hat{H}_T$
CMS [30]	7.0	5.0	133	0.7	$p_T^{\text{jet}} \in [114, 2116]$	$ y < 3.0$	$p_T^{\text{jet}}, \hat{H}_T$
ATLAS [31]	8.0	20.3	2×171	0.4, 0.6	$p_T^{\text{jet}} \in [70, 2500]$	$ y < 3.0$	$p_T^{\text{jet}}, \hat{H}_T$
CMS [32]	8.0	5.6 19.7	248	0.7	$p_T^{\text{jet}} \in [21, 74]$ $p_T^{\text{jet}} \in [74, 2500]$	$ y < 4.7$	$p_T^{\text{jet}}, \hat{H}_T$
ATLAS [33]	13.0	3.2	177	0.4	$p_T^{\text{jet}} \in [100, 3937]$	$ y < 3.0$	$p_T^{\text{jet}}, \hat{H}_T$
CMS [34]	13.0	36.3 33.5	2×78	0.4 0.7	$p_T^{\text{jet}} \in [97, 3103]$	$ y < 2.0$	$p_T^{\text{jet}}, \hat{H}_T$



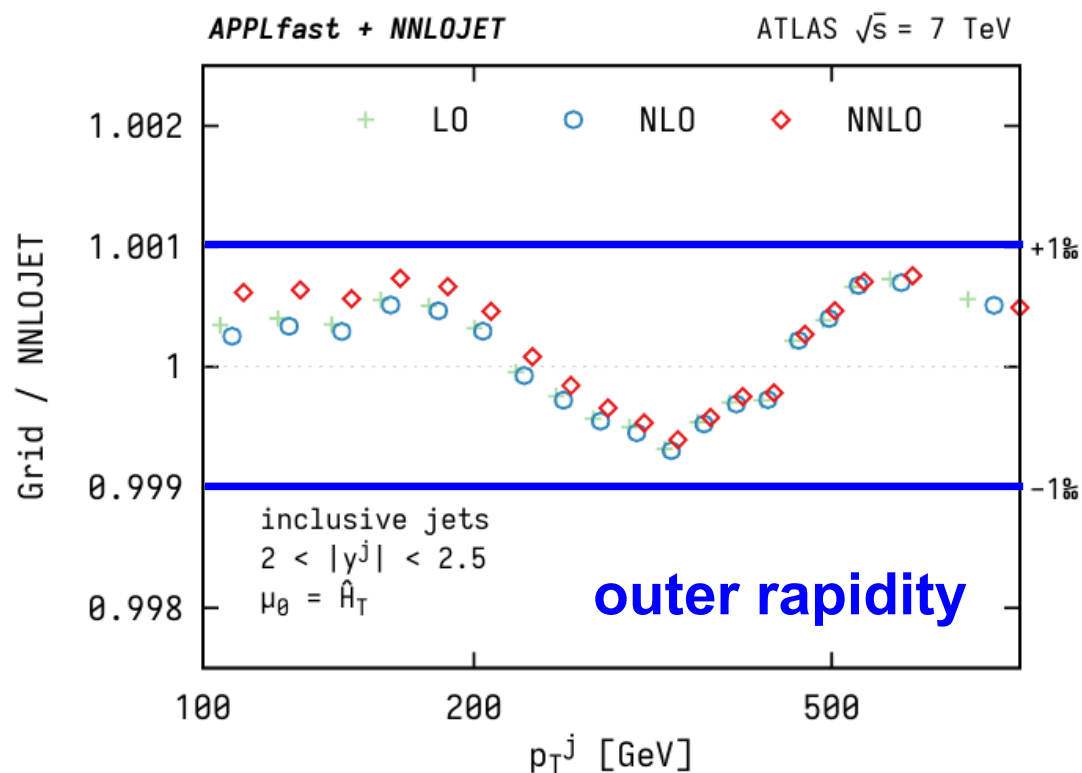
Grid closure vs. NNLOJET



Closure deteriorates somewhat towards phase space limits; exceptionally may exceed 1 ‰ at phase space edges

ATLAS inclusive jets at 7 TeV

Generally aim at closure better than 1 ‰



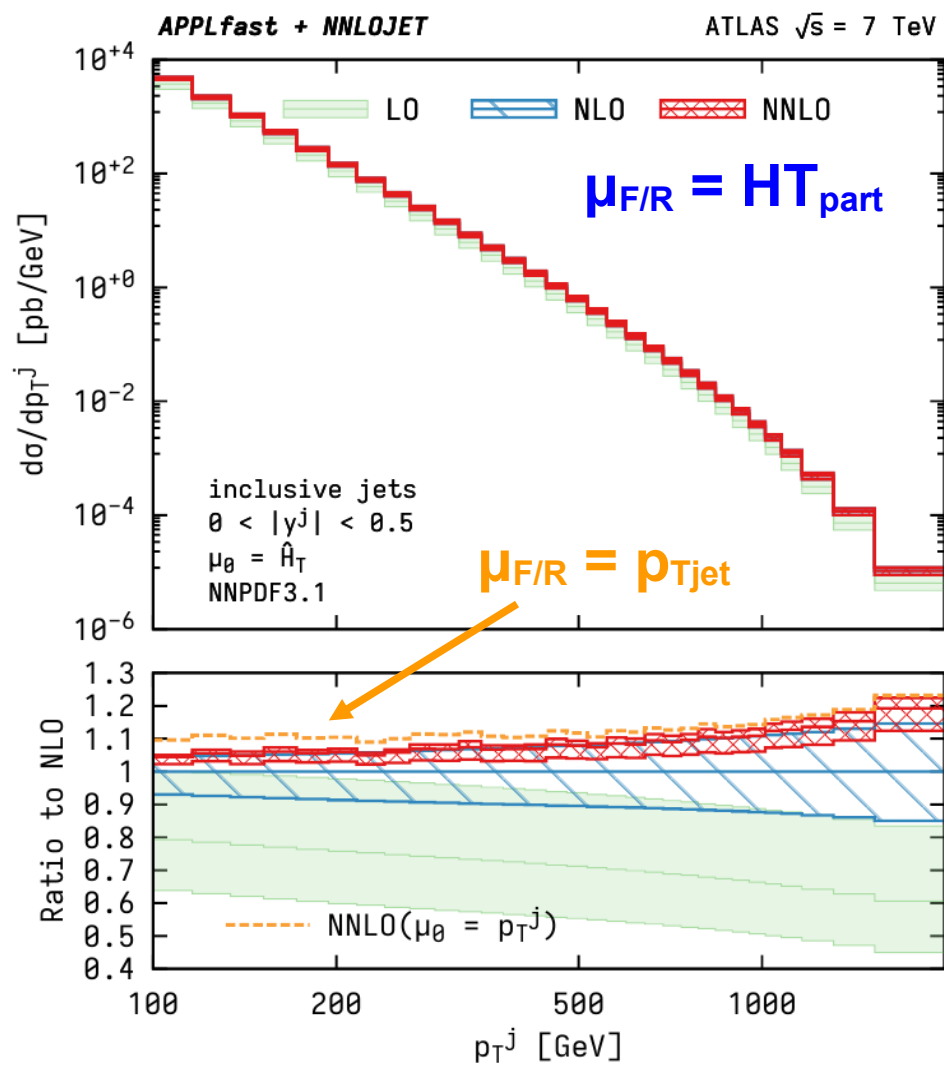


Scale dependence

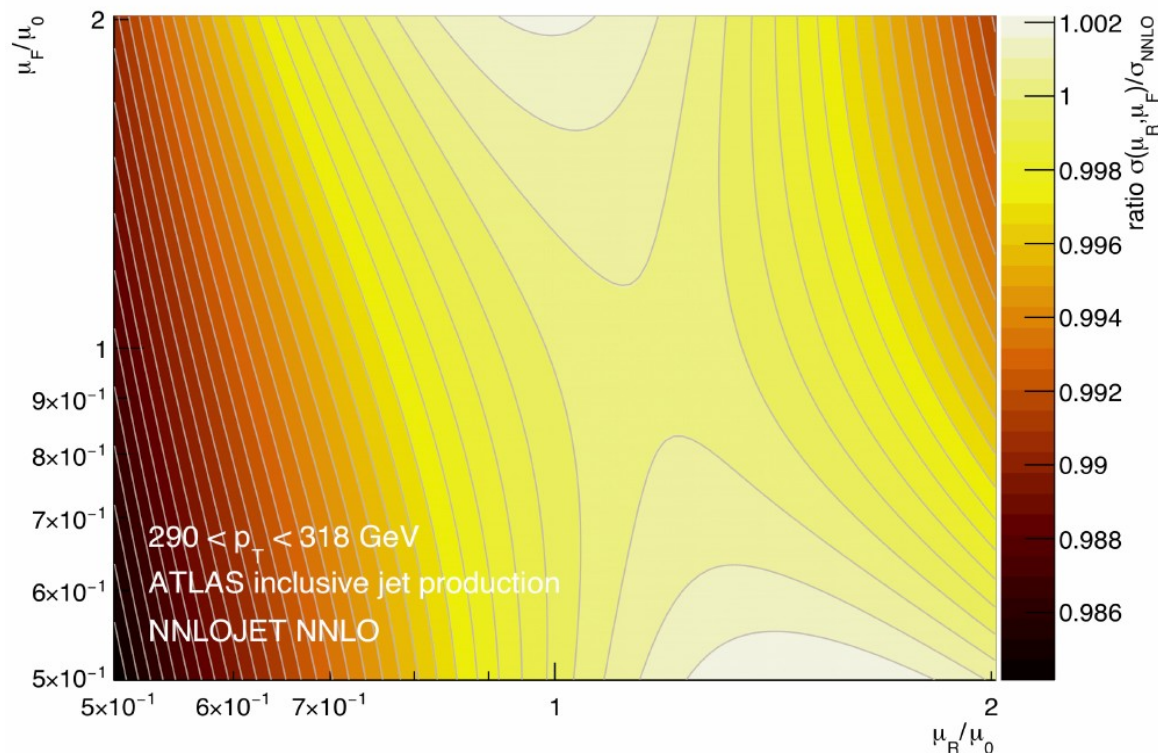


ATLAS inclusive jets at 7 TeV

Scale uncertainty bands: **LO**, **NLO**, **NNLO**



Full 2-dimensional scale dependence in $\mu_{F/R}$ for each bin



Additional scale: **p_{Tjet}** instead of **HT_{part}**



PDF dependence

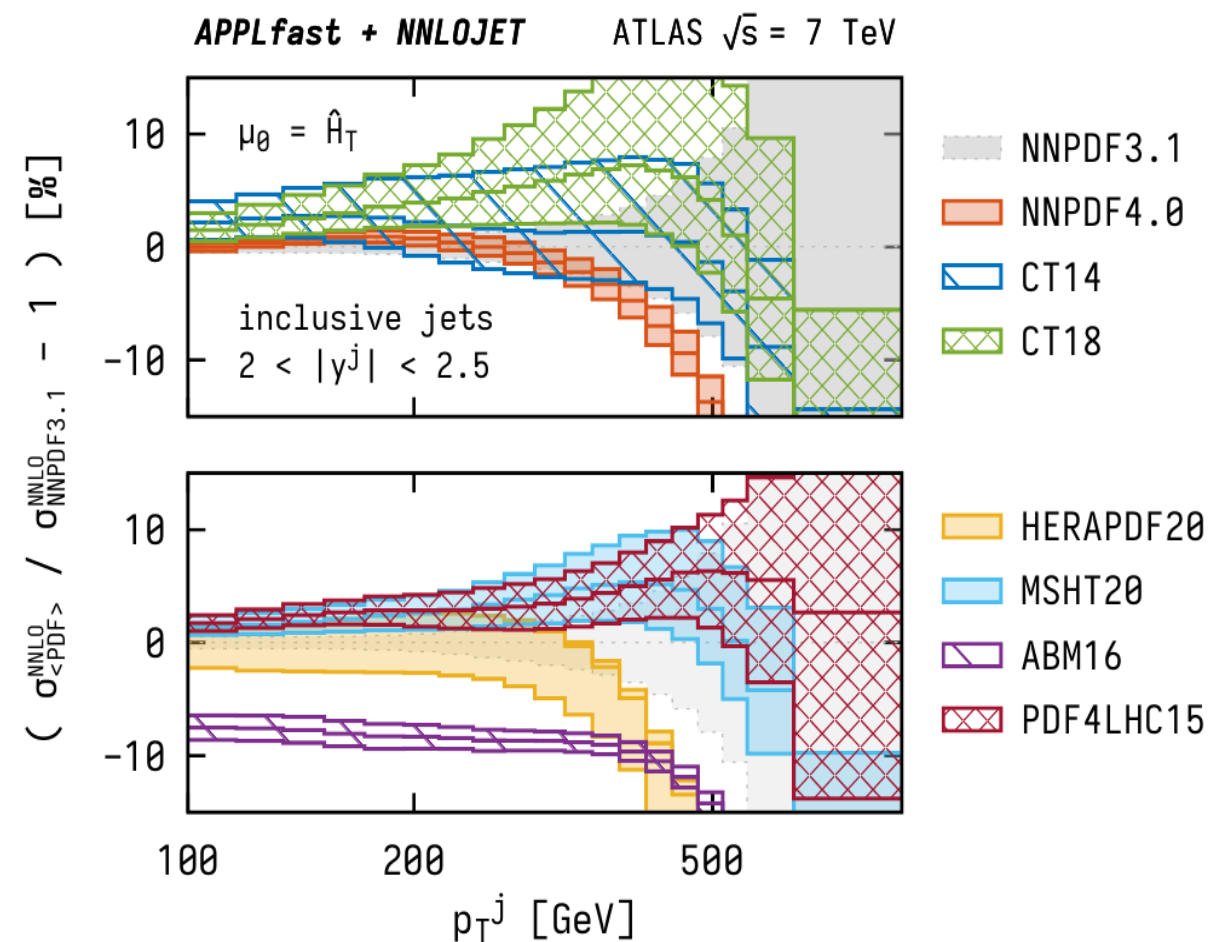
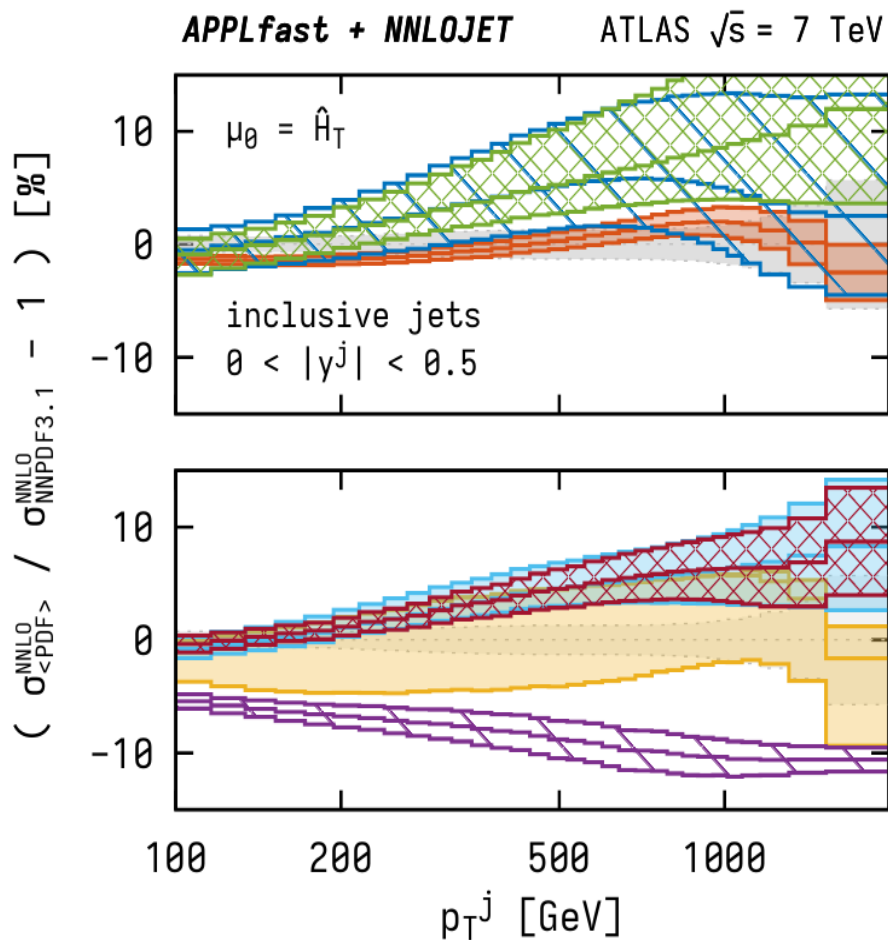


ATLAS inclusive jets at 7 TeV

PDF uncertainty bands for selection of 8 PDF sets

central rapidity

outer rapidity

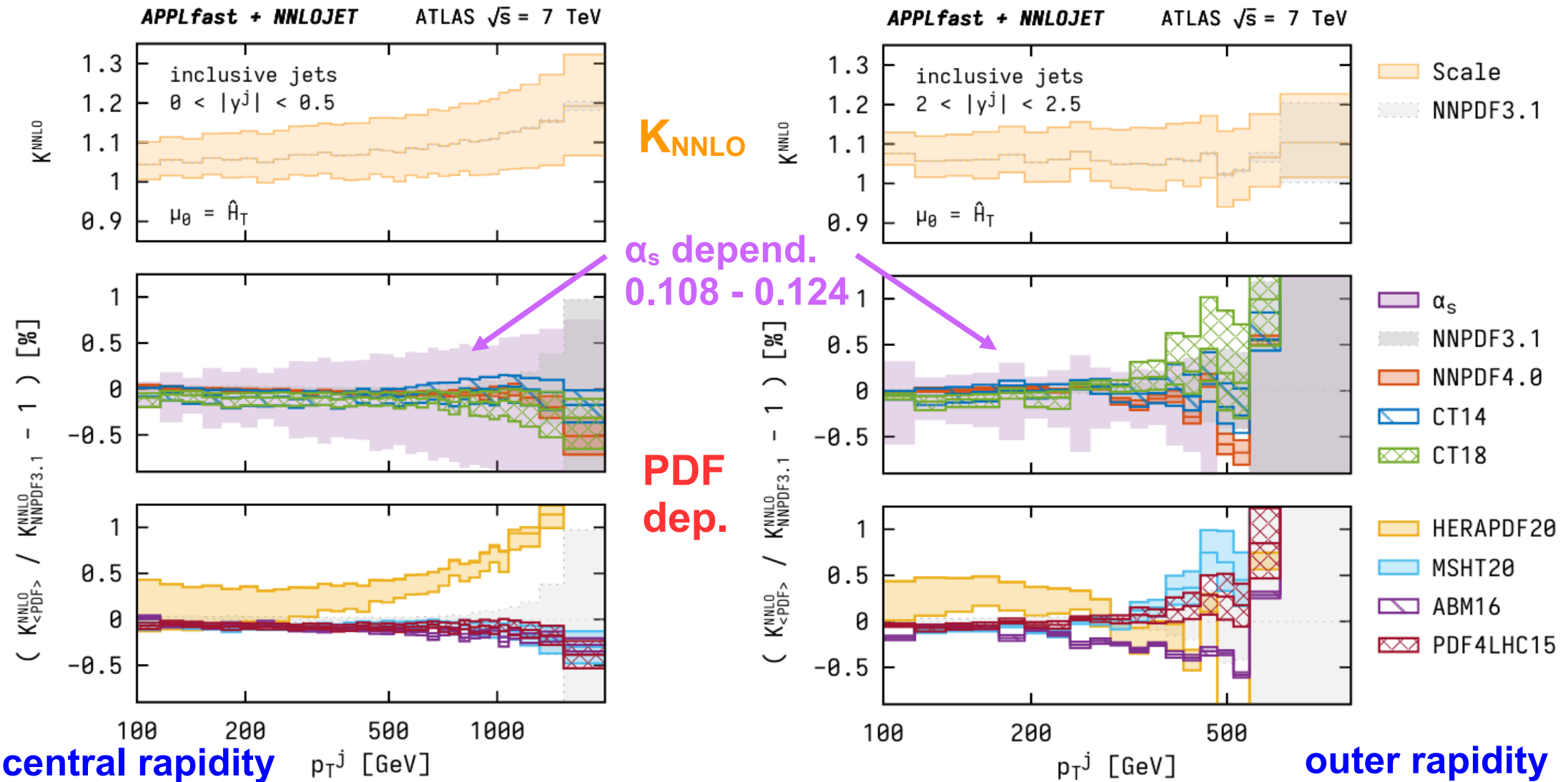




K factor robustness



Dependence of K_{NNLO} on: α_s – negligible
 most PDF sets – $\sim 0.5\%$
 exceptionally (ABM, HERAPDF) – $\sim 1\%$





Dijet datasets



- Four dijet datasets from ATLAS & CMS, 2D in m_{12} and y^* or y_{\max} , or 3D in $\langle p_{T12} \rangle$, y^* , y_b
- Three centre-of-mass energies, three jet radii R
- One central scale for $\mu_{R/F}$, except for 3D data with two

**Sample plots
in this talk**

Data	\sqrt{s} [TeV]	\mathcal{L} [fb ⁻¹]	no. of points	anti- k_T R	kinematic range [GeV]	fiducial cuts	$\mu_{R/F}$ -choice
ATLAS [48]	7.0	4.5	90	0.6	$m_{12} \in [260, 5040]$	$ y_1 , y_2 < 3.0$ $[p_{T,1}, p_{T,2}] > [100, 50]$ GeV $y^* < 3.0$	m_{12}
CMS [30]	7.0	5.0	54	0.7	$m_{12} \in [197, 5058]$	$ y < 5.0$ $[p_{T,1}, p_{T,2}] > [60, 30]$ GeV $ y_{\max} < 2.5$	m_{12}
CMS [47]	8.0	19.7	122	0.7	$\langle p_{T1,2} \rangle \in [133, 1784]$	$ y < 5.0$ $p_{T,1}, p_{T,2} > 50$ GeV $ y_1 , y_2 < 3.0$	$p_{T,1} \exp(0.3 y^*)$ m_{12}
ATLAS [33]	13.0	3.2	136	0.4	$m_{12} \in [260, 9066]$	$ y_1 , y_2 < 3.0$ $p_{T,1}, p_{T,2} > 75$ GeV $\langle p_{T1,2} \rangle > 100$ GeV $y^* < 3.0$	m_{12}



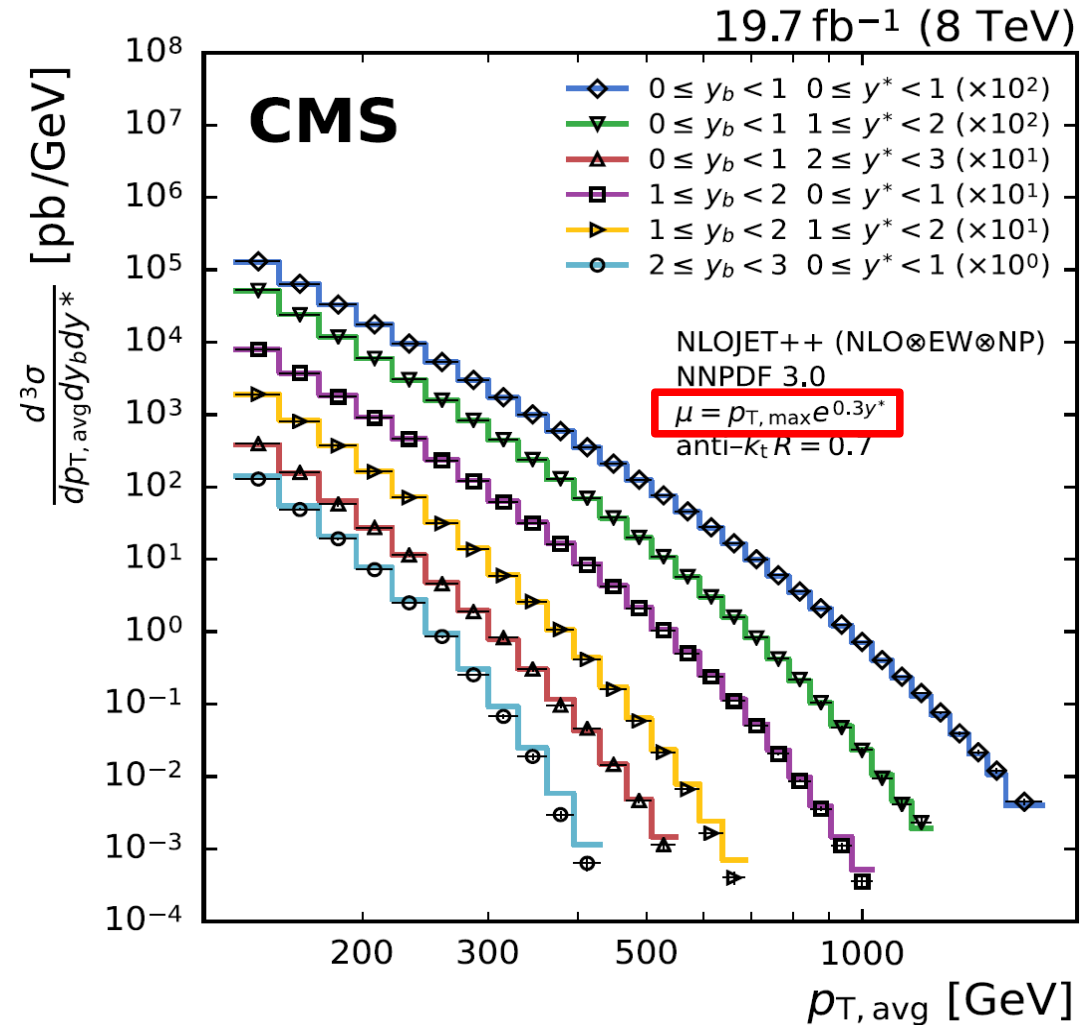
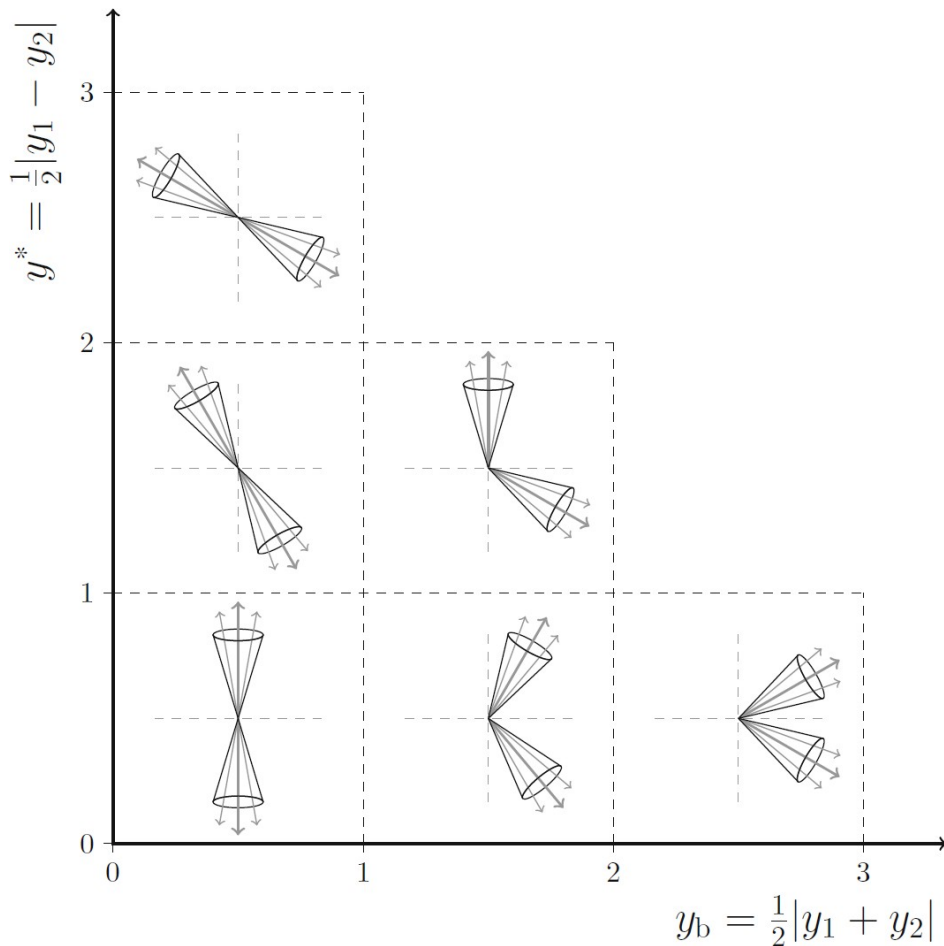
Triple-differential dijets



Most measurements done with respect to dijet mass and either max. rapidity $|y|_{\max}$ (CMS) or rapidity separation y^* (ATLAS). One CMS result 3D in p_{T12} , y^* , y_b :

$$\frac{d^3\sigma}{dp_{T,\text{avg}} dy_b dy^*} \propto \alpha_s^2$$

Illustration of dijet event topologies

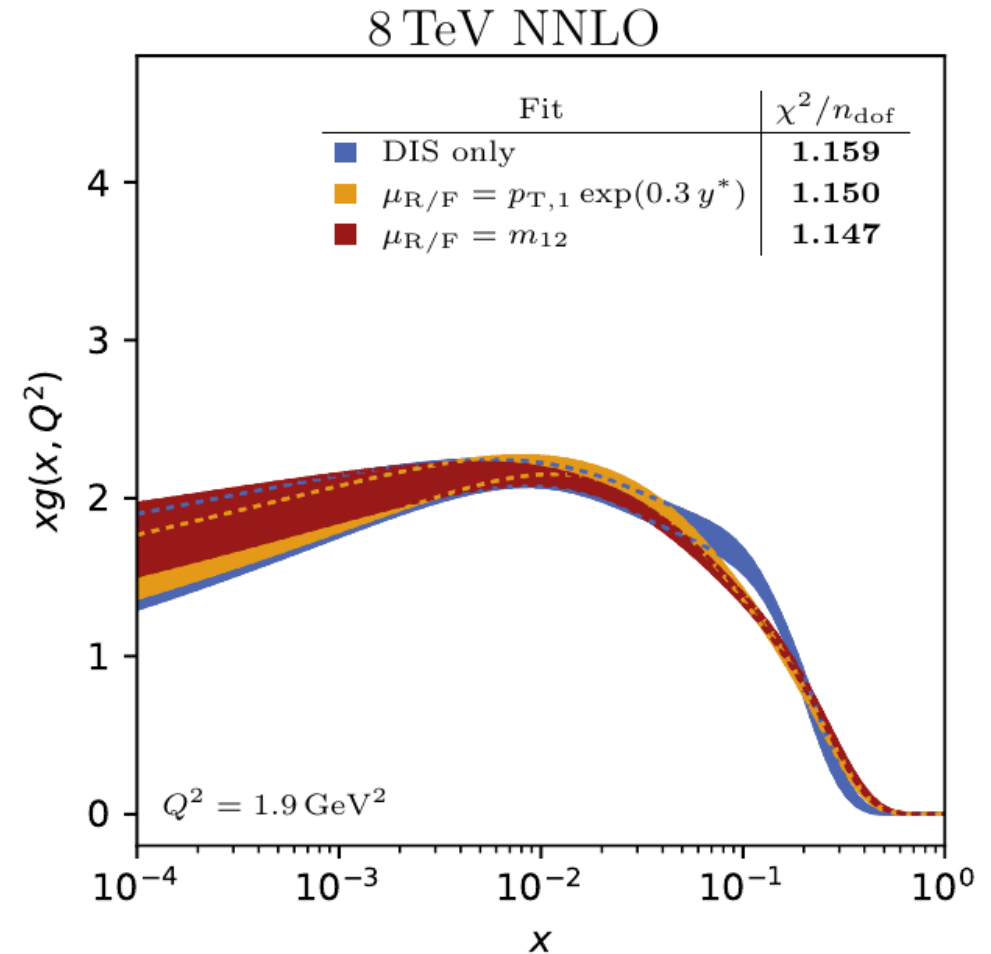
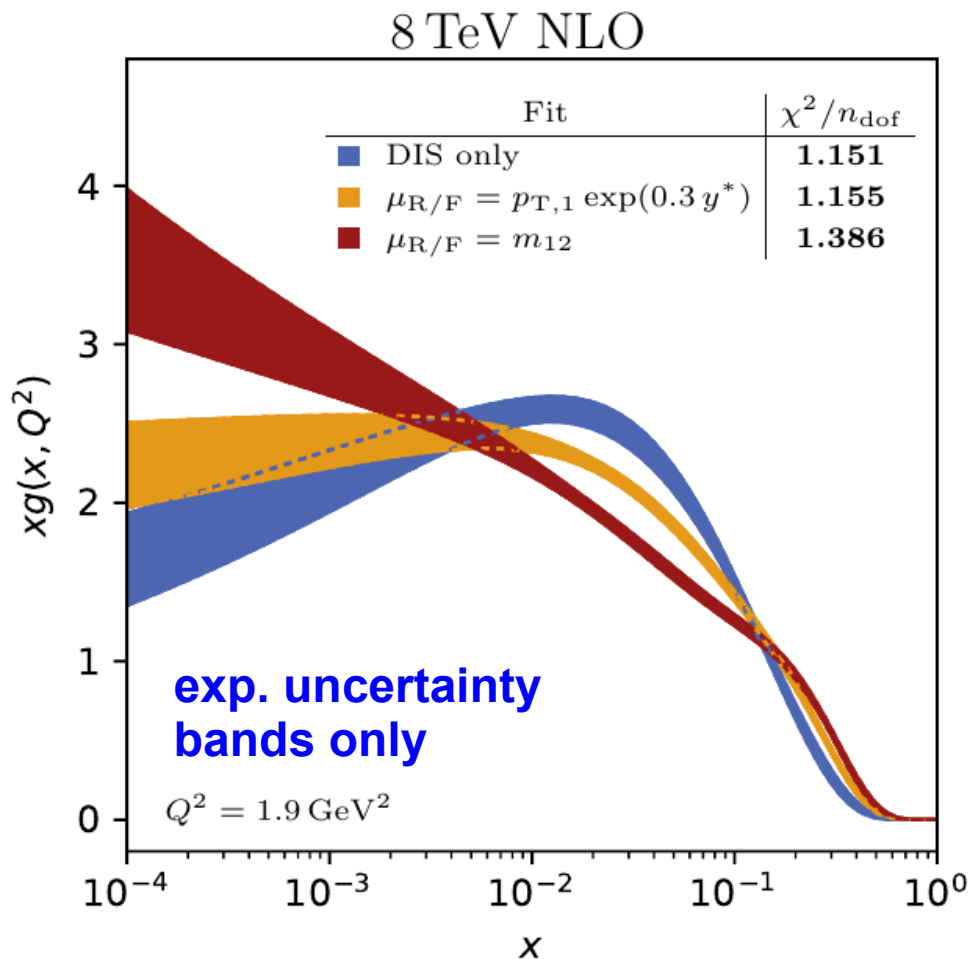




13 parameter fits – NLO vs. NNLO



Gluon from 13-parameter PDF fit with xfitter for two central scale choices



Left: NLO **Significant differences between central scale definitions**
 Right: NNLO **Much improved agreement between scales & better fit quality**

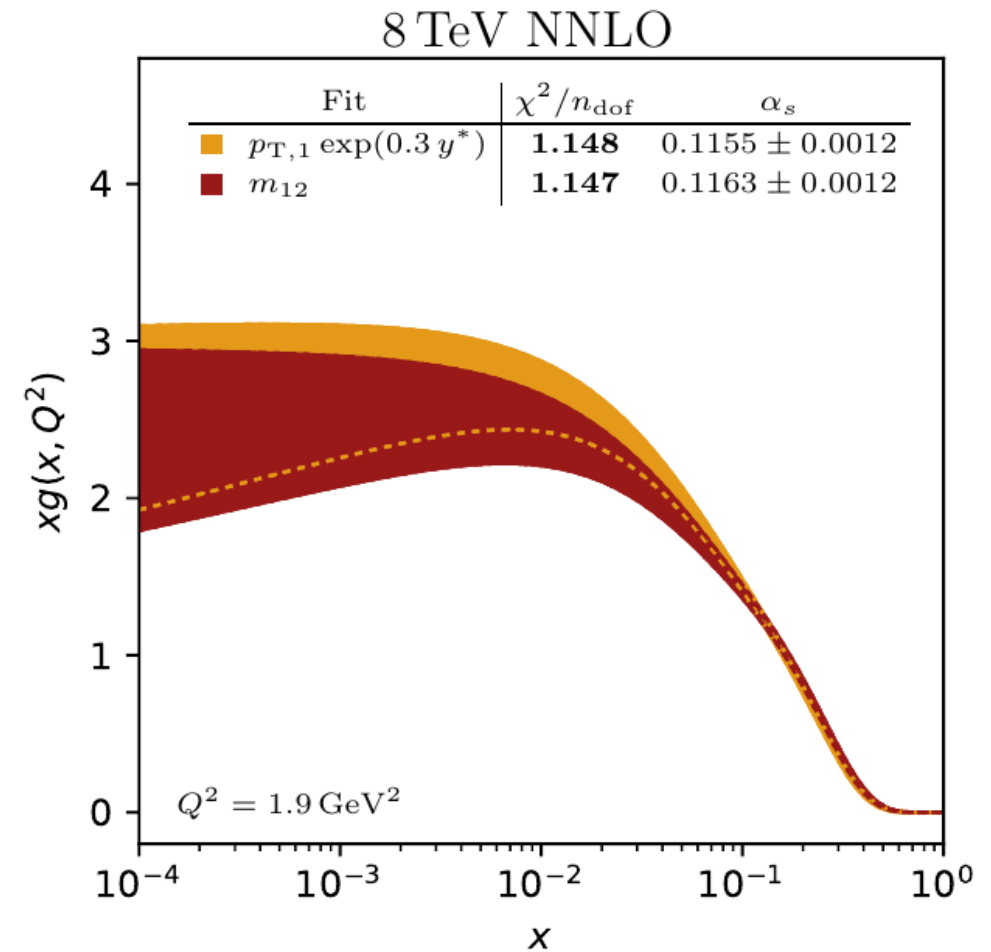
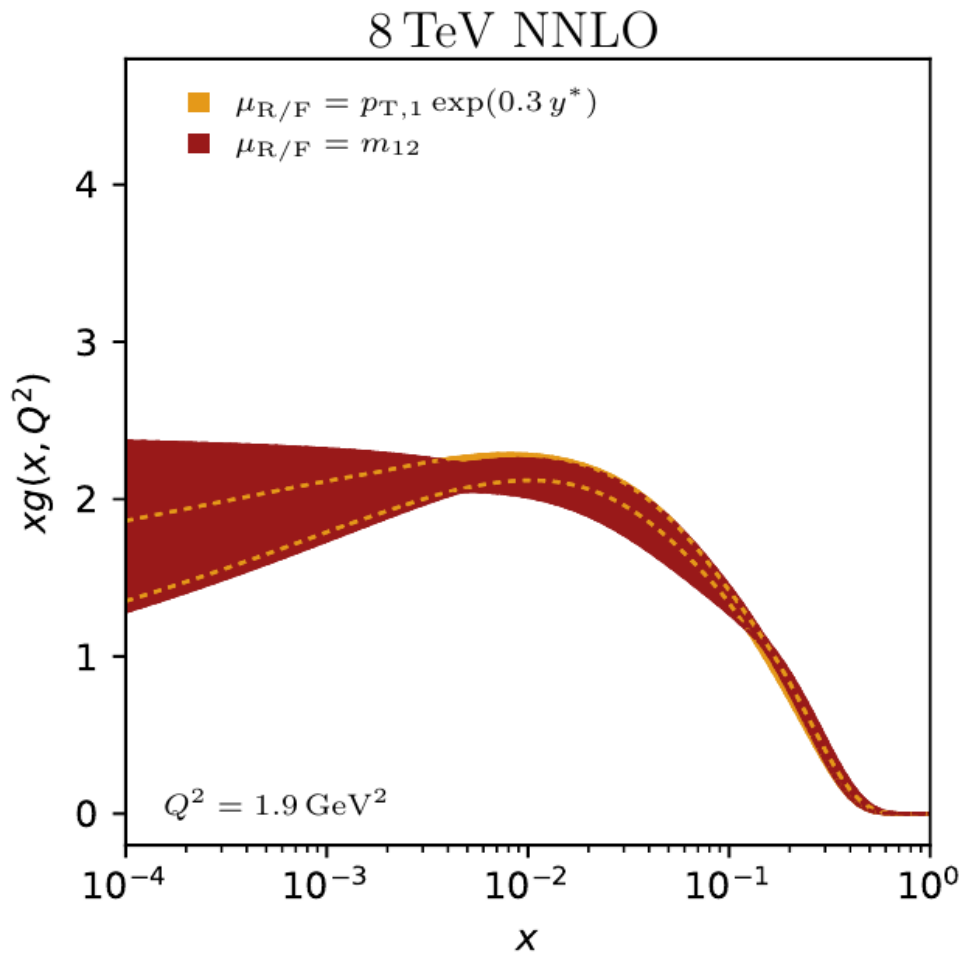


Scale varied fits & PDF+ α_s fit



Glue from 13-parameter PDF fit with scale variation band at NNLO

Glue from 14-parameter PDF fit at NNLO with free α_s



Much reduced scale dependence

Consistent results between scales



Fitted α_s values

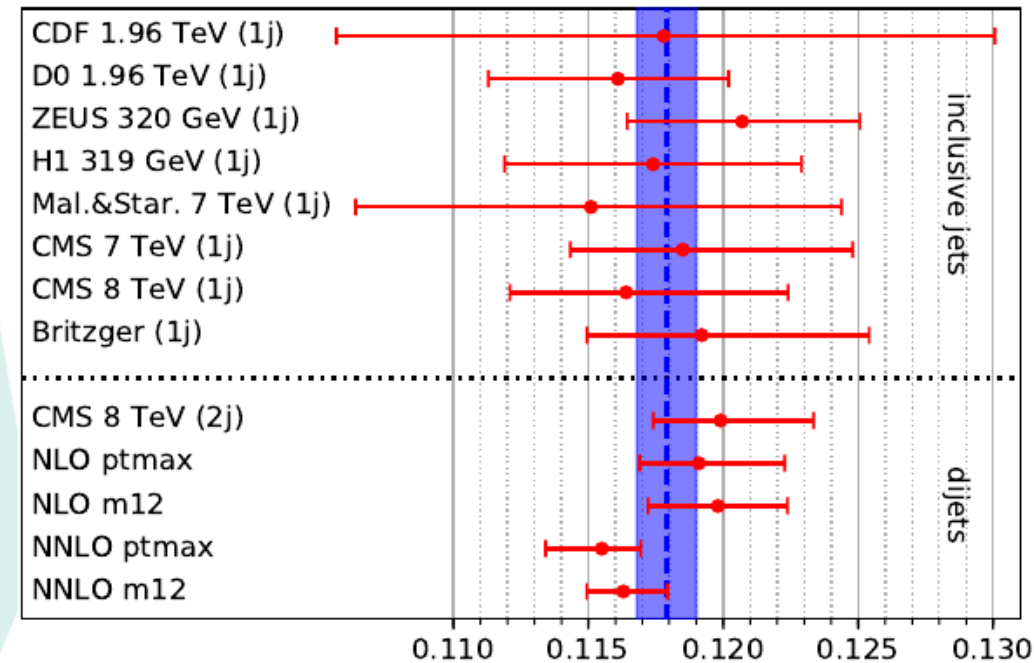


Fitted $\alpha_s(M_Z)$ values

NLO	$\mu = p_{T,1} e^{0.3y^*}$	$0.1191 \pm 0.0015(\text{exp})_{-0.0016}^{+0.0028}(\text{scale})$
	$\mu = m_{12}$	$0.1198 \pm 0.0015(\text{exp})_{-0.0021}^{+0.0021}(\text{scale})$
NNLO	$\mu = p_{T,1} e^{0.3y^*}$	$0.1155 \pm 0.0012(\text{exp})_{-0.0017}^{+0.0008}(\text{scale})$
	$\mu = m_{12}$	$0.1163 \pm 0.0013(\text{exp})_{-0.0004}^{+0.0010}(\text{scale})$

⚠ Only experimental and scale uncertainties

- as expected, smaller α_s values at NNLO
- scale uncertainties: envelope of 6 scale variations
- experimental and especially scale uncertainties smaller at NNLO



Comparison with other values obtained from jet cross sections and the world average (blue)



Summary & Outlook



- Series of interpolation grids for pp → jets produced for ATLAS & CMS:
 - Inclusive jet & dijet production
 - Several centre-of-mass energies and jet sizes R
 - Numerical accuracy of O(‰) and precision of O(%)
- Will be made public via Ploushare rather soon'ish

- Continue work on:
 - Other processes in NNLOJET
 - Simplified interface to new NNLOJET version with numerous performance improvements, also for the grid filling
 - Remark concerning [arXiv:2204.10173](#) and leading color:
 - ➔ Small effect on inclusive jets or 2D dijets
 - ➔ Updated interface also gets full-color result



● ATLAS data:

- ➔ **Inclusive jets:** JHEP 02 153 (2015), JHEP 09 020 (2017), JHEP 05 195 (2018).
- ➔ **Dijets:** JHEP 05 059 (2014), JHEP 05 195 (2018).

● CMS data:

- ➔ **Inclusive jets:** EPJC 76(5) 265 (2016), PRD 87 112002 (2013), JHEP 03 156 (2017), JHEP 02 142 (2022).
- Dijets:** PRD 87 112002 (2013), EPJC 77(11) 746 (2017).

● More details on 3D dijet fits:

- ➔ **G. Sieber, PhD thesis:** IEKP-KA/2016-05, KIT, May 2016.
- ➔ <https://publish.etp.kit.edu/record/21328>
- ➔ **J. Stark, Master thesis:** ETP-KA/2021-2, KIT, Feb. 2021.
- ➔ <https://publish.etp.kit.edu/record/22044>



Backup Slides





Interpolation concept



Implemented in APPLgrid & fastNLO

Use interpolation kernel

- Introduce set of n discrete **x-nodes**, x_i 's being equidistant in a function $f(x)$
- Take set of **Eigenfunctions** $E_i(x)$ around nodes x_i

→ Interpolation kernels

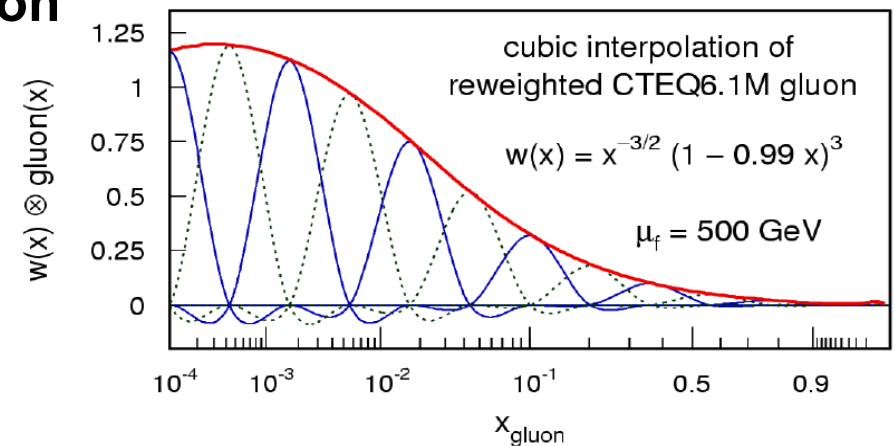
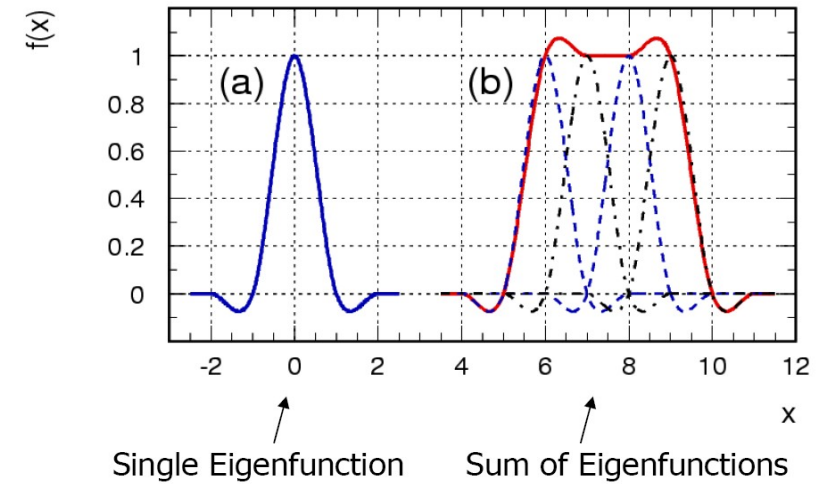
- Actually a rather old idea, see e.g.

C. Pascaud, F. Zomer (Orsay, LAL), LAL-94-42

→ Single PDF is replaced by a linear combination of interpolation kernels

$$f_a(x) \cong \sum_i f_a(x_i) \cdot E^{(i)}(x)$$

- Then the integrals are done only once
- Afterwards only summation required to change PDF



Tabulate the convolution of the perturbative coefficients with the interpolation kernel



PDF parameterisation



$$xg(x) = A_g x^{B_g} (1 - x)^{C_g} (1 + E_g x^2)$$

$$xu_v(x) = A_{u_v} x^{B_{u_v}} (1 - x)^{C_{u_v}} (1 + D_{u_v} x)$$

$$xd_v(x) = A_{d_v} x^{B_{d_v}} (1 - x)^{C_{d_v}}$$

$$x\bar{U}(x) = A_{\bar{U}} x^{B_{\bar{U}}} (1 - x)^{C_{\bar{U}}} (1 + D_{\bar{U}} x)$$

$$x\bar{D}(x) = A_{\bar{D}} x^{B_{\bar{D}}} (1 - x)^{C_{\bar{D}}}$$

Here:
 $f_s = 0.40$

$$A_{\bar{U}} = A_{\bar{D}}(1 - f_s)$$

CMS:
 $f_s = 0.31$

$$xg(x) = A_g x^{B_g} (1 - x)^{C_g} - A'_g x^{B'_g} (1 - x)^{C'_g},$$

$$xu_v(x) = A_{u_v} x^{B_{u_v}} (1 - x)^{C_{u_v}} (1 + D_{u_v} x + E_{u_v} x^2),$$

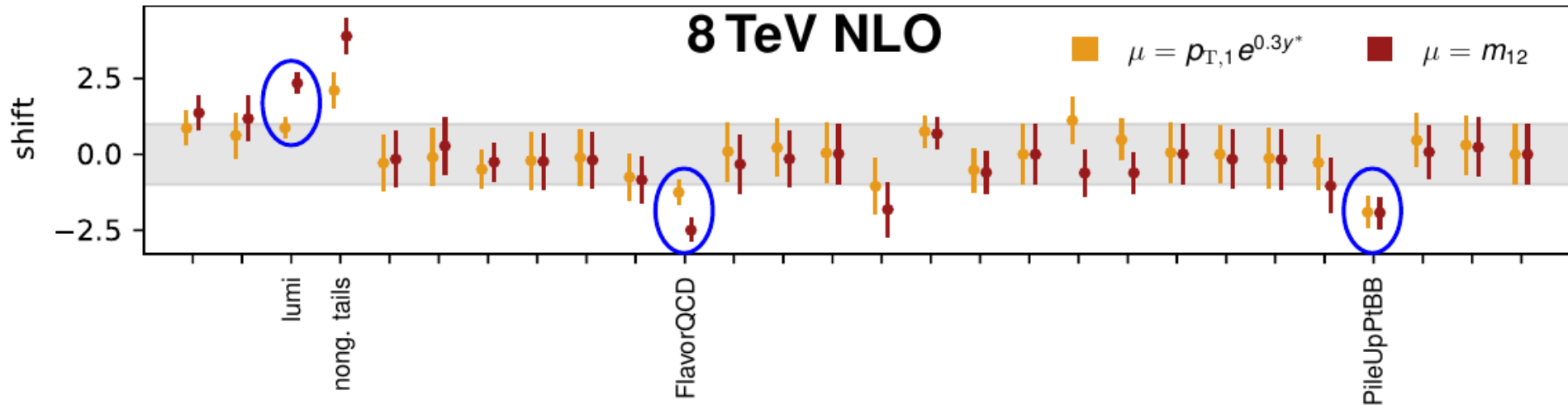
$$xd_v(x) = A_{d_v} x^{B_{d_v}} (1 - x)^{C_{d_v}} (1 + D_{d_v} x),$$

$$x\bar{U}(x) = A_{\bar{U}} x^{B_{\bar{U}}} (1 - x)^{C_{\bar{U}}} (1 + D_{\bar{U}} x),$$

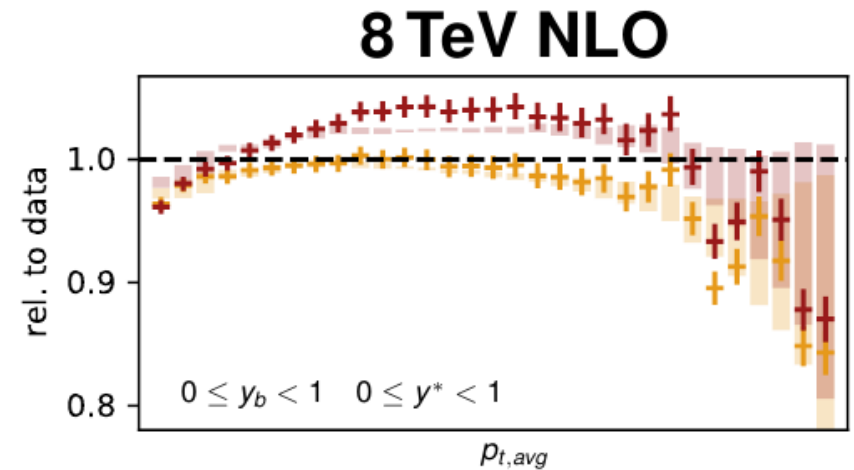
$$x\bar{D}(x) = A_{\bar{D}} x^{B_{\bar{D}}} (1 - x)^{C_{\bar{D}}},$$



Pulls of fitted nuisance parameters

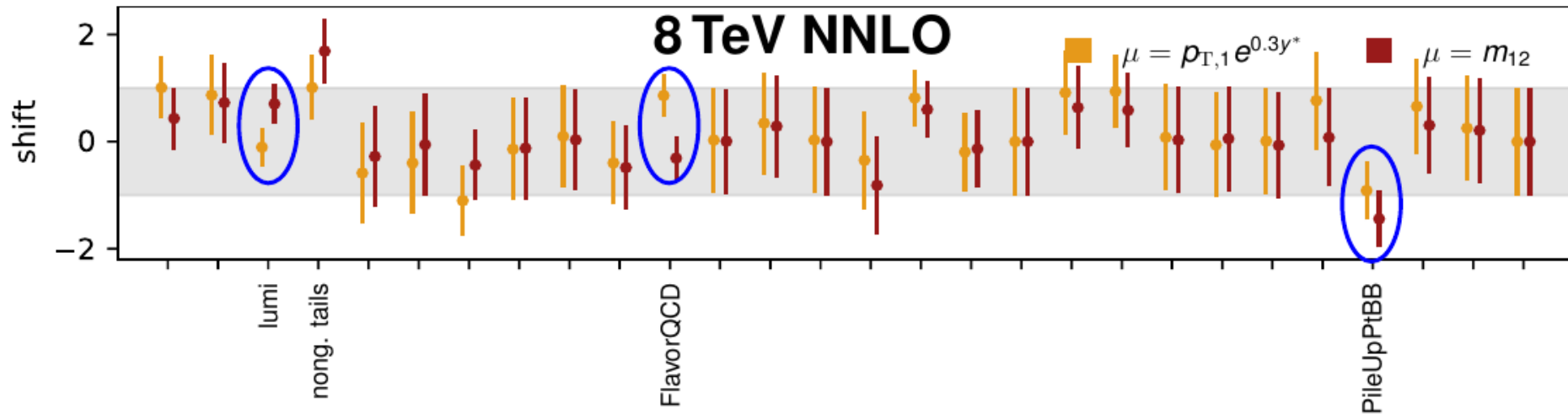


- curved distortion between prediction (crosses) and central data (black line)
- data is shifted towards prediction (transparent bars)
- outliers in shifts produce distortion, lumi globally shifts data

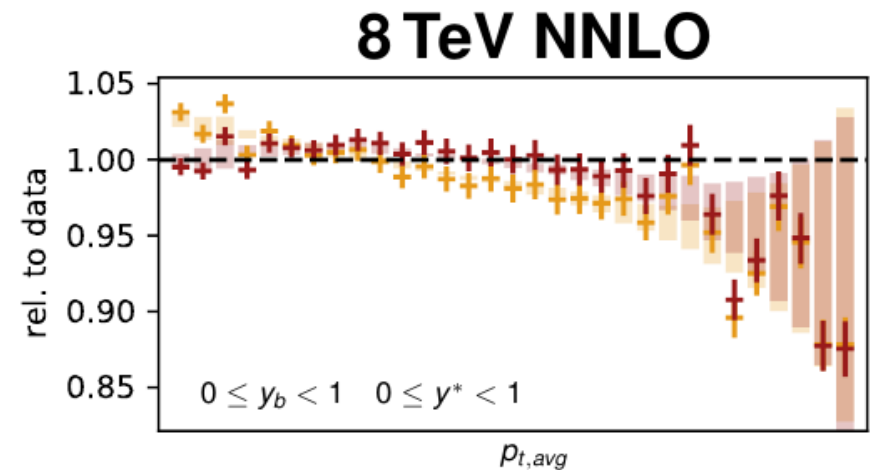




Pulls of fitted nuisance parameters

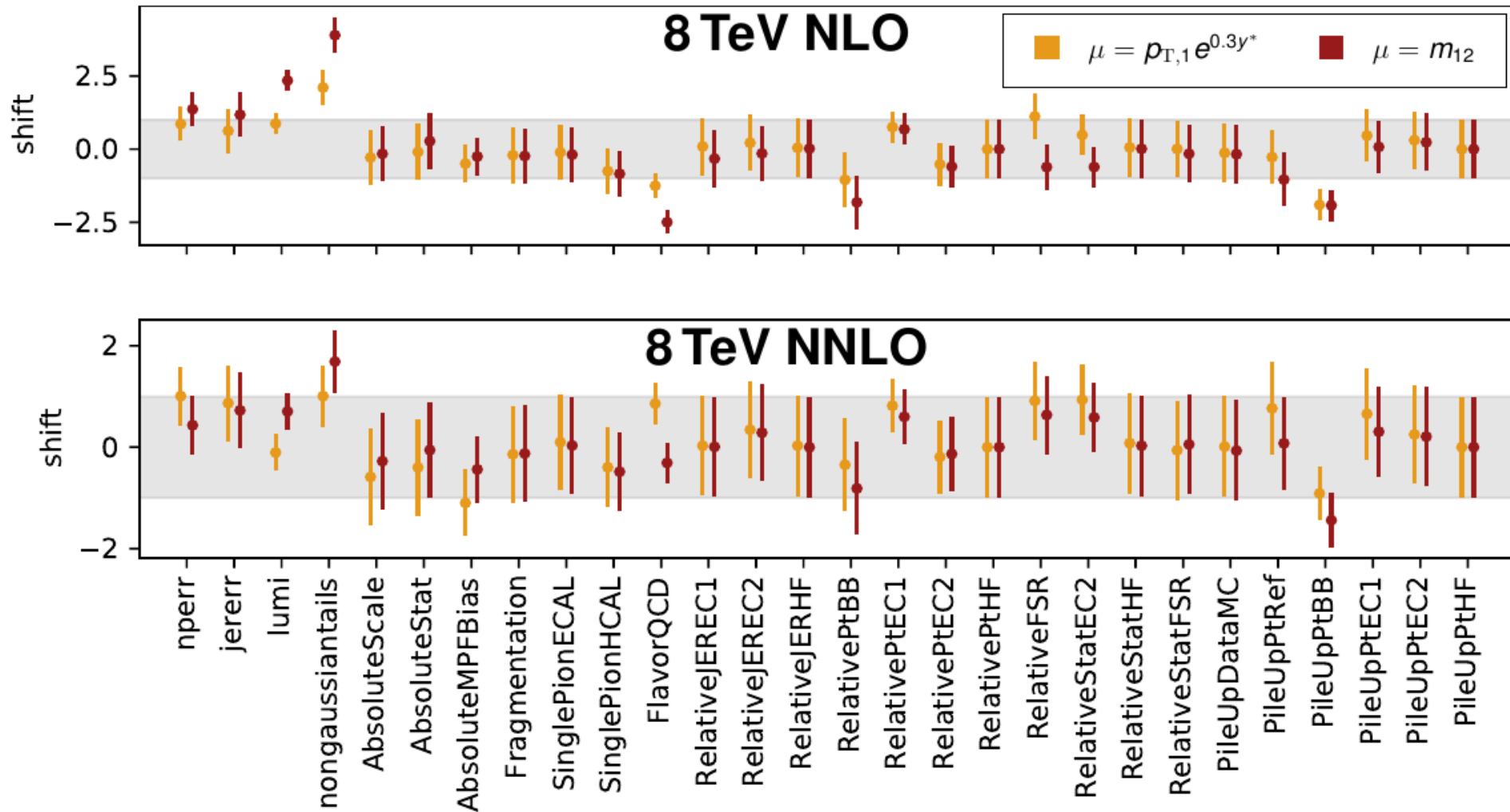


- curved distortion drastically reduced in NNLO
- previous outliers in shifts almost back in the gray 1-sigma region





Pulls of fitted nuisance parameters

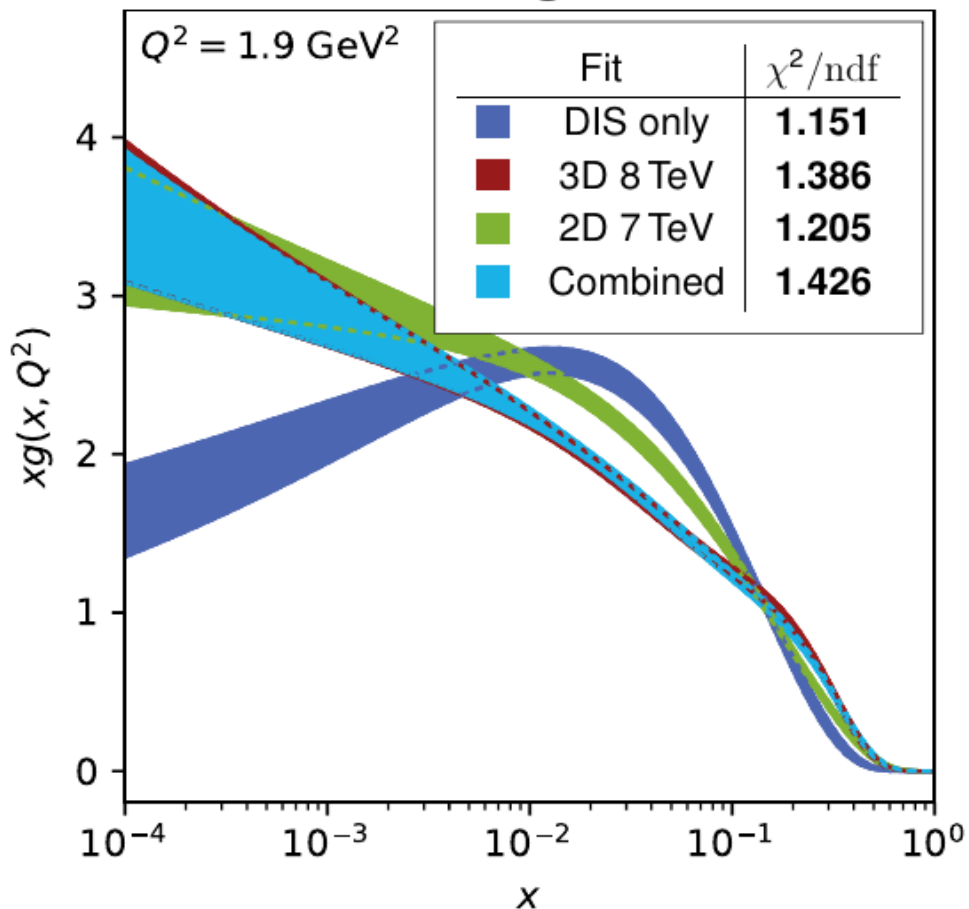




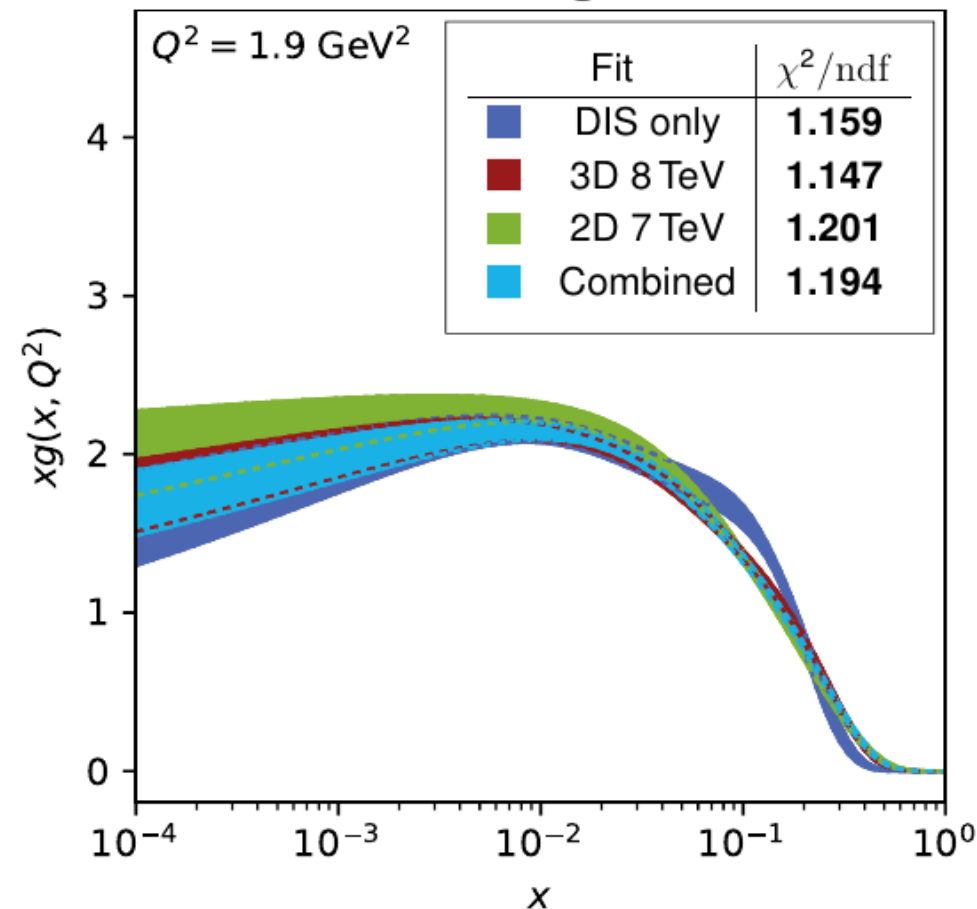
Including 7 TeV 2D dijet data



NLO



NNLO



- use $\mu = m_{12}$ (dijet mass) scale for both datasets
- 7 & 8 TeV data compatible, 3D 8 TeV data dominates the combined fit
- NNLO results much more consistent



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