
QCD at the LHC

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IPPP, Durham University



Physics at the Terascale

Helmholtz-Alliance workshop

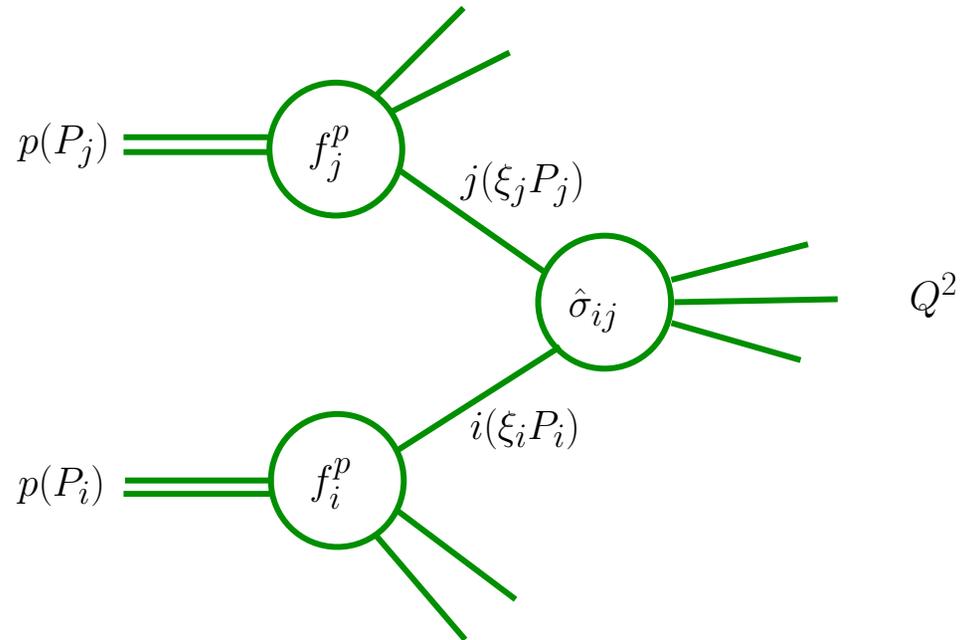
DESY Hamburg

3 December 2007

Present Status of QCD

- ✓ Thanks to LEP, HERA and the TEVATRON
QCD now firmly established theory of strong interactions
- ✓ We have gained a lot of confidence in comparing theoretical predictions with experimental data
- ✓ No major areas of discrepancies
- ✓ Now prepared to enter a new era of precision physics for QCD

Theoretical Framework - Leading Twist



$$\sigma(Q^2) = \int \sum_{i,j} [d\hat{\sigma}_{ij}(\alpha_s(\mu_R), \mu_R^2/Q^2, \mu_F^2/Q^2) \otimes f_i^P(\mu_F) \otimes f_j^P(\mu_F)]$$

- ✓ partonic cross sections $d\hat{\sigma}_{ij}$
- ✓ running coupling $\alpha_s(\mu_R)$
- ✓ parton distributions $f_i(x, \mu_F)$
- ✓ renormalization/factorization scale μ_R, μ_F
- ✓ + parton shower + hadronisation model + underlying event + ...

The challenge

- ✓ Everything at the LHC (signals, backgrounds, luminosity measurement) involves QCD
- ✓ Strong coupling is not small: $\alpha_s(M_Z) \sim 0.12$ and running is important
 - ⇒ events have high multiplicity of hard partons
 - ⇒ each hard parton fragments into a cluster of collimated particles jet
 - ⇒ higher order perturbative corrections can be large
 - ⇒ theoretical uncertainties can be large
- ✓ Processes can involve multiple energy scales: e.g. p_T^W and M_W
 - ⇒ may need resummation of large logarithms
- ✓ Parton/hadron transition introduces further issues, but for suitable (infrared safe) observables these effects can be minimised
 - ⇒ importance of infrared safe jet definition
 - ⇒ accurate modelling of underlying event, hadronisation, ...

What is covered in this talk

Will focus on status of fixed order parton-level predictions

- ✓ Systematic to higher order/higher multiplicity in perturbation theory
- ✓ Appropriate for hard well separated final states
- ✓ Lead to a systematic reduction in renormalisation/factorisation scale uncertainties
- ✓ Many recent theoretical developments and new calculations/numerical programmes available

caveat Parton-level, relies on matching to experimental observables
e.g. merging with parton showers, etc

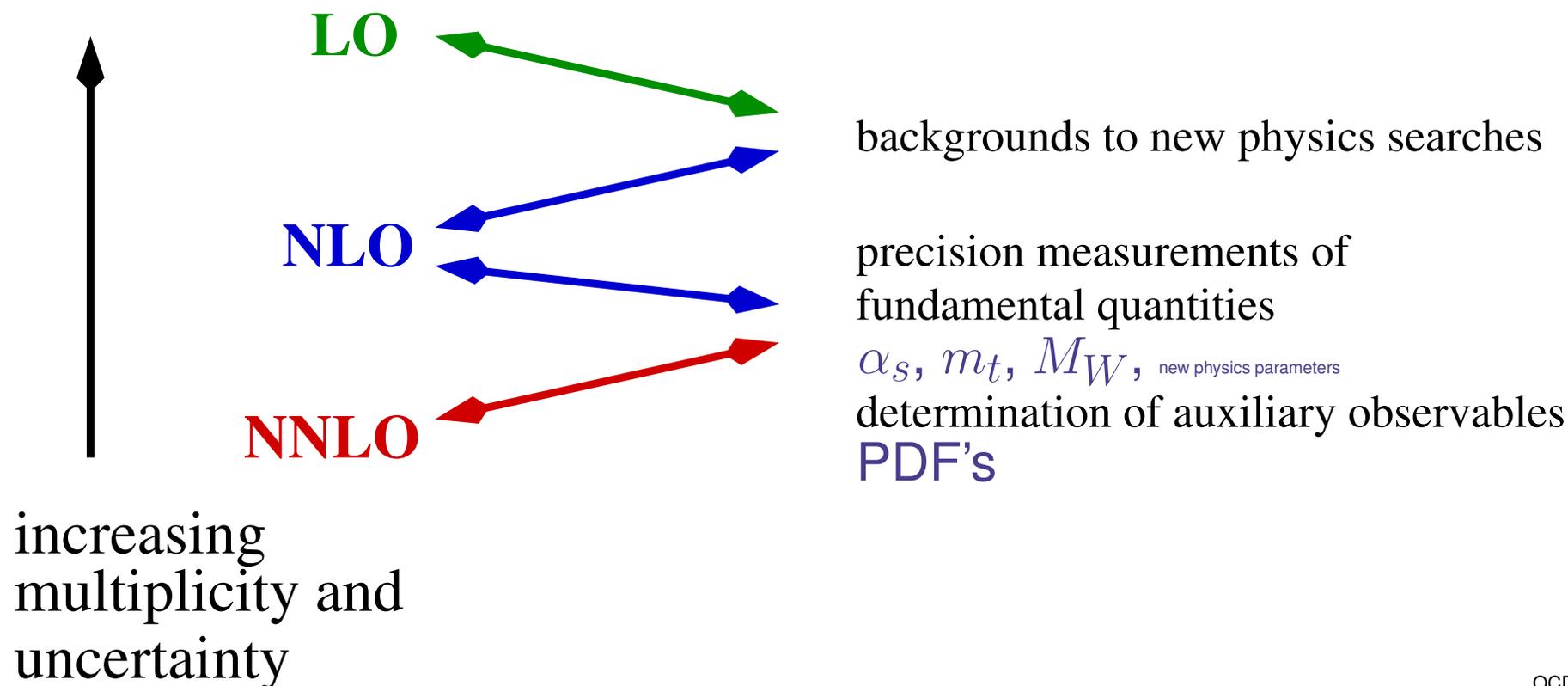
CKKW, MLM, MCNLO, POWHEG

- ✗ No time for many important topics;
- ✗ parton distributions
- ✗ soft gluon resummation
- ✗ small x issues
- ✗ central exclusive diffractive production
- ✗ studies of jet definitions; fast k_T algorithm, infrared safe cone algorithms,...

Matching onto Physics Goals

Twin Goals:

1. Identification and study of New Physics
2. Precision measurements (e.g. α_s , PDF's) leading to improved theoretical predictions



State of the Art - at a glance

Relative Order	$2 \rightarrow 1$	$2 \rightarrow 2$	$2 \rightarrow 3$	$2 \rightarrow 4$	$2 \rightarrow 5$	$2 \rightarrow 6$
1	LO					
α_s	NLO	LO				
α_s^2	NNLO	NLO	LO			
α_s^3		NNLO	NLO	LO		
α_s^4				NLO	LO	
α_s^5					NLO	LO

LO Well under control, even for multiparticle final states

NLO Well understood for $2 \rightarrow 1$ and $2 \rightarrow 2$

NLO Many new $2 \rightarrow 3$ calculations, new developments

NLO Still waiting for first $2 \rightarrow 4$ LHC cross section

NNLO Recent breakthroughs for inclusive and exclusive $2 \rightarrow 1$

NNLO Recent landmark calculation of NNLO splitting functions

Moch, Vermaseren, Vogt

NNLO Still waiting for $2 \rightarrow 2$

Leading order

Many available programs for automatic evaluation of LO cross sections

- ✓ Feynman diagrams: matrix elements automatically generated up to $2 \rightarrow 6$

MADGRAPH, COMPHEP, GRACE, ...

- ✓ Off-shell recursion relations:

Berends, Giele; Caravaglios, Moretti

matrix elements automatically generated up to $2 \rightarrow 8$ or more

HELAC, AMEGIC++, ALPHA, ...

- ✓ (Twistor inspired) On-shell recursion relations:

Cachazo, Svrcek, Witten; Britto, Cachazo, Feng, Witten

AMEGIC++; Dinsdale, Ternick, Weinzierl

- ✓ plus automatic integration over phase space

HELAC/PHEGAS, MADGRAPH/MADEVENT, SHERPA/AMEGIC++, ALPHA/ALPGEN, ...

- ✓ very good for estimating importance of various processes in different models - properly populate phase space with multiple hard objects

- ✓ able to interface with parton showers CKKW in SHERPA, MLM in ALPGEN, ...

Comparison of algorithms

- ✓ On-shell recursion relations (CSW, BCF) yield compact analytic results
- ✓ Numerical implementations show that Berends-Giele (BG) is faster

Final state	BG		BCF		CSW	
	CO	CD	CO	CD	CO	CD
2g	0.24	0.28	0.28	0.33	0.31	0.26
3g	0.45	0.48	0.42	0.51	0.57	0.55
4g	1.20	1.04	0.84	1.32	1.63	1.75
5g	3.78	2.69	2.59	7.26	5.95	5.96
6g	14.20	7.19	11.90	59.10	27.80	30.60
7g	58.50	23.70	73.60	646	146	195
8g	276	82.10	597	8690	919	1890
9g	1450	270	5900	127000	6310	29700
10g	7960	864	64000		48900	

Duhr, Hoche, Maltoni

- ✓ Remains to be seen whether hybrid can be even faster

Example at LO

Multi-jet production at the LHC using HELAC/PHEGAS

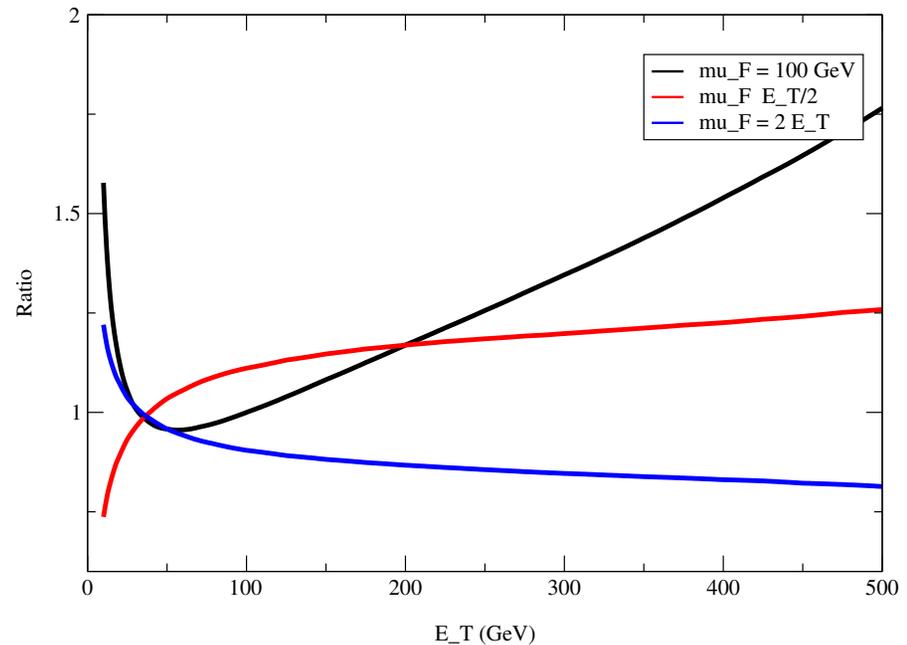
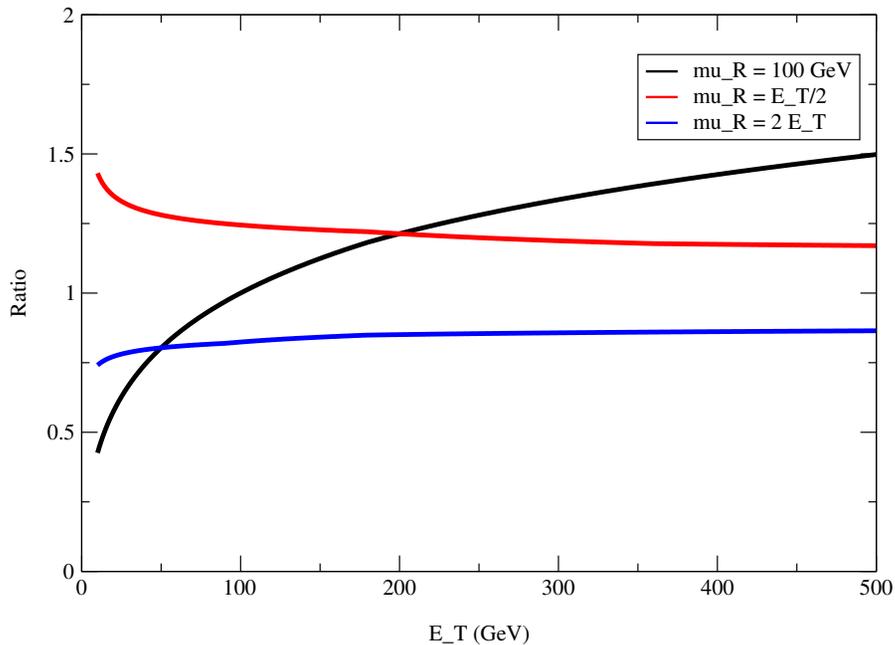
Draggiotis, Kleiss, Papadopoloulos

# of jets	2	3	4	5	6	7	8
# of dist.processes	10	14	28	36	64	78	130
total # of processes	126	206	621	861	1862	2326	4342
$\sigma(nb)$	-	91.41	6.54	0.458	0.030	0.0022	0.00021
% Gluonic	-	45.7	39.2	35.7	35.1	33.8	26.6

- ✓ For each final state, there are many distinct contributing processes
e.g. $gg \rightarrow gg, gg \rightarrow q\bar{q}, q\bar{q} \rightarrow gg, qg \rightarrow qg, q\bar{q} \rightarrow Q\bar{Q}, qQ \rightarrow qQ$ etc
- ✓ Assigning different quark flavours gives even more
- ✓ Bookkeeping, phase space generation and evaluation done automatically
- ✓ ALPGEN also very fast for multiparticle SM processes
- ✓ MADGRAPH slower, but adapted for other models, effective H, MSSM, 2HDM, ...

Limitations of LO

Very large uncertainty for multiparticle final states

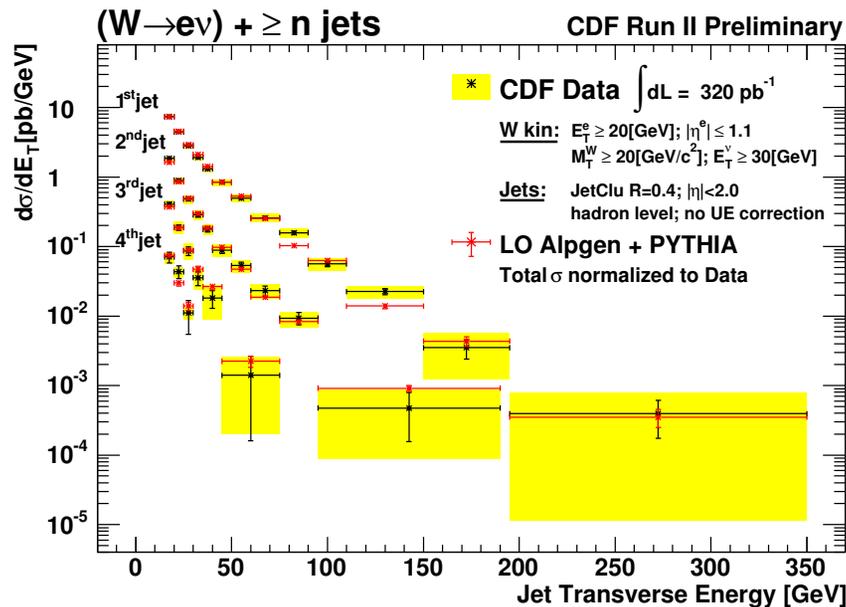


scale uncertainty on α_s^2

parton luminosity uncertainty

- ✓ New channels open up at higher orders qg + large gluon PDF
- ✓ Increased phase space
- ✓ Large π^2 coefficients in s -channel \Rightarrow large NLO corrections 30% - 100%

W + Jets at CDF Run II with 320 pb⁻¹

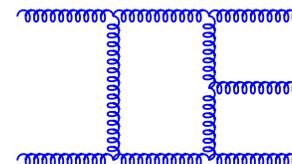


cross sections for the leading jet in $W + \geq 1$ jet events, second jet in $W + \geq 2$ jets events, etc

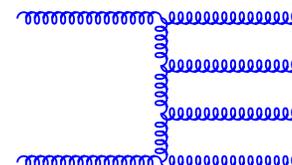
- ✓ ALPGEN+PYTHIA merged **LO+PS** prediction normalised to the inclusive cross section for each jet multiplicity
- ✓ Excellent **qualitative** agreement

Anatomy of a NLO calculation

✓ one-loop $2 \rightarrow 3$ process
looks like 3 jets in final state



✓ tree-level $2 \rightarrow 4$ process
looks like 3 or 4 jets in final state



✓ plus method for combining the infrared divergent parts - dipole subtraction

Catani, Seymour; Dittmaier, Trocsanyi, Weinzierl, Phaf

✓ automated dipole subtraction

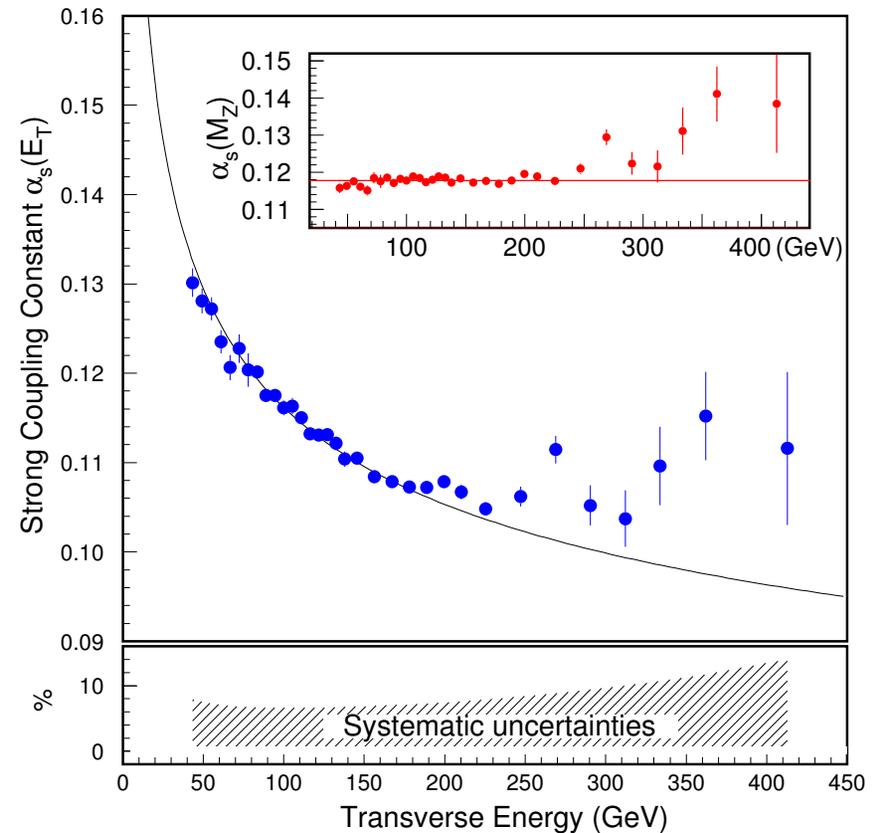
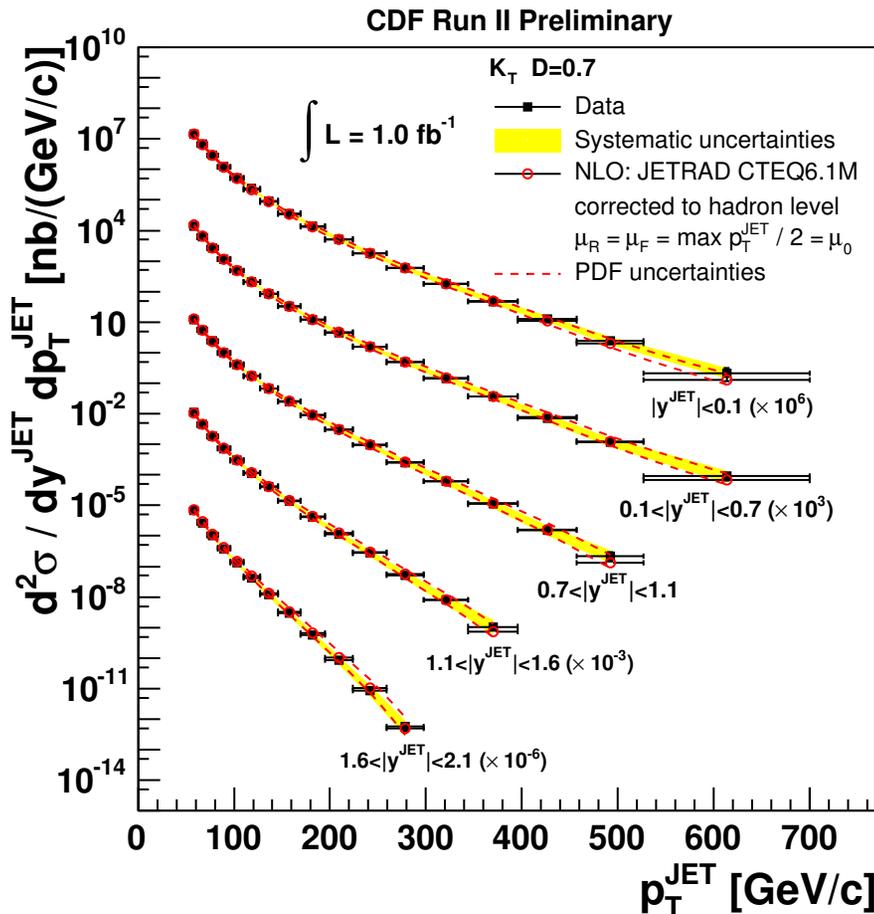
Gleisberg, Krauss; Weinzierl

Bottleneck: one-loop matrix elements

Availability of NLO calculations

- ✓ $2 \rightarrow 2$ processes
 - ✓ parton level **integrators** available for all $2 \rightarrow 2$ Standard Model and MSSM processes for some time
 - ✓ extensively used at LEP, TEVATRON and HERA
EVENT, JETRAD, MCFM, DISENT, DIPHOX, HQQB, NLOJET++, VBFNLO etc
 - ✓ can be matched with parton shower MC@NLO, POWHEG – Frixione, Webber; Nason, Oleari, Ridolfi; Krämer, Soper
 - ✓ $2 \rightarrow 3$ processes
 - ✓ many $2 \rightarrow 3$ processes now available at NLO
e.g. backgrounds $pp \rightarrow 3 \text{ jets}, V + 2 \text{ jets}, \gamma\gamma + \text{jet}, V + b\bar{b}, VV + \text{jet}, t\bar{t} + \text{jet}$
as well as signals $pp \rightarrow t\bar{t}H, b\bar{b}H, H + 2 \text{ jets}, HHH, t\bar{t} + \text{jet}$
- <http://www.cedar.ac.uk/hepcode>
- ✗ no $2 \rightarrow 4$ LHC cross sections known (yet)

Inclusive Jet Production using the Kt Algorithm



Single jet inclusive differential cross section in different rapidity slices

- ✓ Described by NLO QCD
- ✓ Excellent quantitative agreement \implies Run I α_s measurement

LHC priority wish list, Les Houches 2005

process	background	status
$pp \rightarrow VV + 1 \text{ jet}$	WBF $H \rightarrow VV$	$W^+W^- + 1 \text{ jet}, (07)$
$pp \rightarrow t\bar{t} + b\bar{b}$	$t\bar{t}H$	
$pp \rightarrow t\bar{t} + 2 \text{ jets}$	$t\bar{t}H$	$t\bar{t} + 1 \text{ jet}, (07)$
$pp \rightarrow VV + b\bar{b}$	WBF $H \rightarrow VV, t\bar{t}H$, new physics	
$pp \rightarrow VV + 2 \text{ jets}$	WBF $H \rightarrow VV$	
$pp \rightarrow V + 3 \text{ jets}$	new physics	
$pp \rightarrow VVV$	SUSY trilepton	$ZZZ, (07)$

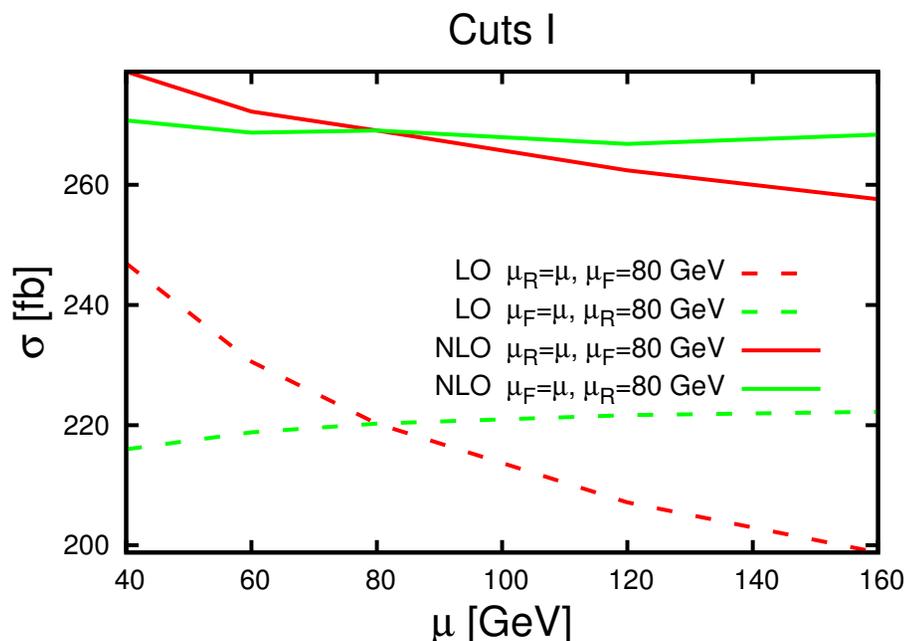
- ✓ A lot of progress in past 18 months - plus
- ✓ $pp \rightarrow H + 2 \text{ jets}$ via gluon fusion Campbell, Ellis, Zanderighi, hep-ph/0608194
- ✓ $pp \rightarrow VV + 2 \text{ jets}$ via WBF Bozzi, Jäger, Oleari, Zeppenfeld, hep-ph/0701105
- ✓ $pp \rightarrow H + 2 \text{ jets}$ via WBF, electroweak and QCD corrections Ciccolini, Denner, Dittmaier, arXiv/0710.4749
- ✓ $pp \rightarrow H + 3 \text{ jets}$ via WBF, Figy, Hankele, Zeppenfeld, arXiv/0710.5621
- ✓

Vector boson pair plus jet

QCD corrections to $pp \rightarrow W^+W^-j + X$ recently completed

Dittmaier, Kallweit, Uwer, arXiv/0710.1577; Campbell, Ellis, Zanderighi, arXiv/0710.1832

✓ Background to Higgs in both WBF, GF channels - $H \rightarrow W^+W^-$ with one jet missed, or Higgs recoiling against jet



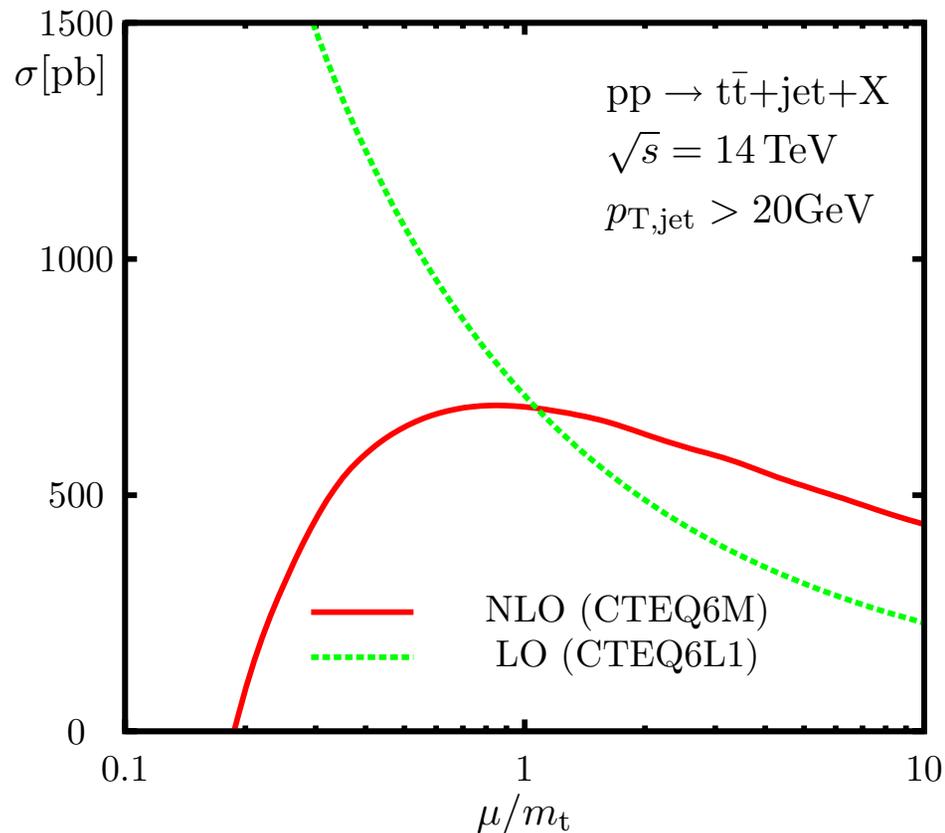
- ✓ For inclusive cuts, NLO increases cross section by about **25%**
- ✓ Factorisation scale uncertainty small, renormalisation scale uncertainty reduced by **$\sim 50\%$**
- ✓ Shapes of NLO inclusive distributions very similar to LO
- ✓ For WBF cuts, with one or both jets forward, WW_j is one of dominant backgrounds
NLO increased by **$\sim 70\%$** cf LO

Top pair plus jet

QCD corrections to $pp \rightarrow t\bar{t}j + X$

Dittmaier, Uwer, Weinzierl hep-ph/0703120

- ✓ Background to Higgs in WBF, $t\bar{t}H$ channels
- ✓ measurement of t properties



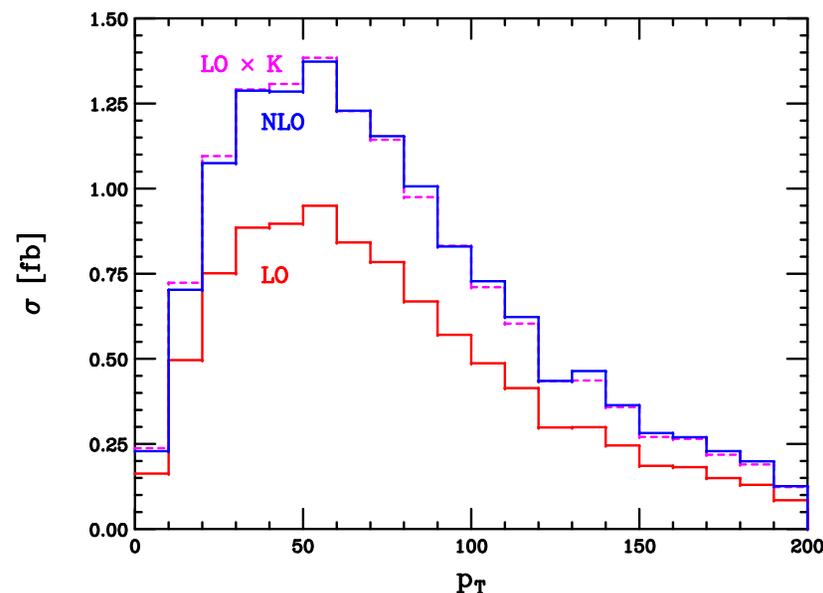
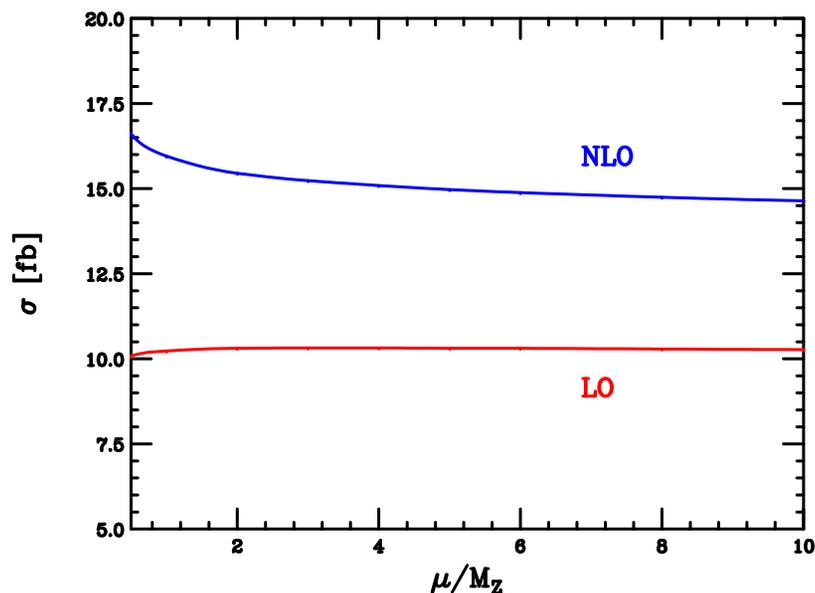
- ✓ Residual scale dependence reduced
- ✓ NLO corrections essentially eliminate forward-backward charge asymmetry at Tevatron

Triple Vector Boson Production

QCD corrections to $pp \rightarrow ZZZ + X$

Lazopoulos, Melnikov, Petriello, hep-ph/0703273

- ✓ Background to various SUSY tri-lepton signatures, gauge boson coupling measurements,



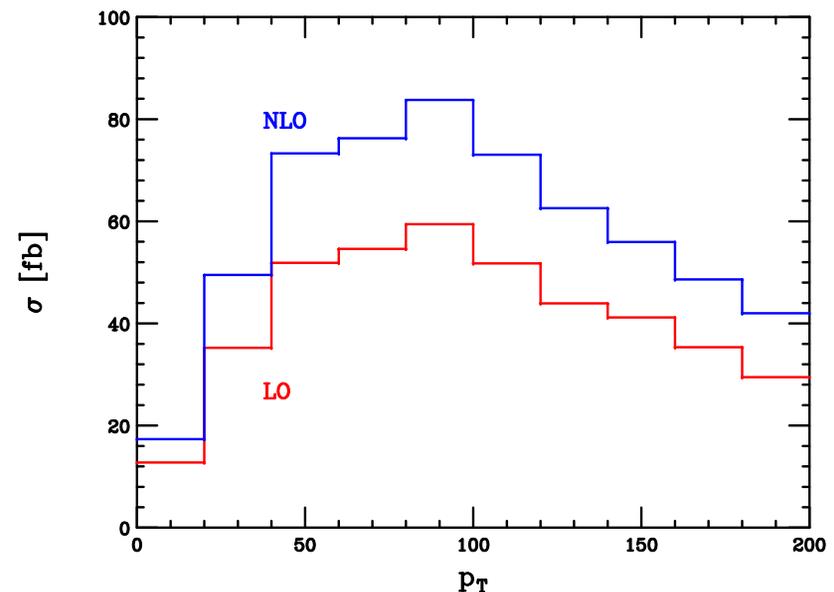
- ✓ Large, 50% corrections not seen by LO scale variation!
15% shift from pdfs, 35% shift from π^2 terms

Top pair plus Z Production

QCD corrections to $gg \rightarrow t\bar{t}Z + X$

Lazopoulos, Melnikov, Petriello, arXiv/0709.4044

- ✓ Background to various SUSY tri-lepton signatures, gauge boson coupling measurements,
- ✓ Fully numerical calculation - using sector decomposition and contour deformation
- ✓ First step towards $pp \rightarrow t\bar{t}Z$
- ✓ For reasonable choices of μ , corrections as large as **75%**



The one-loop problem

Any one-loop integral can be written as

$$\mathcal{M} = \sum a(D)\text{boxes} + \sum b(D)\text{triangles} + \sum c(D)\text{bubbles} + \sum d(D)\text{tadpoles}$$

- ✓ most of the scalar loop integrals **boxes** etc are known analytically
- ✓ only problem is to compute the D -dimensional coefficients $a(D)$ etc.

Sometimes its better to compute

$$\mathcal{M} = \sum a(4)\text{boxes} + \sum b(4)\text{triangles} + \sum c(4)\text{bubbles} + \sum d(4)\text{tadpoles} + \mathbf{R}$$

where the coefficients are now 4-dimensional and **R** is a rational (non-logarithmic) term

The only problem is **complexity** - the number of terms generated is too large to deal with, even with computer algebra systems, and there can be very large cancellations.

The one-loop problem - continued

Lots of ideas and strategies

- ✓ Improved tensor reduction: Denner, Dittmaier; Binoth, Guillet, Heinrich, Pilon, Schubert, ...
- ✓ Numerical evaluation of recursion relations Giele, Ellis, Zanderighi
- ✓ 4-d Unitarity and cut constructibility Bern, Dixon, Dunbar, Kosower; Britto, Cachazo, Feng; ...
- ✓ D-dimensional unitarity Anastasiou, Britto, Cachazo, Feng, Kunszt, Mastrolia
- ✓ Numerical loop integration: accuracy only has to match real emission contribution Nagy, Soper, [hep-ph/0610028](#)
Sector decomposition plus contour deformation automated by Anastasiou, Beerli, Daleo, [hep-ph/0703282](#)
- ✓ Reduction of the integrand
Ossola, Papadopoulos, Pittau, [hep-ph/0609007](#); Ellis, Giele, Kunszt, [arXiv/0708.2398](#)
- ✓

Testing ground: Six-photon amplitude [hep-ph/0610028](#), [hep-ph/0703311](#)
[hep-ph/0704.1271](#)

One-loop six gluon amplitude

✓ Analytic computation

Bedford, Berger, Bern, Bidder, Bjerrum-Bohr, Brandhuber, Britto, Buchbinder, Cachazo, Dixon, Dunbar, Feng, Forde, Kosower, Mastrolia, Perkins, Spence, Travaglini, Xiao, Yang, Zhu

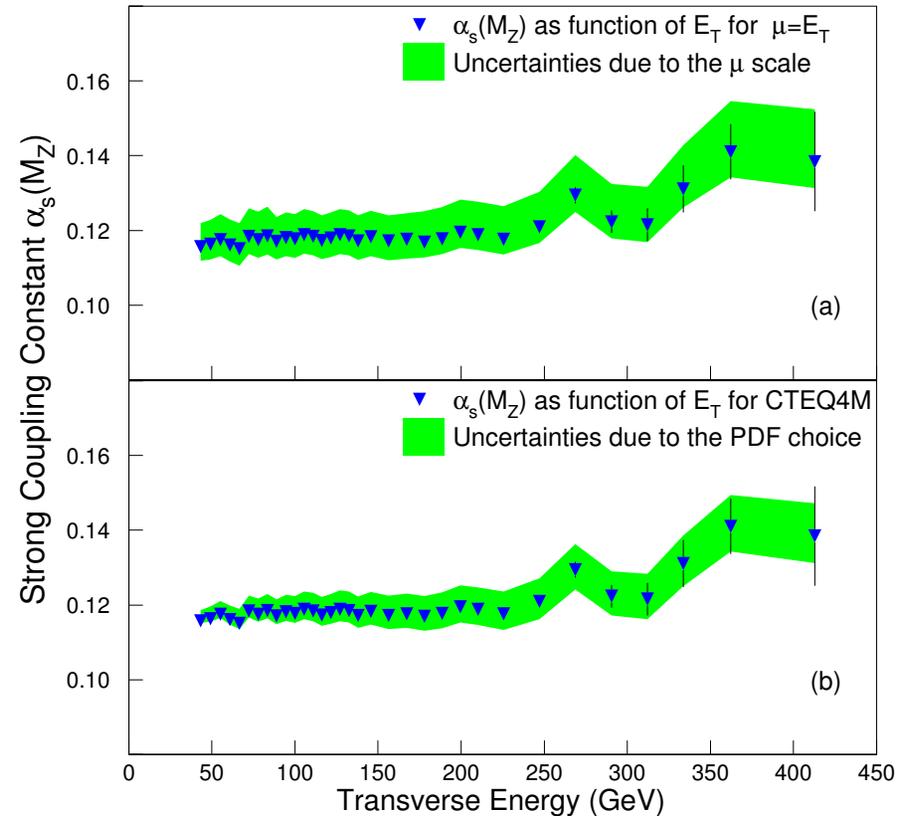
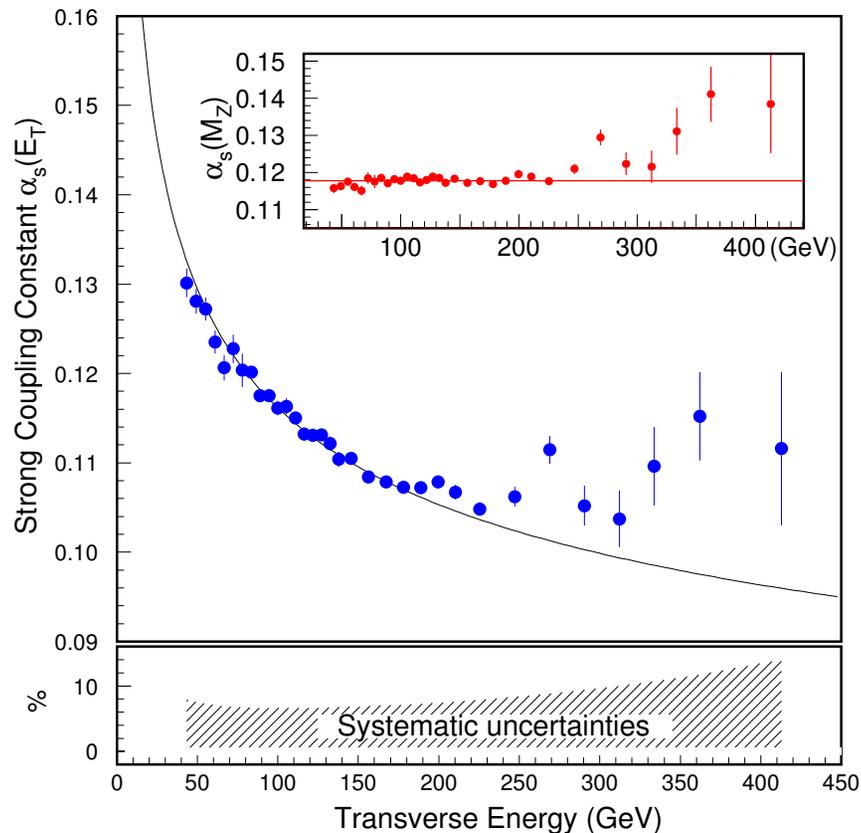
Amplitude	$\mathcal{N} = 4$	$\mathcal{N} = 1$	$\mathcal{N} = 0$ (cut)	$\mathcal{N} = 0$ (rat)
− − + + + +	BDDK (94)	BDDK (94)	BDDK (94)	BDK (94)
− + − + + +	BDDK (94)	BDDK (94)	BBST (04)	BBDFK (06), XYZ (06)
− + + − + +	BDDK (94)	BDDK (94)	BBST (04)	BBDFK (06), XYZ (06)
− − − + + +	BDDK (94)	BDDK (94)	BBDI (05), BFM (06)	BBDFK (06), XYZ (06)
− − + − + +	BDDK (94)	BBDP (05), BBCF (05)	BFM (06)	XYZ (06)
− + − + − +	BDDK (94)	BBDP (05), BBCF (05)	BFM (06)	XYZ (06)

✓ Numerical evaluation via recursion Ellis, Giele, Zanderighi (06)

✓ Numerical evaluation based on unitarity Ellis, Giele, Kunszt (07)

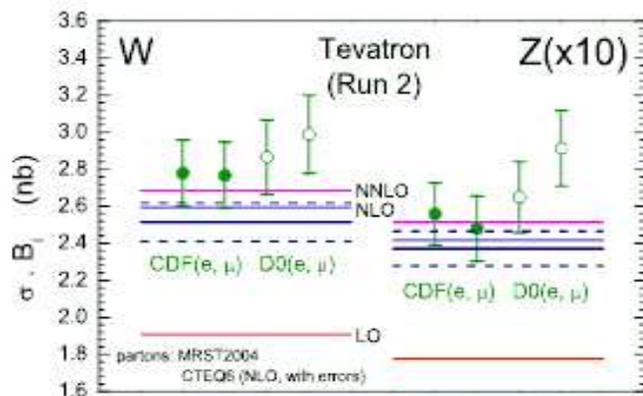
Why go beyond NLO?

In many cases, the uncertainty from the pdf's and from the choice of renormalisation scale still give NLO uncertainties that are as big or bigger than the experimental errors.



$$\alpha_s(M_Z) = 0.1178 \quad +6\% \quad -4\% (scale) \quad +5\% \quad -5\% (pdf)$$

Drell Yan production



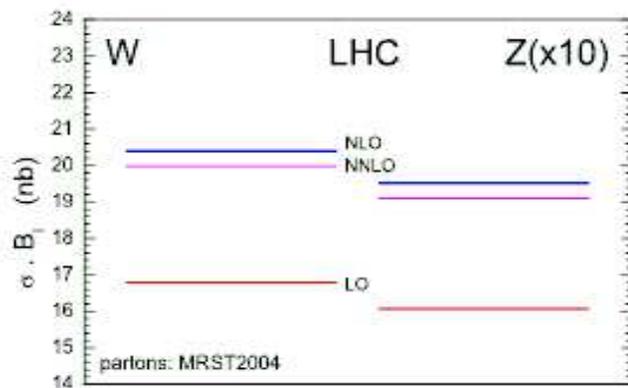
Most accurate prediction yet

- ✓ NNLO splitting functions
- ✓ NNLO PDF fits
- ✓ NNLO Drell-Yan cross section

⇒ High precision

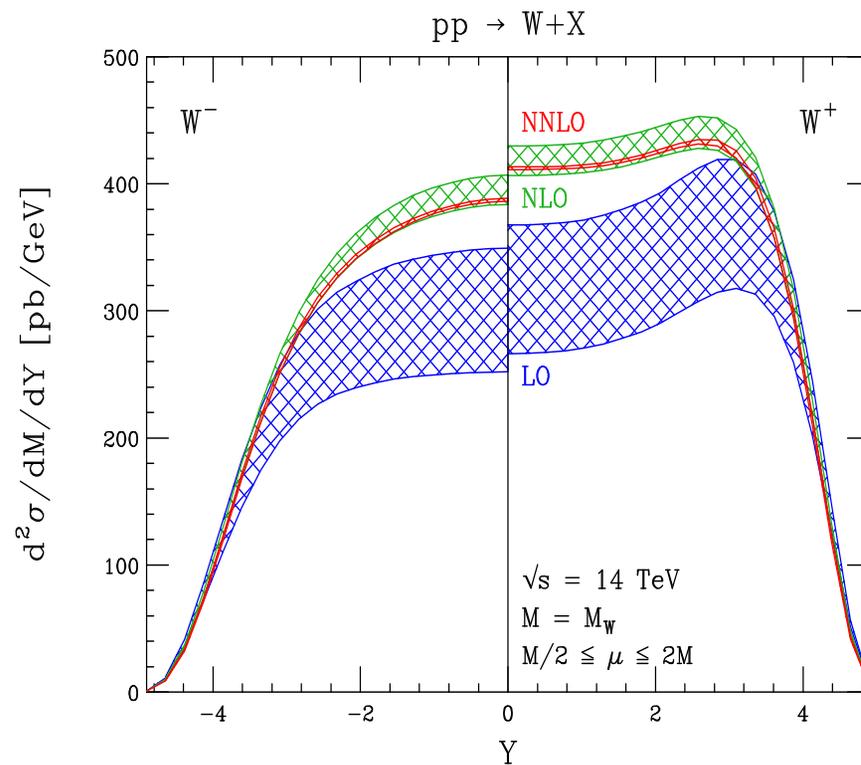
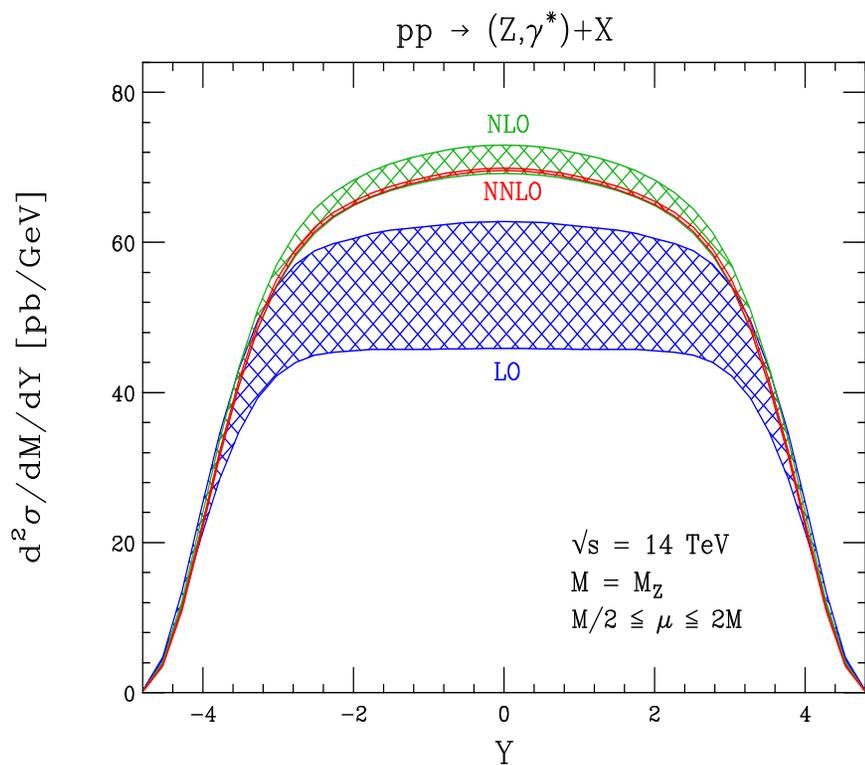
Total error of 4% – –5.5%

Martin et al



Aim to be able to use as **Standard Candle** for luminosity measurements.

Gauge boson production at the LHC



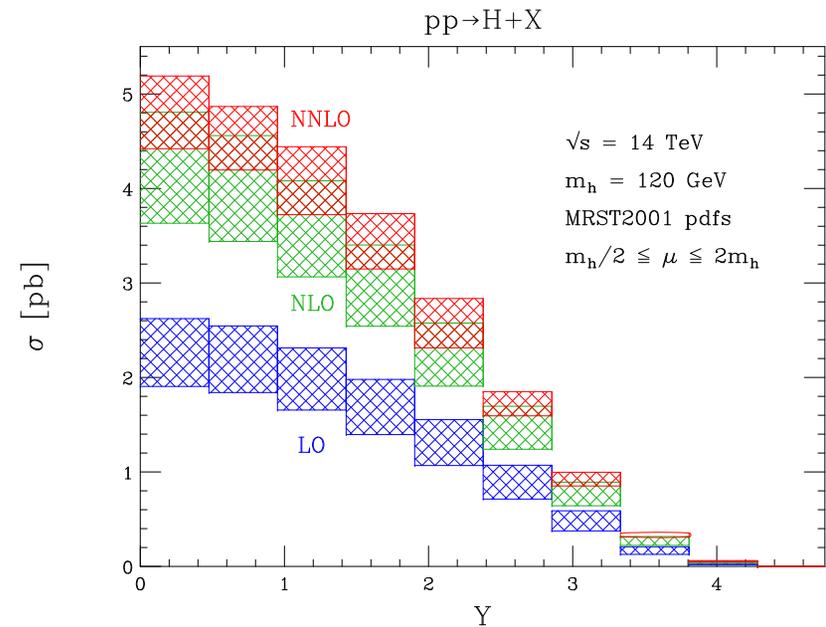
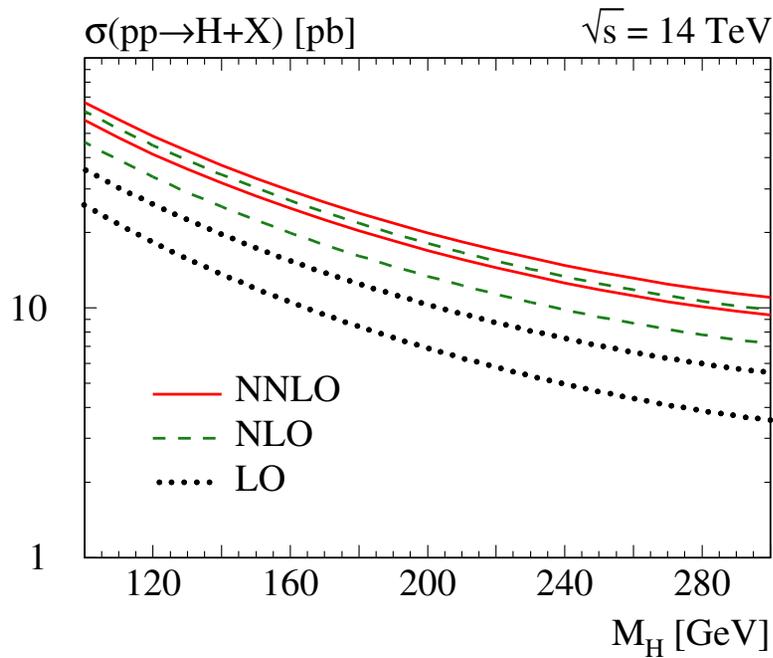
Gold-plated process

Anastasiou, Dixon, Melnikov, Petriello

At LHC NNLO perturbative accuracy better than 1%

⇒ use to determine parton-parton luminosities at the LHC

Higgs boson production at the LHC



Total cross section

Harlander, Kilgore; Anastasiou, Melnikov, Petriello; . . .

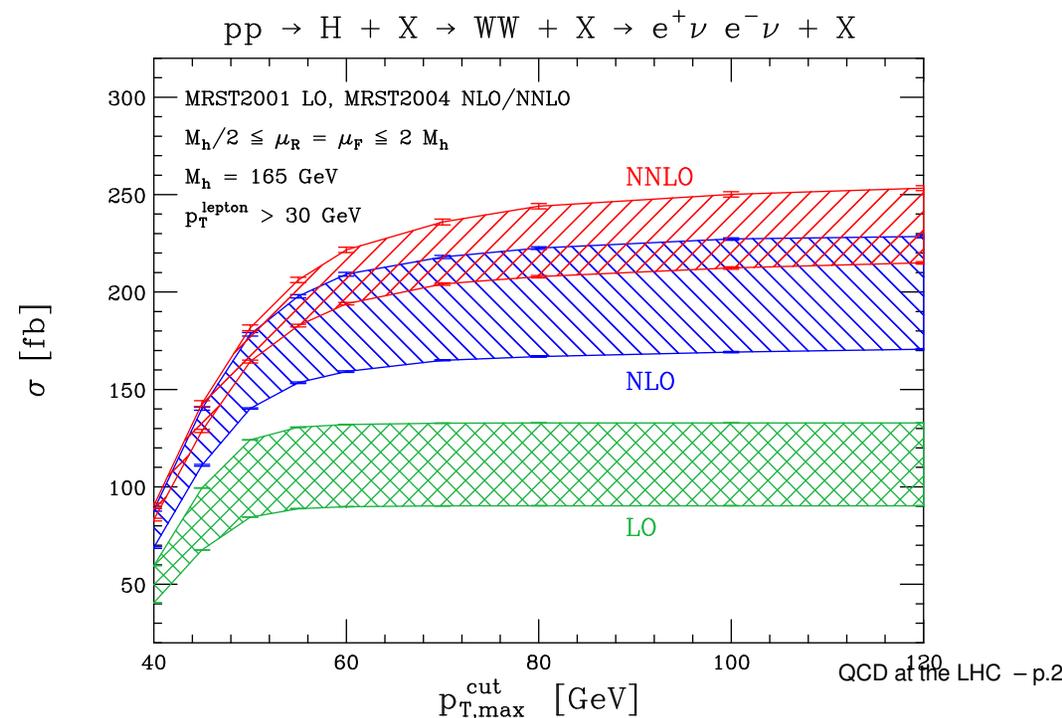
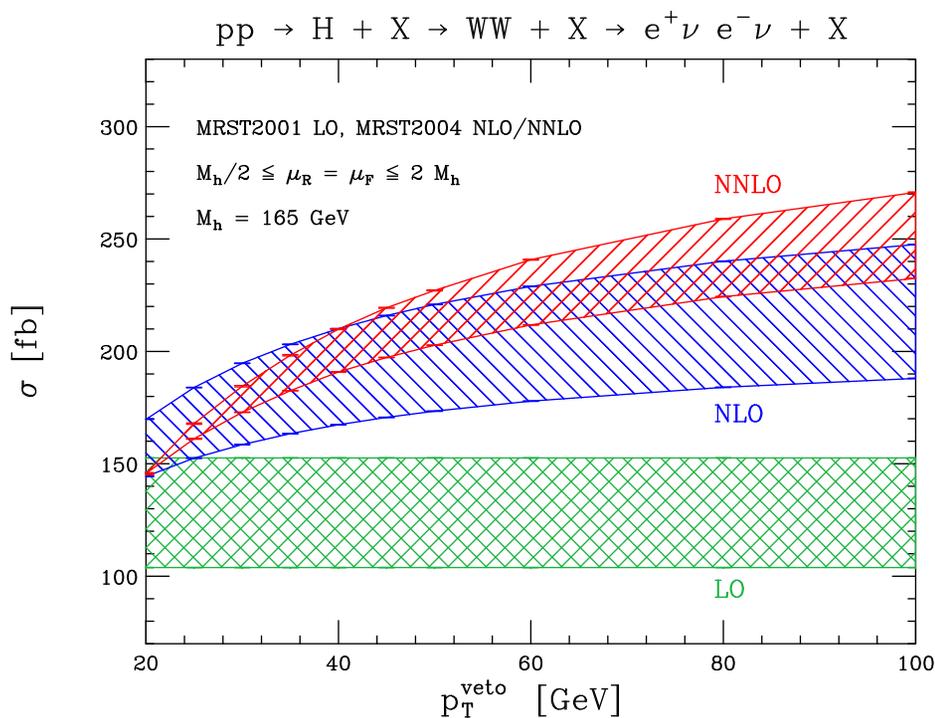
Fully differential

Anastasiou, Melnikov, Petriello

NNLO needed for reliable predictions

Higgs boson production at the LHC

- ✓ First study of fully inclusive $pp \rightarrow H \rightarrow WW \rightarrow \ell\nu\ell\nu$ with $m_H \sim 165$ GeV
Anastasiou, Dissertori, Stöckli, arXiv/0707.2373
 - ✓ Apply experimental cuts to reduce backgrounds from $t\bar{t}$, non-resonant W^+W^- production
 - ✓ Cuts affect LO/NLO/NNLO cross sections differently
- ⇒ shouldn't use inclusive **K**-factor



Other NNLO calculations on horizon

✓ $pp \rightarrow jet + X$

- ✓ needed to constraint PDF's and fix strong coupling
- ✓ matrix elements known for some time

Anastasiou et al, Bern et al

- ✓ antenna subtraction terms worked out

Daleo, Gehrmann, Maitre

✓ $pp \rightarrow t\bar{t}$

- ✓ necessary for precise m_t determination
- ✓ matrix elements recently worked out

Czakon, Mitov, Moch, arxiv/0707.4139

✓ $pp \rightarrow VV$

- ✓ signal: to study the gauge structure of the Standard Model
- ✓ background: for Higgs boson production and decay in the intermediate mass range
- ✓ large NLO corrections

Chachamis, Czakon, Eiras

Summary

QCD A lot still to do, but progress being made towards main targets

LO largely solved (plus BSM models)

✓ high multiplicity merged with parton shower, **ALPGEN, SHERPA, ...**

✗ large theoretical uncertainty

NLO **QCD** corrections generally large **30% – 100%** - much larger than scale variation suggests

✓ Cuts tend to spoil use of inclusive **K**-factor

✓ Serious effort on **Les Houches NLO** wish list, several new NLO calculations this year,
WWj, ttj, ZZZ, ttZ

✗ ✓ $2 \rightarrow 4$ barrier yet to be breached for LHC, but several new techniques available

NNLO Inclusive and exclusive results for *H, W, Z* production

✓ DGLAP splitting kernels \implies NNLO PDF fits

✗ ✓ $2 \rightarrow 2$ calculations coming onto horizon

✓ Crucial role of **Loops and Legs** workshops in stimulating community