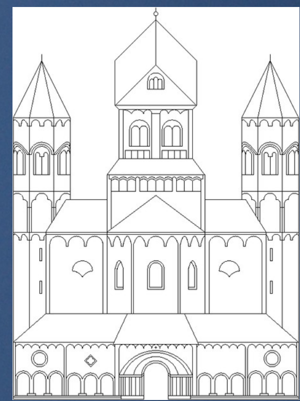
# QCD and Jets at the LHC

## V01 – QCD: From quarks & gluons to jets

**Herbstschule Maria Laach**




**Klaus Rabbertz**

Karlsruhe Institute of Technology

Institut für Experimentelle Teilchenphysik

# Outline

- Historic recap
- Quarks and gluons
- Event shapes
- Jets

# The subnuclear zoo 1957

**First of the PDG Reviews 1957**

**Authors:**
**M. Gell-Mann**
**A. Rosenfeld**

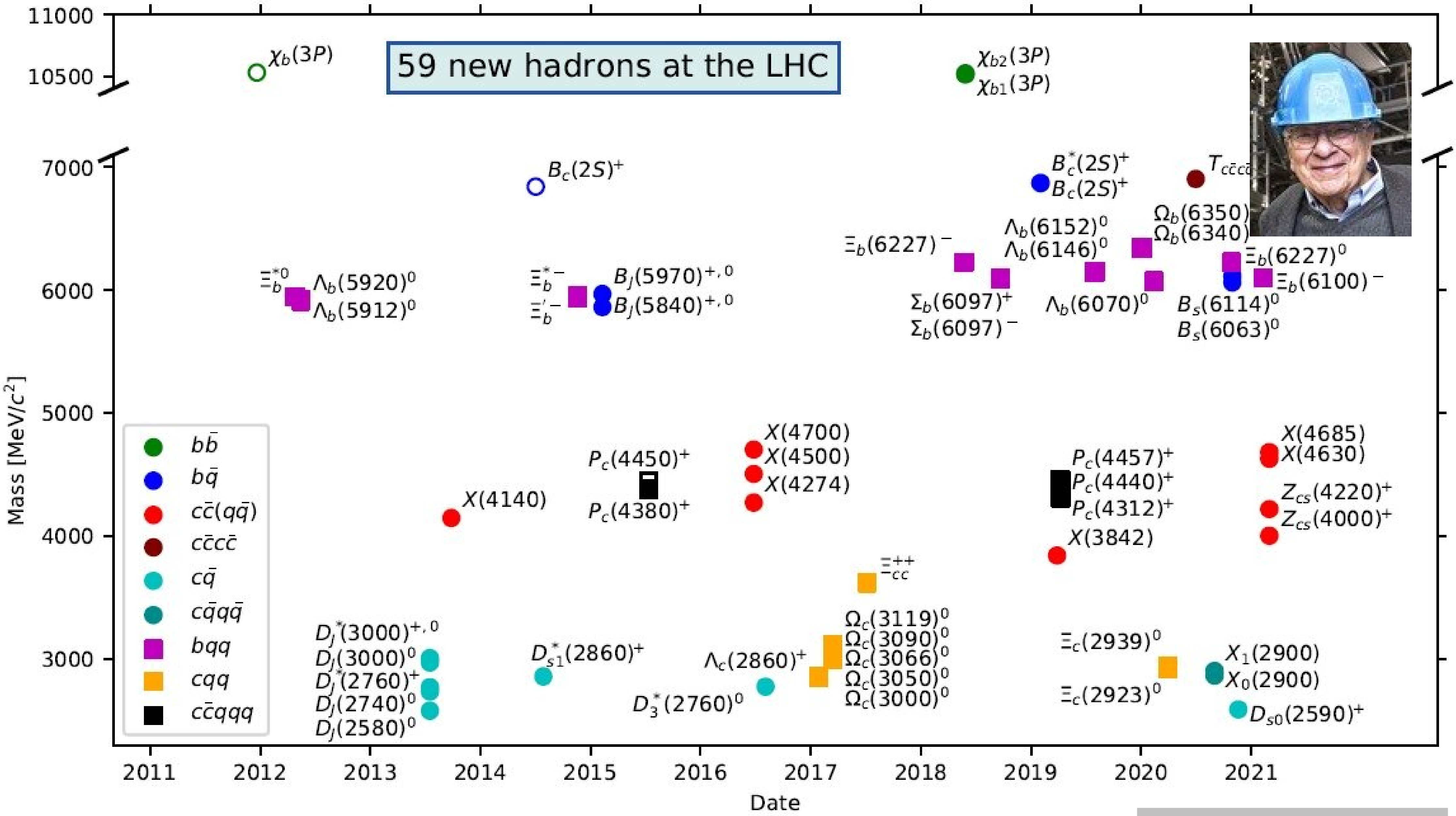
MASSES AND LIFETIMES OF ELEMENTARY PARTICLES

| | Particle | Spin | Mass (Errors represent standard deviation) (Mev) | Mass difference (Mev) | Mean life (sec) | Decay rate (number per second) |
|---|---|---|---|---|---|---|
| tons | $\gamma$ | 1 | 0 | | stable | 0.0 |
| tons and | $\nu$, $\bar{\nu}$ | ½ | 0 | | stable | 0.0 |
| ıtileptons | $e^-$, $e^+$ | ½ | 0.510976* | | stable | 0.0 |
| | $\mu^-$, $\mu^+$ | ½ | 105.70 ± 0.06* | | $(2.22 \pm 0.02) \times 10^{-6}$* | $0.45 \times 10^6$ |
| ons | $\pi^\pm$ | 0 | 139.63 ± 0.06* | 4.6* | $(2.56 \pm 0.05) \times 10^{-8}$* | $0.39 \times 10^8$ |
| | $\pi^0$ | 0 | 135.04 ± 0.16* | | $(0.0 < \tau < 0.4) \times 10^{-15}$ (O) | $> 2.5 \times 10^{15}$ |
| | $K^\pm$ | 0 | 494.0 ± 0.20 (a) | | $(1.224 \pm .013) \times 10^{-8}$ (b) | $0.815 \times 10^8$ |
| | | | | 1 ± 5 | $K_1$: $(0.95 \pm .08) \times 10^{-10}$ (P) | $1.05 \times 10^{10}$ |
| | $K^0$ | 0 | 493 ± 5 (Th) | | $K_2$: $(3 < \tau < 100) \times 10^{-8}$ (L)(P) | $(> 0.01 < 0.3) \times 10^8$ |
| 'ons† | $p$ | ½ | 938.213 ± 0.01* | | stable | 0.0 |
| | $n$ | ½ | 939.506 ± 0.01* | | $(1.04 \pm 0.13) \times 10^{+3}$ * | $0.96 \times 10^{-8}$ |
| | $\Lambda$ | ½? | 1115.2 ± 0.13 (B) | | $(2.77 \pm 0.15) \times 10^{-10}$ (d) | $0.36 \times 10^{10}$ |
| | $\Sigma^+$ | ½? | 1189.3 ± 0.35 (B) | 7.1 ± 0.4 | $(0.78 \pm 0.074) \times 10^{-10}$ (e) | $1.28 \times 10^{10}$ |
| | $\Sigma^-$ | ½? | 1196.4 ± 0.5 (B) | $7.6_{-2}^{+3}$ | $(1.58 \pm 0.17) \times 10^{-10}$ (f) | $0.64 \times 10^{10}$ |
| | $\Sigma^0$ | ½? | $1188.8_{-1}^{+2}$ (g) | | $(< 0.1) \times 10^{-10}$ (A) | $> 10 \times 10^{10}$ |
| | | | | | theoretically $\sim 10^{-19}$ | theoretically $\sim 10^{19}$ |
| | $\Xi^-$ | ? | 1321 ± 3.5* | | $(4.6 < \tau < 200) \times 10^{-10}$ (Tr) | $(> 0.005, < 0.2) \times 10^{10}$ |
| | $\Xi^0$ | ? | ? | | ? | |

From compilations by Cohen, Crowe, and DuMond, *Nuovo cimento*, 5, 541 (1957), and "Fundamental Constants of Phys- to be published by Interscience, New York, 1957. They include all data available before January 1, 1957.

M.Gell-Mann, A. Rosenfeld, Ann.Rev.Nucl.Sc. 7 (1957) 407-478.

# New hadrons just at LHC




P. Traczyk, CERN, on phys.org

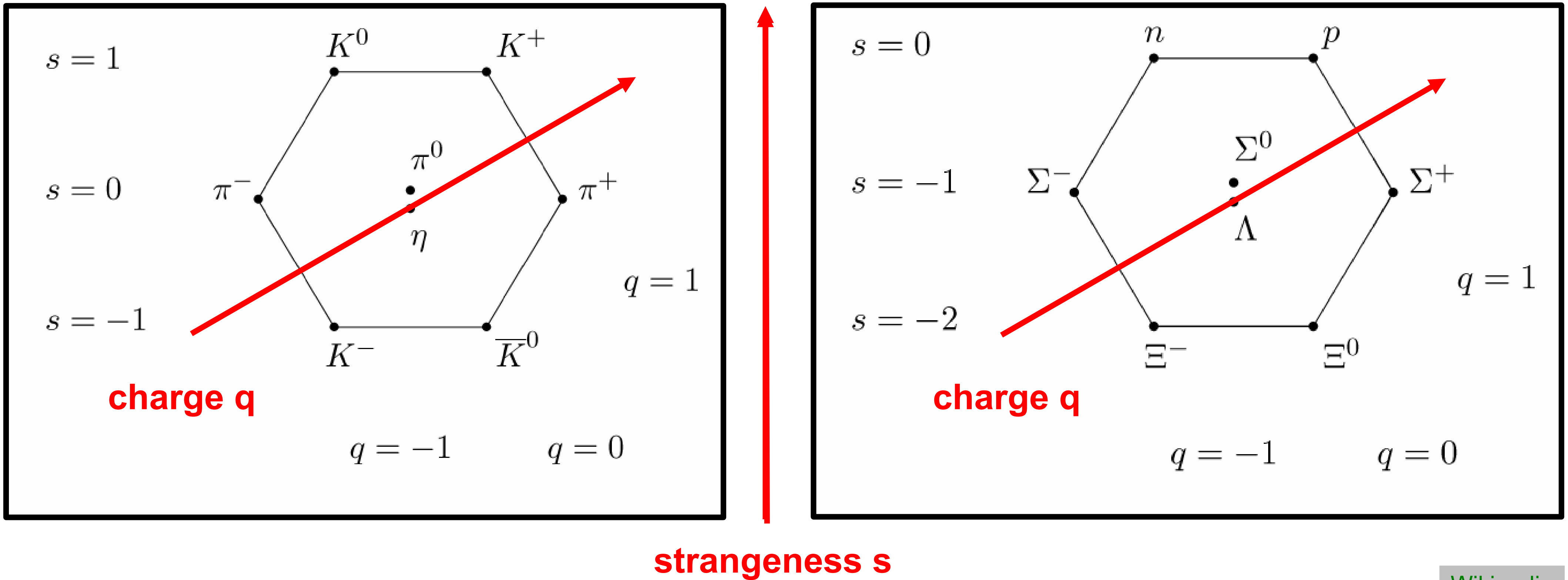
# Order to chaos

- **Cosmic ray & accelerator experiments 1947 – 1970**
  - **many new "elementary" particles?! And some with "flavor"**
  - **M. Gell-Mann, 1964: Eightfold Way**
    - **order known particles of equal spin into multiplets of charge q and "strangeness" s**

**Nobel prize 1969**

nobelprize.org

**Mesons spin 0** **Baryons spin ½**




Wikipedia

# And new quantum numbers

- J=3/2 fermions with symmetric space, spin and flavor wave function
  - Spin-statistics-problem: Contradiction to Pauli's exclusion rule!
- A way out:
  - O.W. Greenberg, 1964: Additional degree of freedom "color"
  - M. Gell-Mann, 1972: Color = tristate, RedGreenBlue

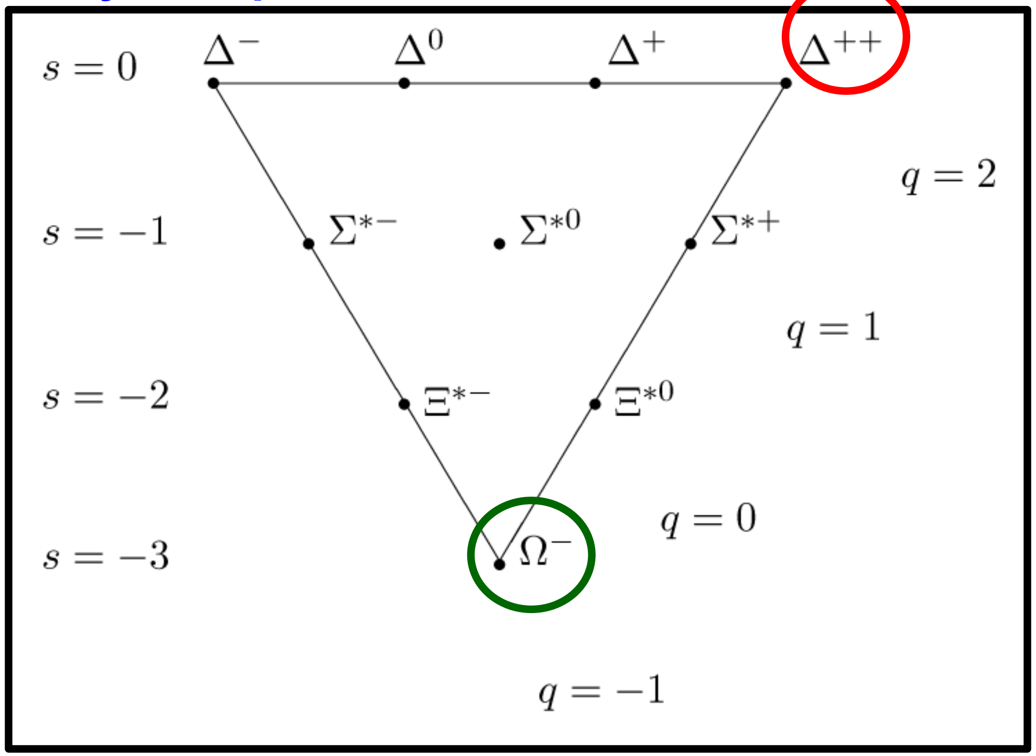
solved

Originally the names were red, blue & white ...

predicted

At Rochester conference (ICHEP) 1962, also Y. Ne'eman

Baryons spin 3/2




Great progress, but no dynamics yet.

Wikipedia

# Quark-Parton-Model

- **M. Gell-Mann:** Mesons: quark-antiquark pairs
  Baryons: three “quarks”
  (J. Joyce “Finnegan's Wake”: “Three quarks for Muster Mark.”)
- **G. Zweig:** Analogous idea, his name “aces” did not stick.
  - Quarks/Aces seen as hypothetical mathematical constructs; charges coming in thirds were never observed
- **R. Feynman:** Measurements of deep-inelastic electron-proton scattering at the SLAC-MIT experiment explained:
  Point-like scattering centres inside the protons: “partons”
  - Later: Identification of the partons with (anti-)quarks and gluons

Sakurai prize 2015

Scienceworld

nobelprize.org

Nobel prize 1965
for QED with
J. Schwinger,
S.-I. Tomonaga

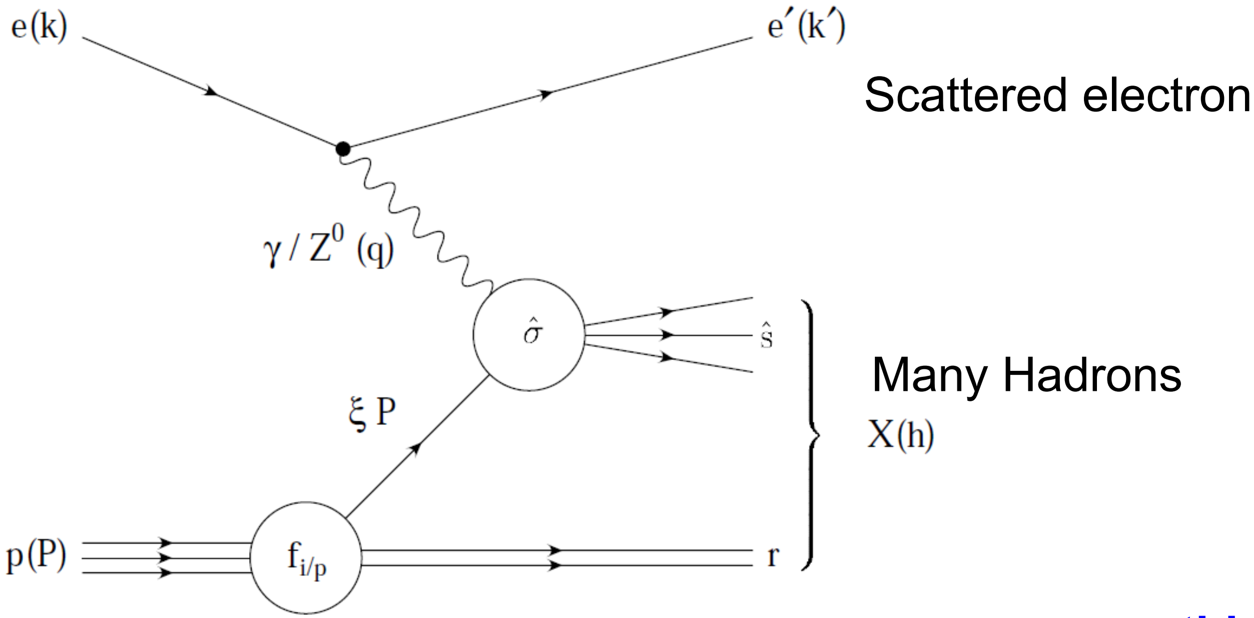
Historically interesting:
Abraham Pais, “Inward bound”, Clarendon Press, Oxford 1986.

→ “lepton”, “baryon”

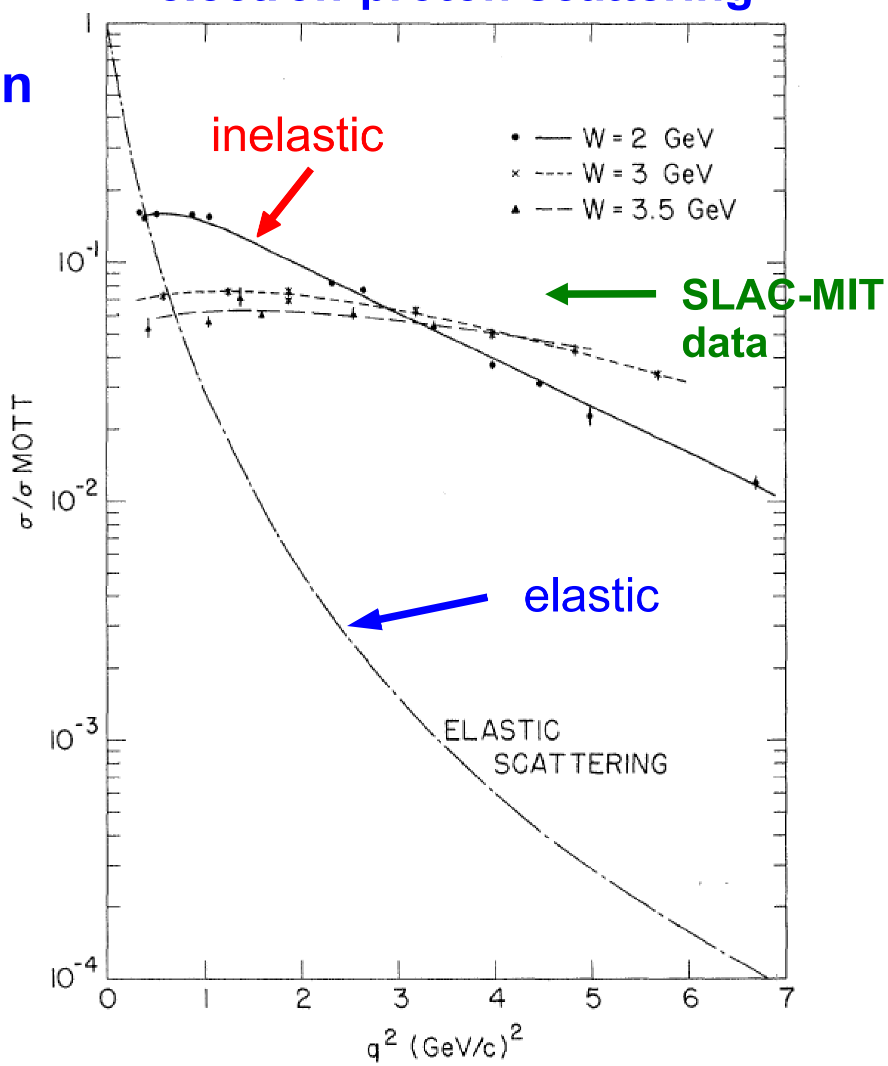
# Scale invariance

- **Inelastic >> elastic cross section**
- **Inelastic cross section ~ const. • Mott x section**
  - **approximately independent of resolution ~ $q^2$**
  - **scale invariant, i.e. no natural length scale**
  - **like scattering at point-like objects**

## Deep-inelastic scattering (DIS)




**electron-proton scattering**



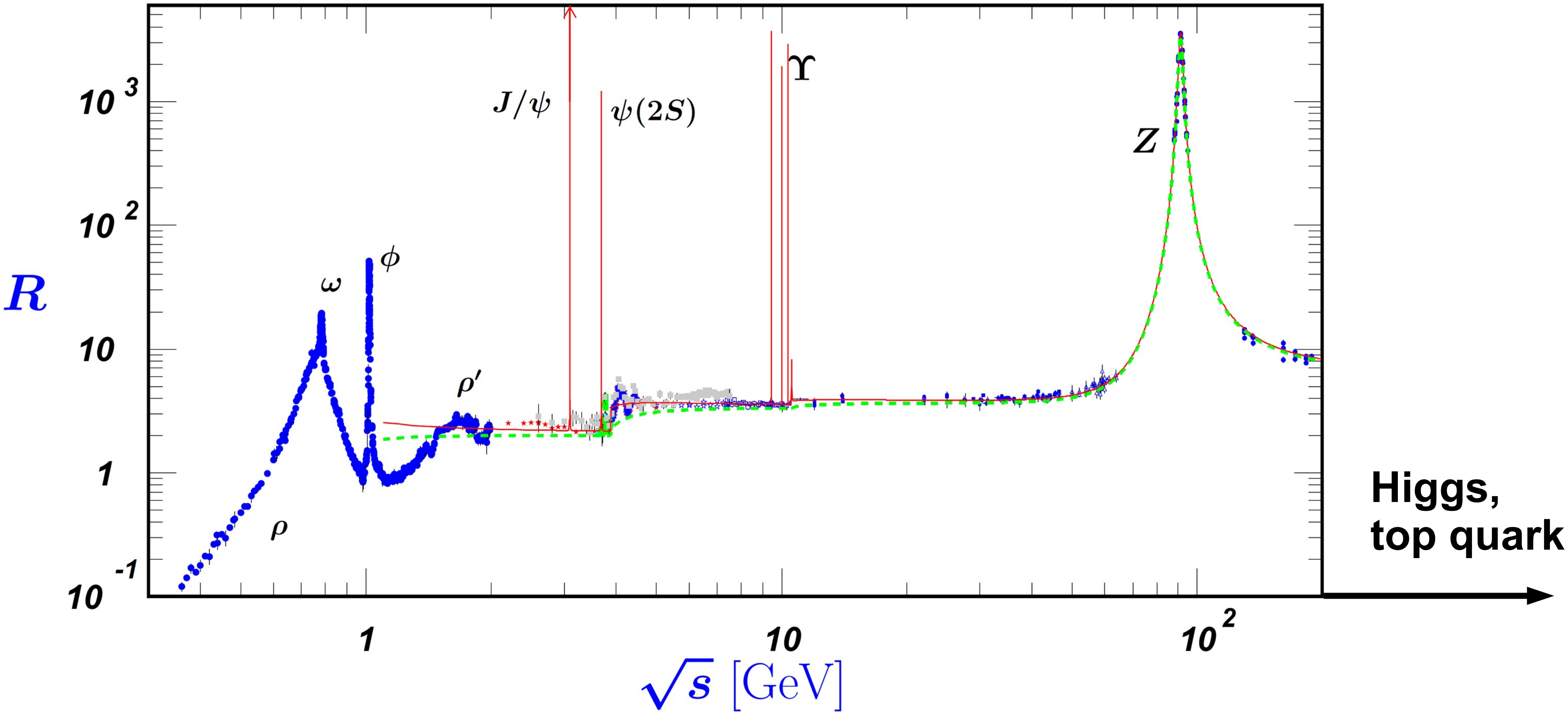

**more on this later ...**

PRL 23 (1969) 935.

# More evidence for “color”

**Hadronic branching ratio in elektron-positron annihilation**

$$R(s) = \frac{\sigma(e^+e^- \to hadrons, s)}{\sigma(e^+e^- \to \mu^+\mu^-, s)}$$




PDG

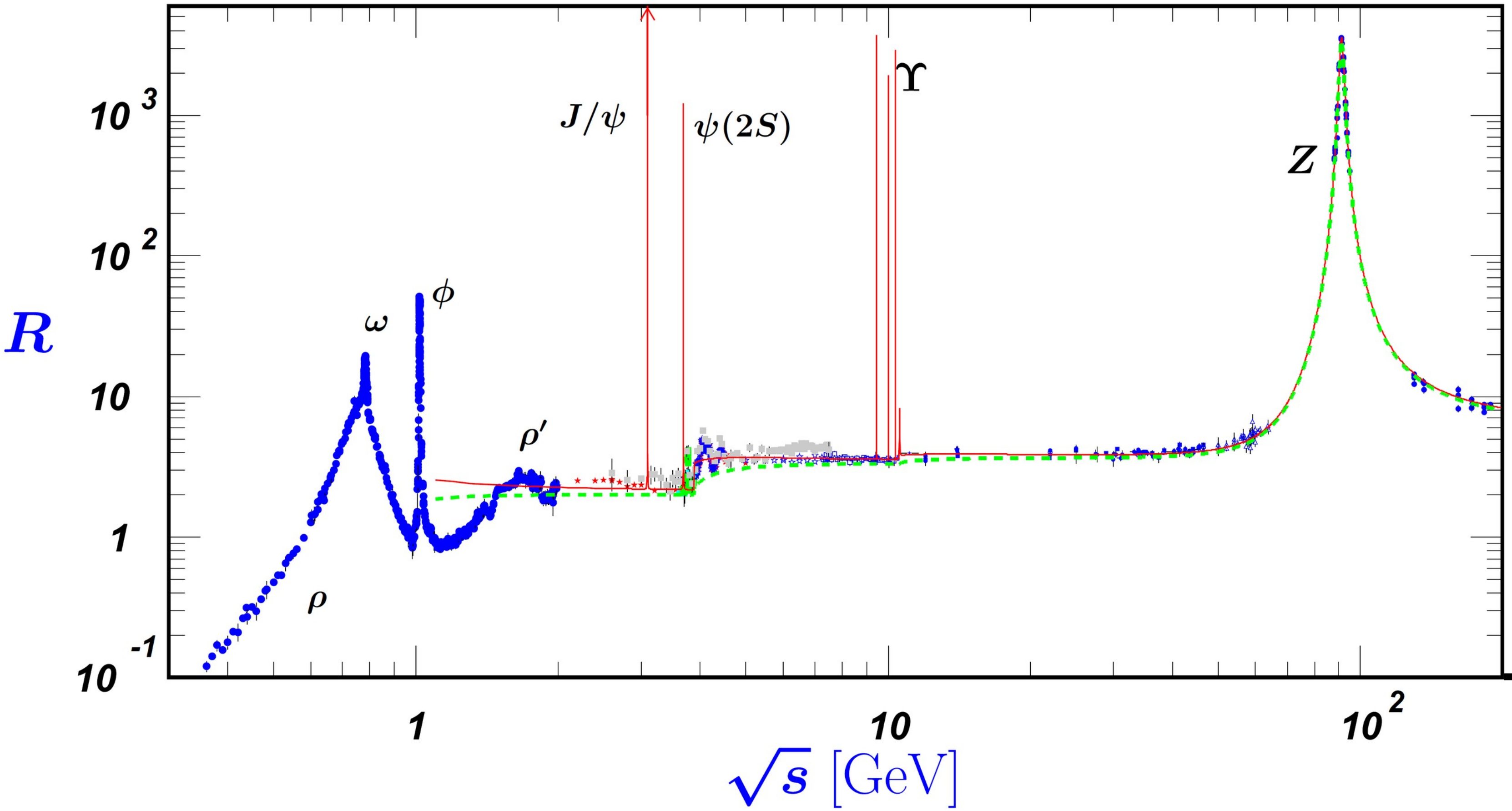
**Hadronic branching ratio in elektron-positron annihilation**

$$R(s) = \frac{\sigma(e^+e^- \to hadrons, s)}{\sigma(e^+e^- \to \mu^+\mu^-, s)} = 3 \cdot \sum_q Q_q^2$$

Color factor $N_C$

Squared quark charges

$$R_{\mathrm{uds}} = 3 \cdot \left(\frac{4}{9} + \frac{1}{9} + \frac{1}{9}\right) = 2$$




Higgs, top quark

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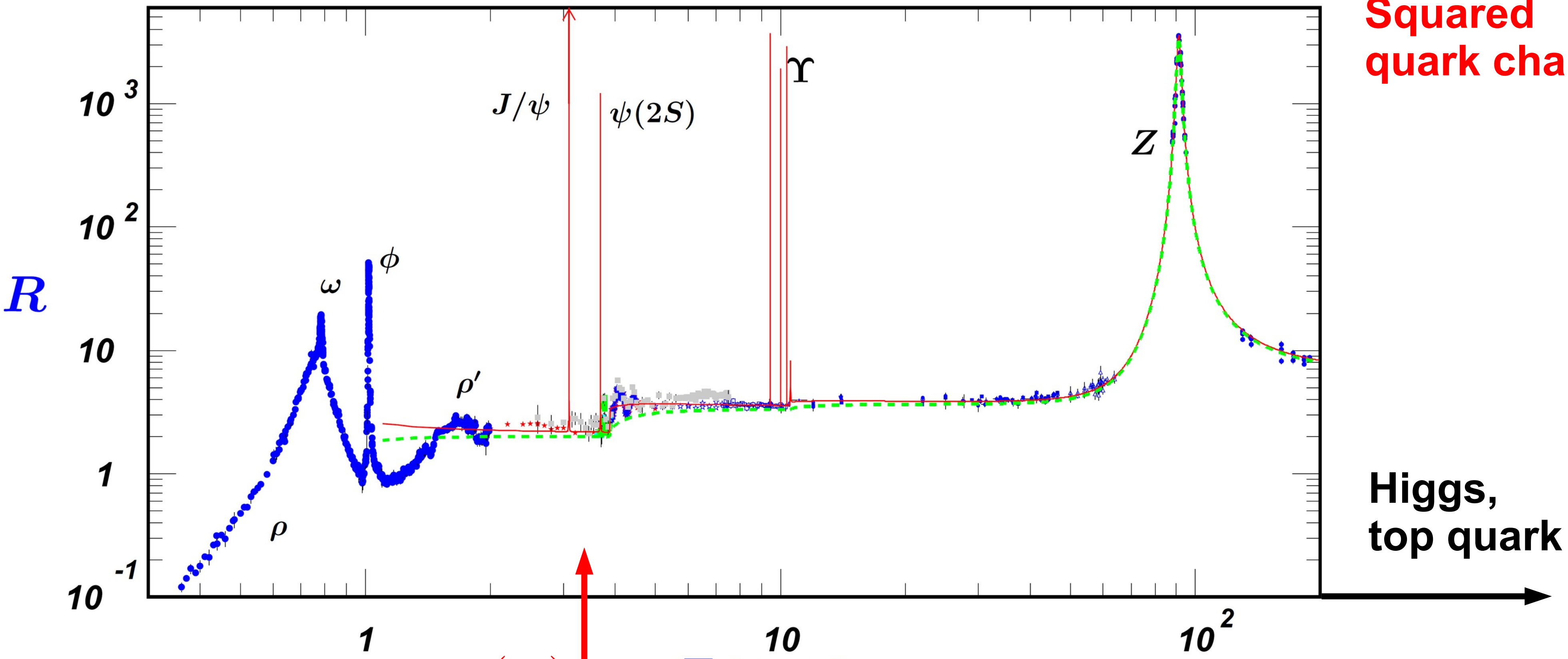


charm threshold $(c\bar{c})$ $\sqrt{s}$ [GeV]

$R \to \frac{10}{3}$

PDG

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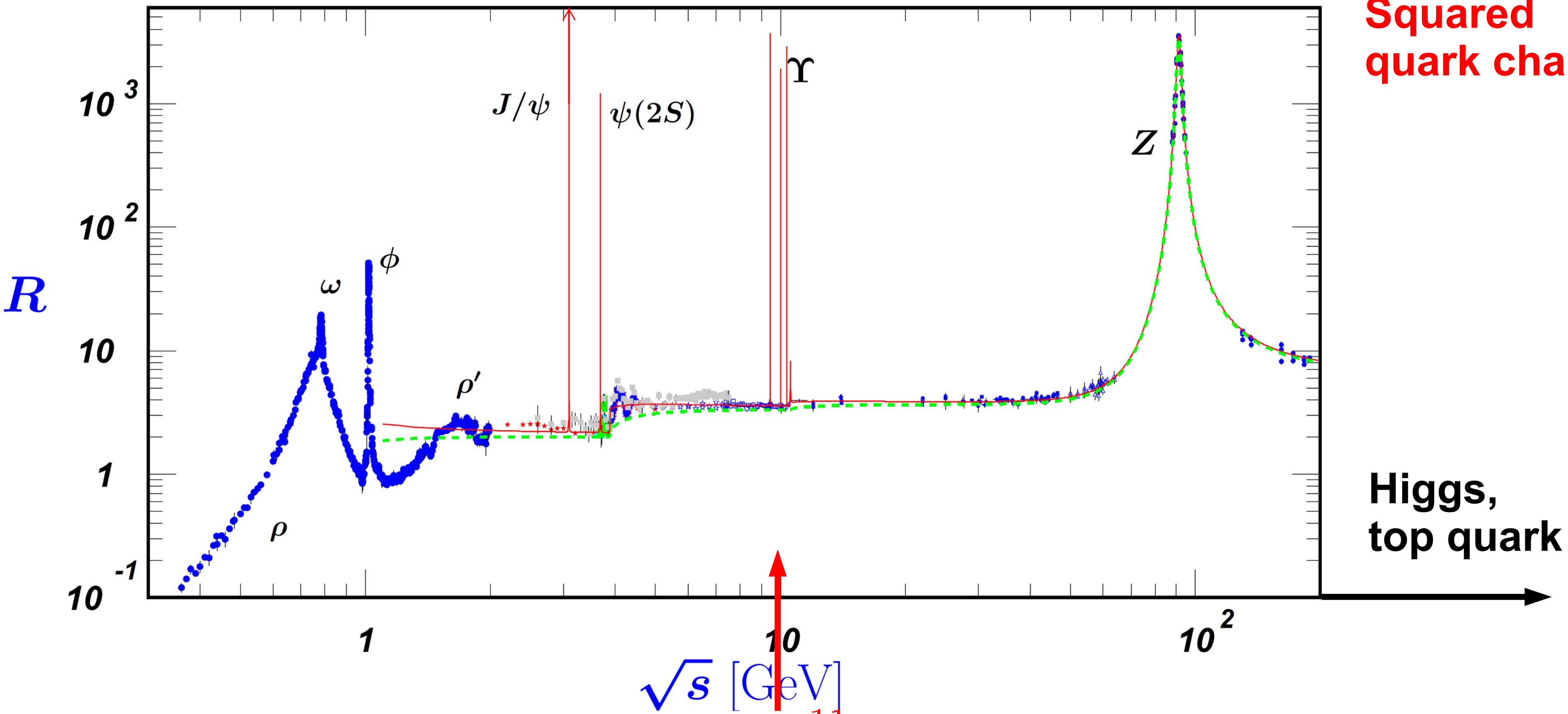


$R \to \frac{11}{3}(b\bar{b})$ **bottom threshold**

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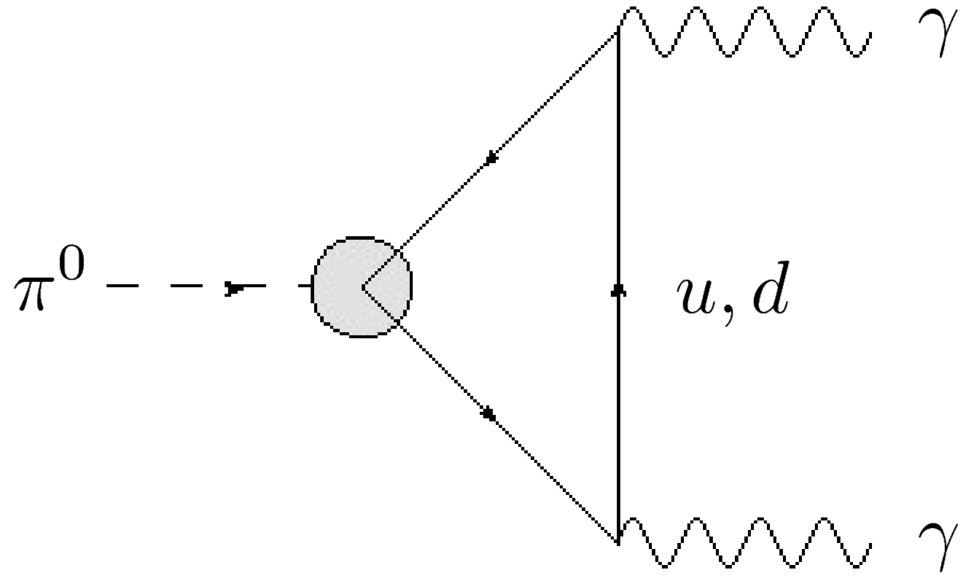
# More evidence for "color"

## Pion decay rate into two photons




**LO amplitude of the decay**

Color factor $N_c$

mass

$$\Gamma(\pi^0 \to \gamma\gamma) = N_c^2 (Q_u^2 - Q_d^2)^2 \frac{\alpha^2 m_\pi^3}{64\pi^3 f_\pi^2}$$

Squared quark charges

Decay constant (from charged pions)

**Evaluation from independent measurements of other observables:**

$$\Gamma(\pi^0 \to \gamma\gamma) = 7.33\,\mathrm{eV} \left(\frac{N_c}{3}\right)^2$$

**Measurement:**

$$\Gamma(\pi^0 \to \gamma\gamma) = 7.84 \pm 0.56\,\mathrm{eV}$$

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# More evidence for “color”

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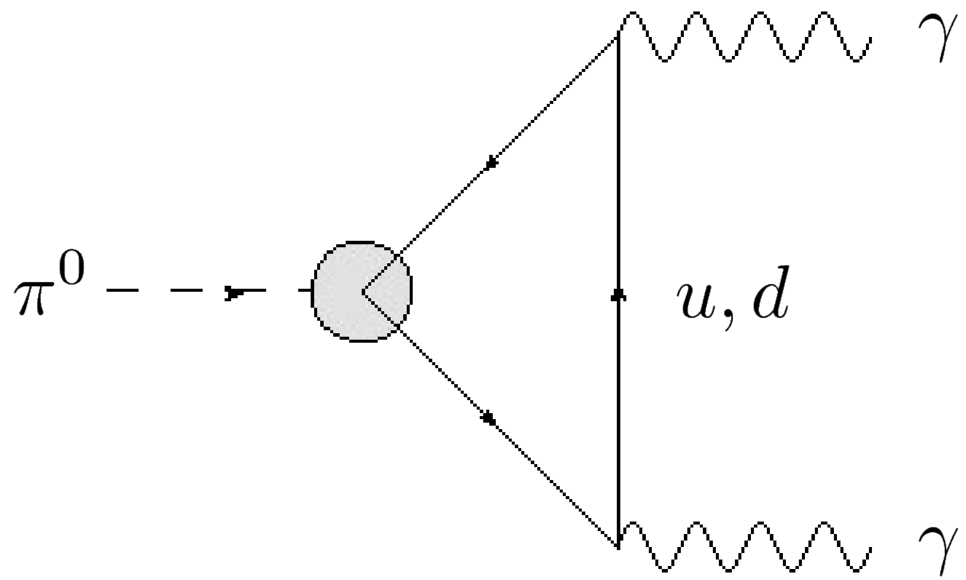


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**Attention, not the only choice!**
**$N_C = 1$, $Q_u = 1$, $Q_d = 0$ ...**

**Evaluation from independent measurements of other observables:**

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**Measurement:**

$$\Gamma(\pi^0 \to \gamma\gamma) = 7.84 \pm 0.56\,\mathrm{eV}$$

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# QCD Lagrangian

- **Covariant derivative:** $(D_\mu)_{ab} = \partial_\mu \delta_{ab} + i g_s \tau^A_{ab} \mathcal{A}^A_\mu$

  Color indices of octet representation A,... = 1,…,8

  Color indices of triplet representation a,b,c = 1,2,3

  Gluon field $\mathcal{A}^A_\mu$

- **Field strength tensors:**

$$\mathcal{G}^A_{\mu\nu} = \partial_\mu \mathcal{A}^A_\nu - \partial_\nu \mathcal{A}^A_\mu - \boxed{g_s f^{ABC} \mathcal{A}^B_\mu \mathcal{A}^C_\nu}$$

  → leads to triple (TGC) and quartic (QGC) gauge couplings

- **Lagrangian of SU(3)$_C$:**

$$\mathcal{L}_{\mathrm{QCD}} = \sum_q \overline{\psi}_a (i\gamma^\mu (D_\mu)_{ab} - m_q)\psi_b - \frac{1}{4}\mathcal{G}^A_{\mu\nu}\mathcal{G}^{\mu\nu}_A$$

- **The gluon remains massless → SU(3)$_C$ exact symmetry of nature!**

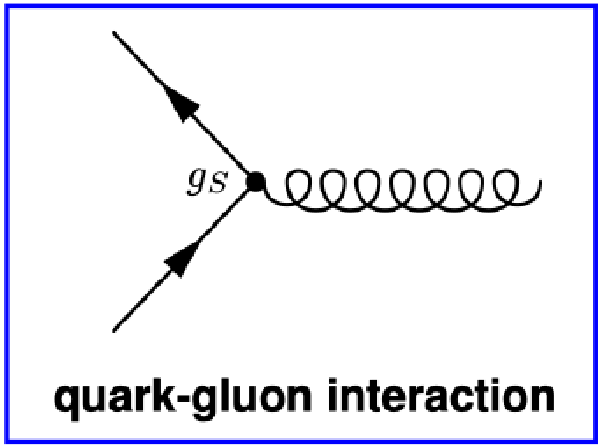
# QCD Lagrangian

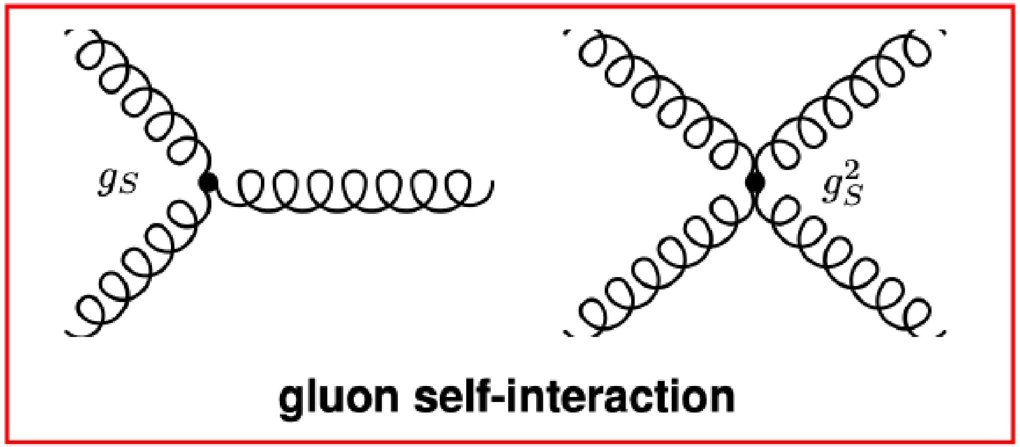
## Invariance under local $SU(3)_c$ transformations

- Three color charges a = 1, 2, 3 → Red, Green, Blue
  (as analogue to electric charge in QED)
- Eight vector fields (gluons) $\mathcal{A}_\mu^A$ carry color charge and color anti-charge
- The gluons are massless
  → exact symmetry
  → in principal infinite range of strong force

$$\mathcal{G}_{\mu\nu}^A = \partial_\mu \mathcal{A}_\nu^A - \partial_\nu \mathcal{A}_\mu^A - \boxed{g_s f^{ABC} \mathcal{A}_\mu^B \mathcal{A}_\nu^C}$$

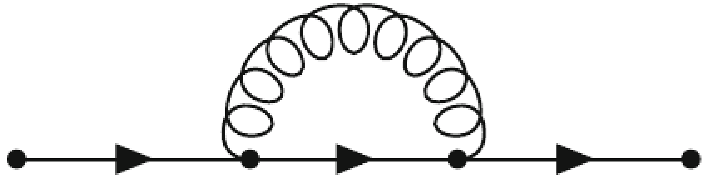
- Non-zero commutator leads to gluon self-interactions via triple and quartic gauge couplings

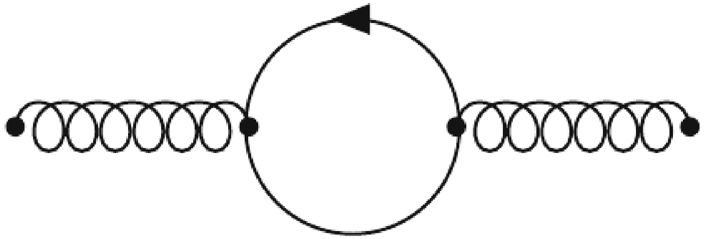


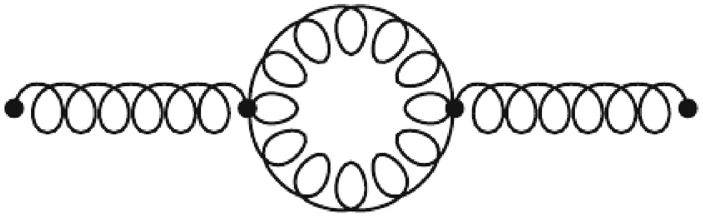
# Quantum corrections

- **Quark (left) and gluon (middle & right) self-energy corrections:**

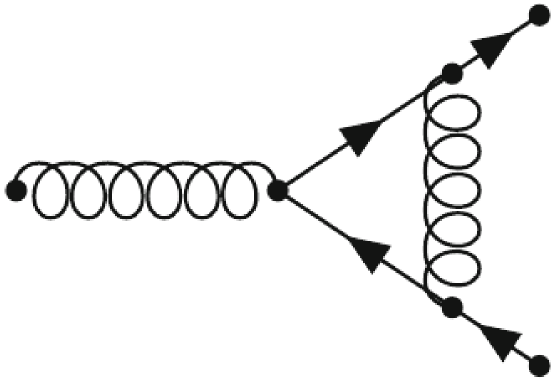


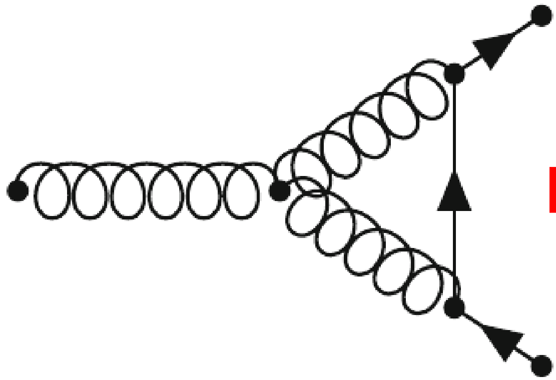


**Not in QED!**



- **Quark-gluon vertex corrections:**





**Not in QED!**

**→ lead to anti-screening**

# Beta functions

- **In (renormalisable) QFT the beta function encodes the dependence of the coupling parameter g on the energy (or distance) scale μ:**

$$\alpha_i := \frac{g_i^2}{4\pi}$$

$$\beta(g) = \frac{\partial g}{\partial \log(\mu^2)}$$

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- **Beta function of QED (1-loop):** $\beta(\alpha) = \frac{1}{3\pi}\alpha^2$
  - **The coupling increases with energy scale**
  - **The coupling decreases with larger distances**
    - **Infinite range, Coulomb potential:** $V(r) \propto \frac{1}{r}$

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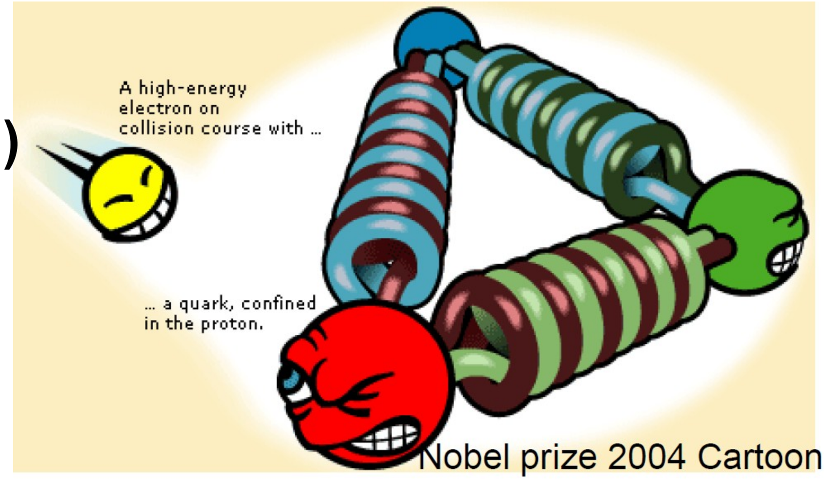
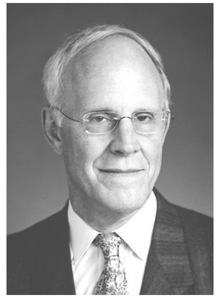
    - **Infinite range, Coulomb potential:** $V(r) \propto \frac{1}{r}$
- **Beta function of QCD (1-loop):** $\beta(\alpha_s) = -\left(\frac{11N_C - 2N_f}{12\pi}\right)\alpha_s^2$
  - **The coupling decreases with energy scale, if** $N_C = 3, \quad N_f \leq 16$
    - **Asymptotic freedom**
  - **The coupling increases with larger distances**
    - **Confinement, string potential:** $V(r) \approx \sigma \cdot r$ **with tension** $\sigma \approx 1\,\mathrm{GeV/fm}$

## Nobel prize 2004

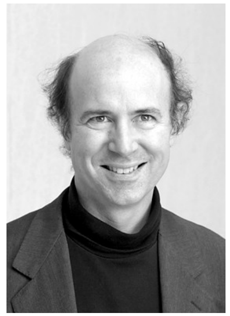
- **Theory:**
  - Renormalisation group equation (RGE)
  - Solution of 1-loop equation
  - Running coupling constant



Nobel prize 2004 Cartoon


D. Gross


D. Politzer


F. Wilczek

nobelprize.org

$$\alpha_s(Q^2) = \frac{\alpha_s(\mu^2)}{1 + \alpha_s(\mu^2)\beta_0 \ln\left(\frac{Q^2}{\mu^2}\right)}$$

$$\alpha_s(Q^2) = \frac{1}{\beta_0 \ln\left(\frac{Q^2}{\Lambda^2}\right)}$$

- 'Strong' coupling weak for $Q^2 \rightarrow \infty$ , i.e. small distances
- Asymptotic freedom
- Perturbative methods usable

$$\beta_0 = \frac{33 - 2 \cdot N_f}{12\pi}$$

Physik Journal 3 (2004) Nr. 12

- **What happens at large distances?**
  - $Q^2 \rightarrow 0$ ?
  - Cannot be answered here!
    For $Q^2 \rightarrow \Lambda^2$ perturbation theory not applicable anymore!

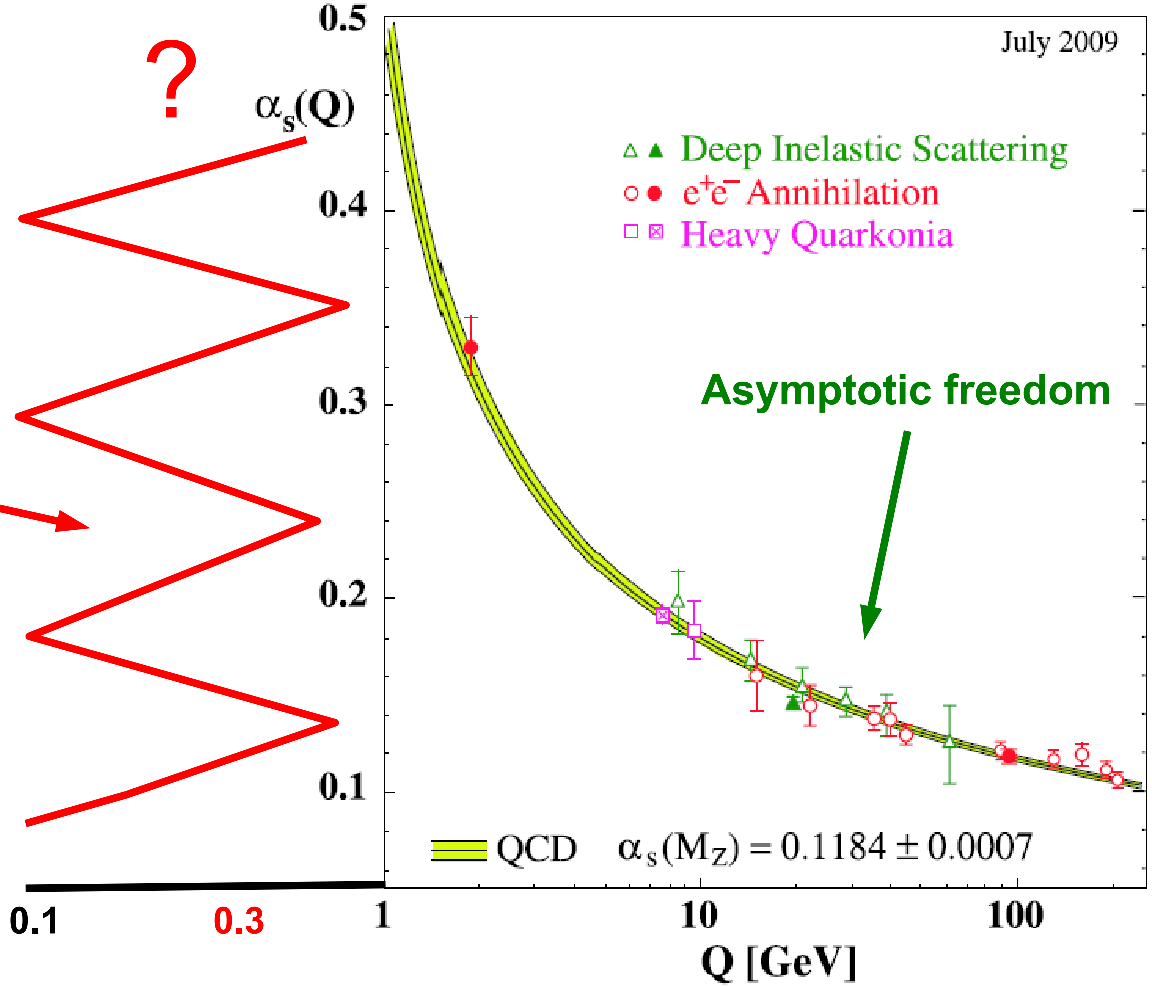
# Running coupling constant

$$\alpha_s(Q^2) = \frac{1}{\beta_0 \ln\left(\frac{Q^2}{\Lambda^2}\right)}$$

**with $\Lambda$ typically ≈ 200 – 300 MeV**

**Non-perturbative regime**

**pre-LHC**




S. Bethke, EPJC 64 (2009).

**QCD potential grows linearly with larger distances:**

$$V = \sigma \cdot r \approx 1\text{GeV/fm} \cdot r$$

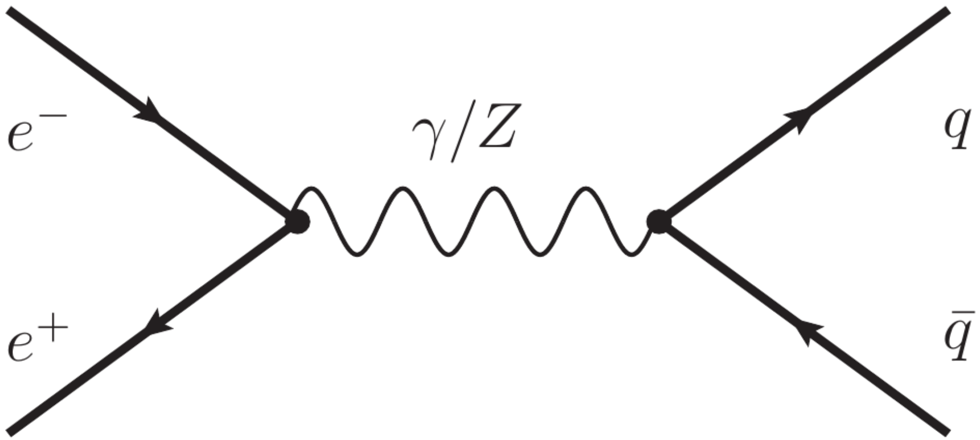
**→ No free quarks (or gluons)**

**→ Confinement**

# So what do we expect to see?

- **Well not quarks (or gluons) thanks to confinement**
- **Searches for particles with non-integer charge unsuccessful**

# How to see "hypothetical" quarks?




**Leading order:**
Quark-Antiquark partons showing up in opposite event hemispheres with energy fractions:

$$x_1 = \frac{2E_q}{\sqrt{s}} \quad x_2 = \frac{2E_{\bar{q}}}{\sqrt{s}} \qquad 0 \leq x_1, x_2 \leq 1$$

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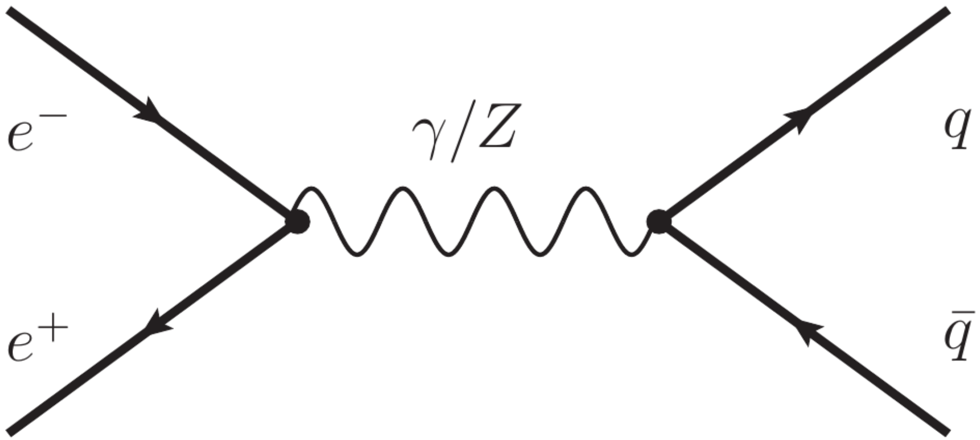


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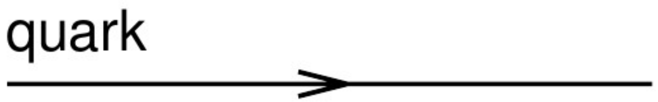

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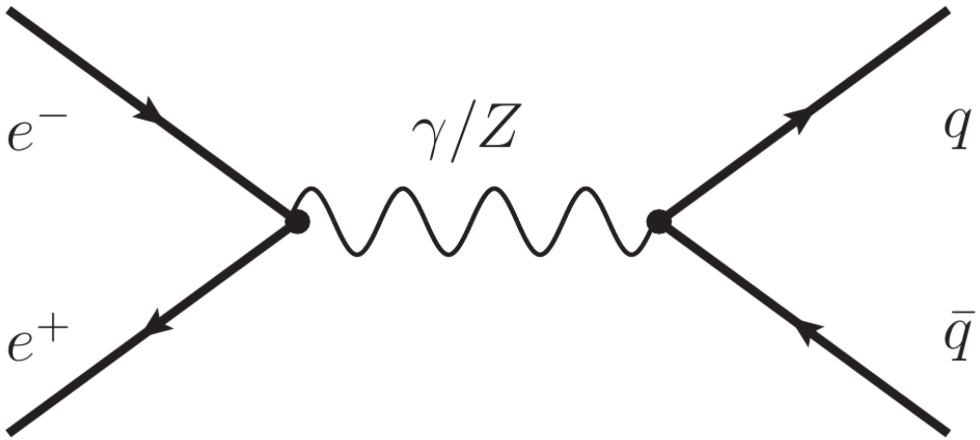


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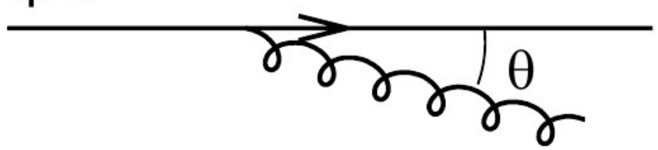


Gluon emission:

$$\int \alpha_s \frac{dE}{E} \frac{d\theta}{\theta} \gg 1$$

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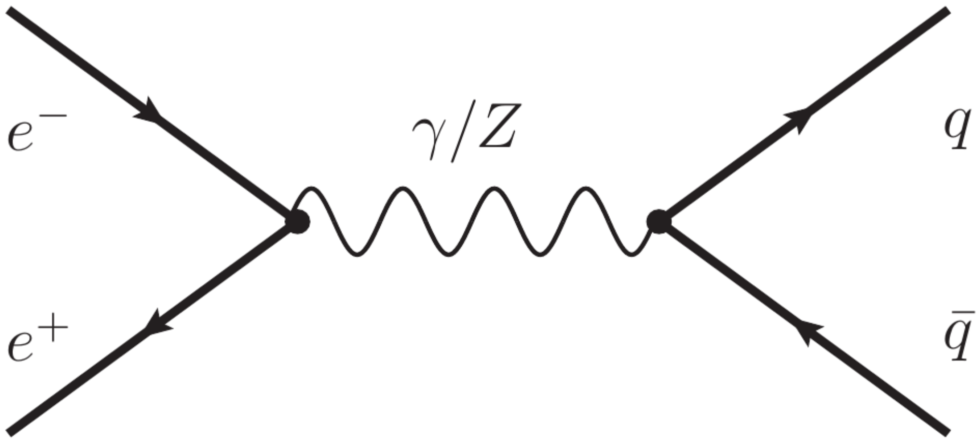


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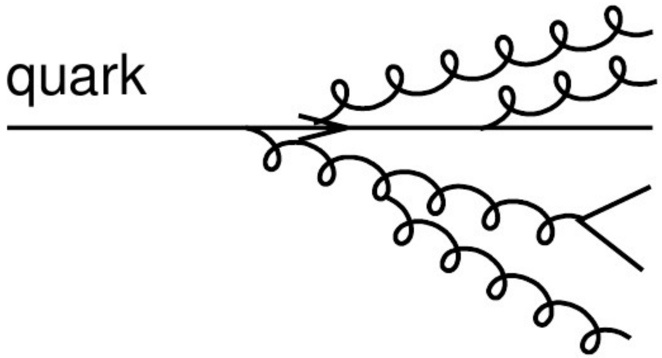


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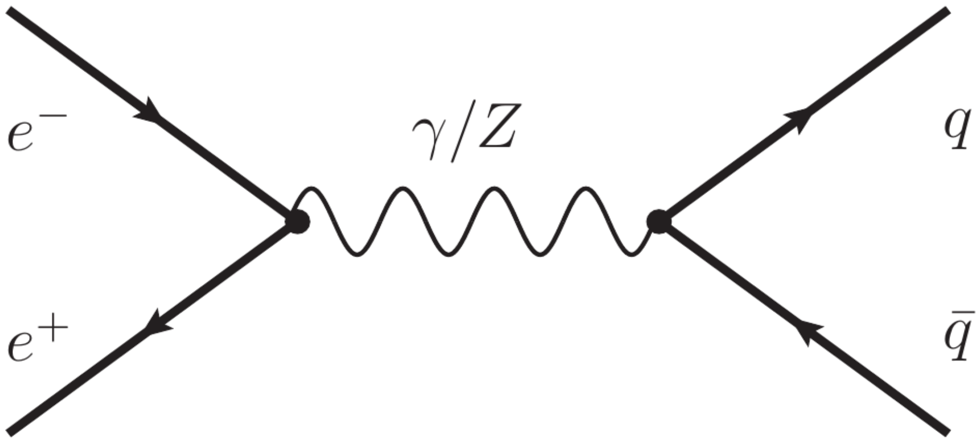


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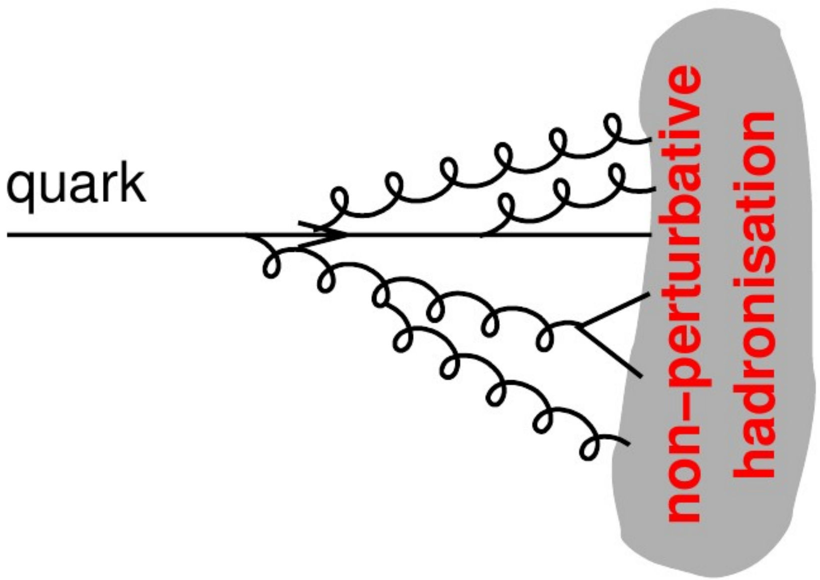
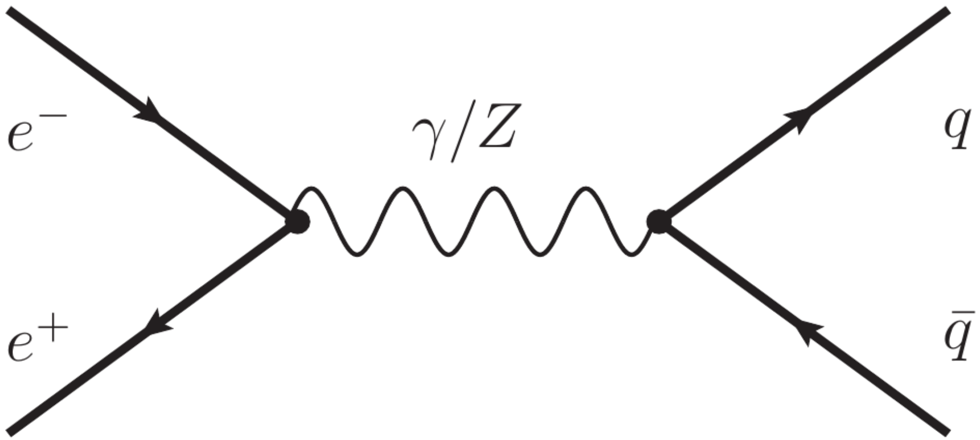



Gluon emission:

$$\int \alpha_s \frac{dE}{E} \frac{d\theta}{\theta} \gg 1$$

At low scales:

$$\alpha_s \to 1$$




**Leading order:**
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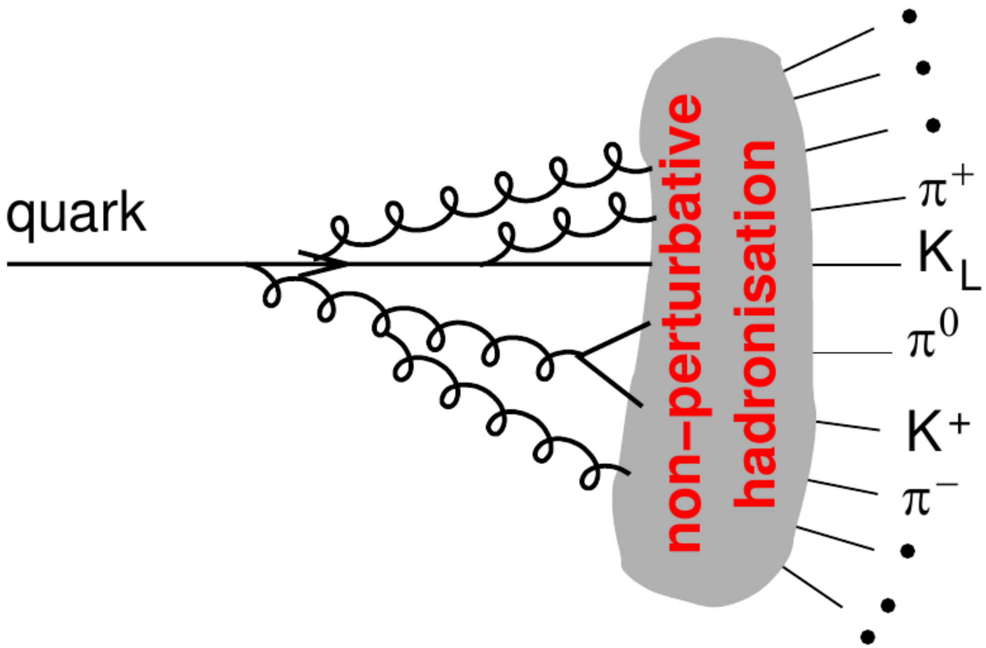



Gluon emission:

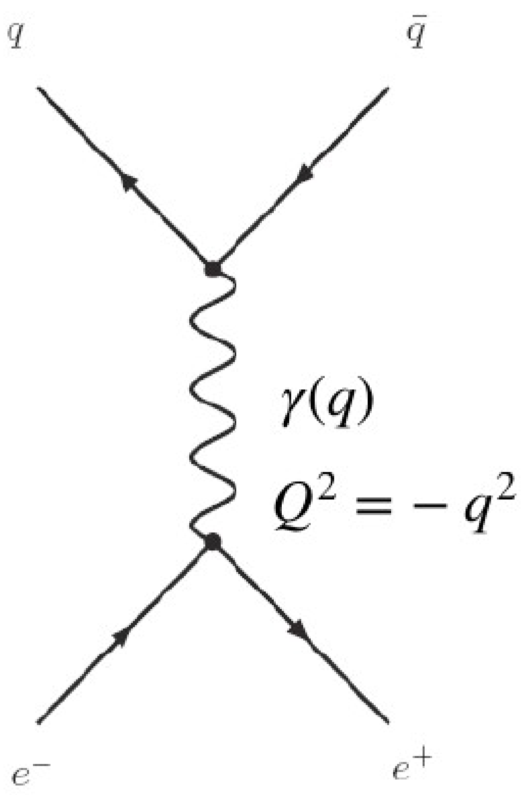
$$\int \alpha_s \frac{dE}{E} \frac{d\theta}{\theta} \gg 1$$

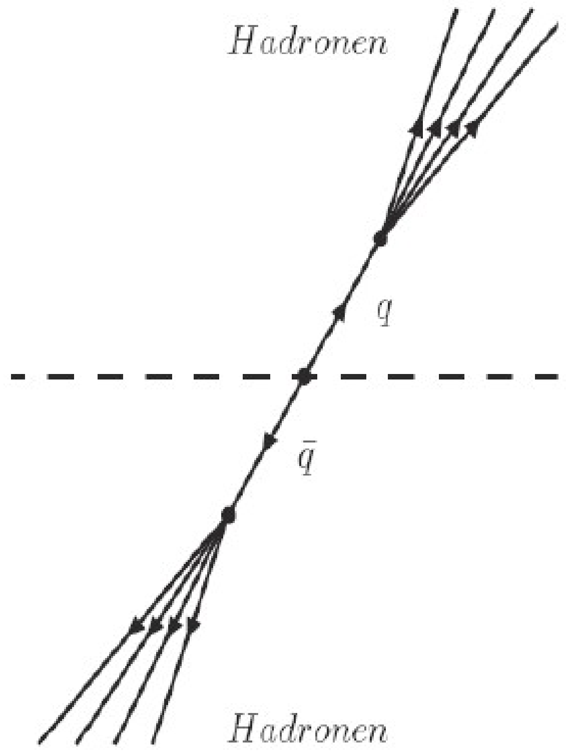
At low scales:

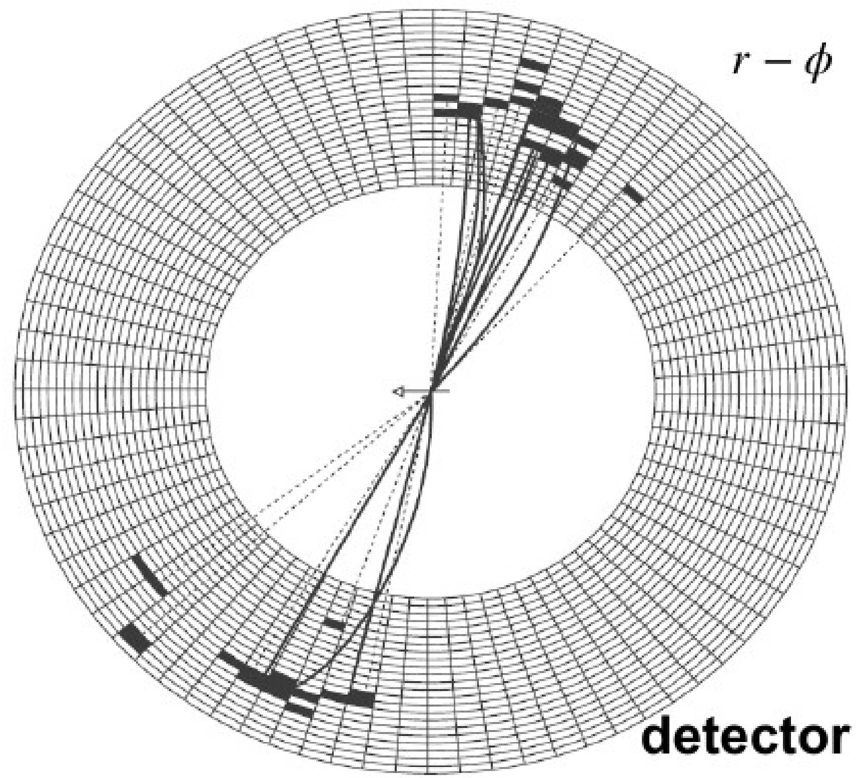
$$\alpha_s \rightarrow 1$$

G. Salam

# The look of two quarks

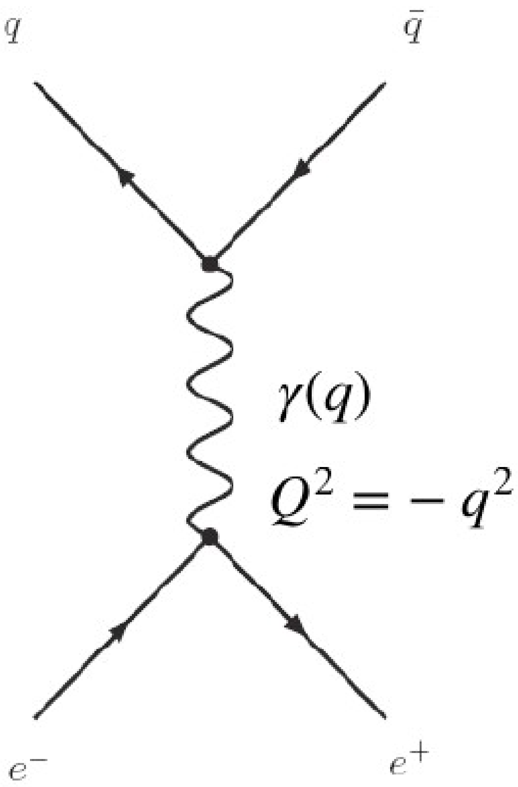


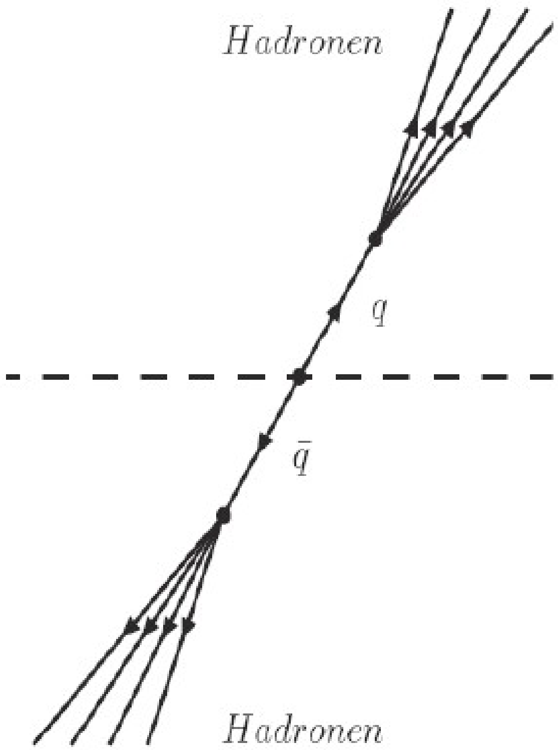



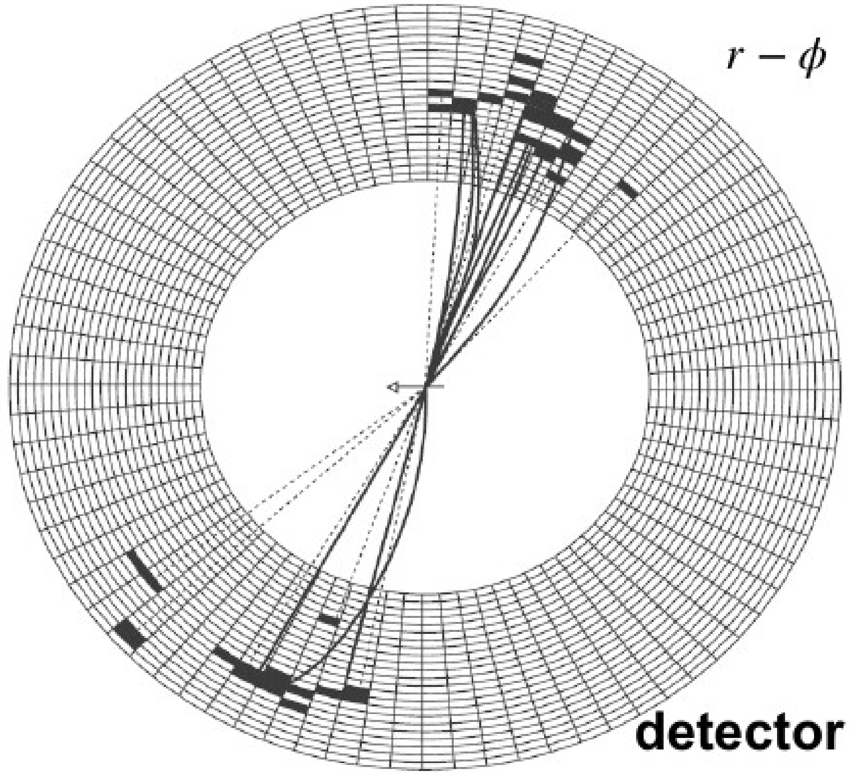




K. Lipka

# The look of two quarks



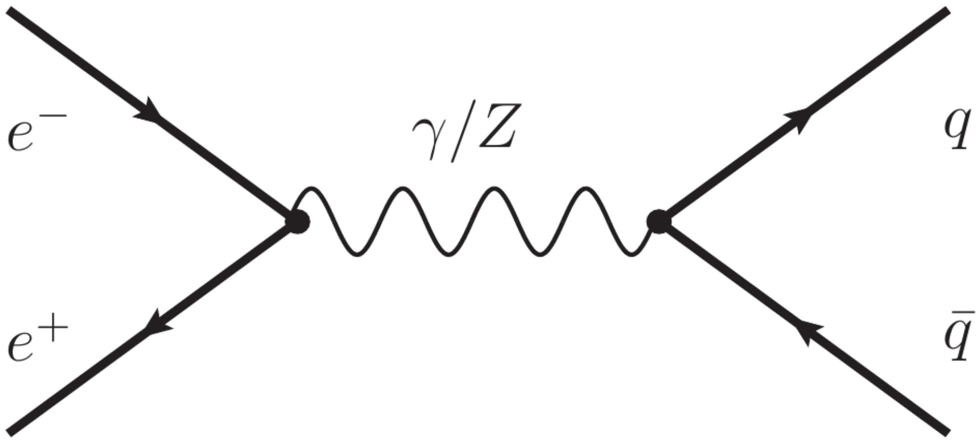




**BUT initially cms energy too small for such clear pictures!**
**Hadronisation with typical energies of $\Lambda_{QCD} \approx 330$ MeV smears out the partonic structure.**

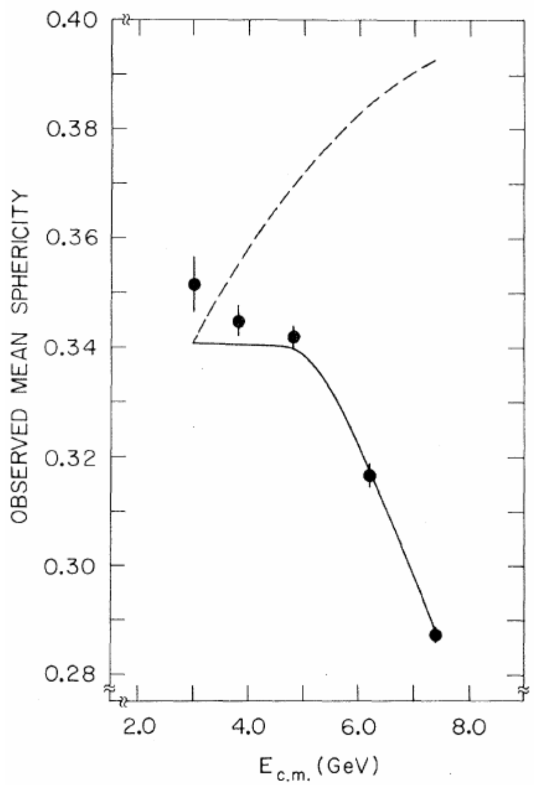
K. Lipka

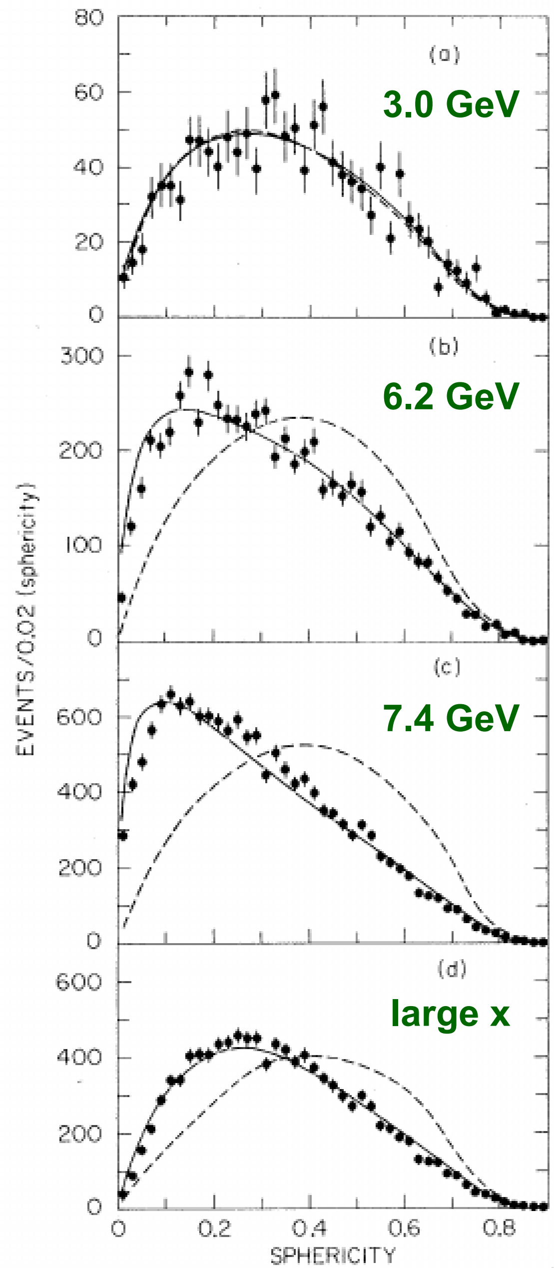
# $e^+e^- \rightarrow qq$ at low energy




Increasing cms energy:

jet model
phase space




**Examine the energy flow**
**Inside the event**
**→ define sphericity S**
**→ compare to other models than QCD**

$$S = 3\sum_i \left(p_{\mathrm{T}i}^2\right) \ / \ 2\sum_i |\vec{p}_i|^2$$

(not safe for pQCD)

SPEAR: Hanson, G. et al., PRL 35 (1975) 1609.

# So what do we expect to see?

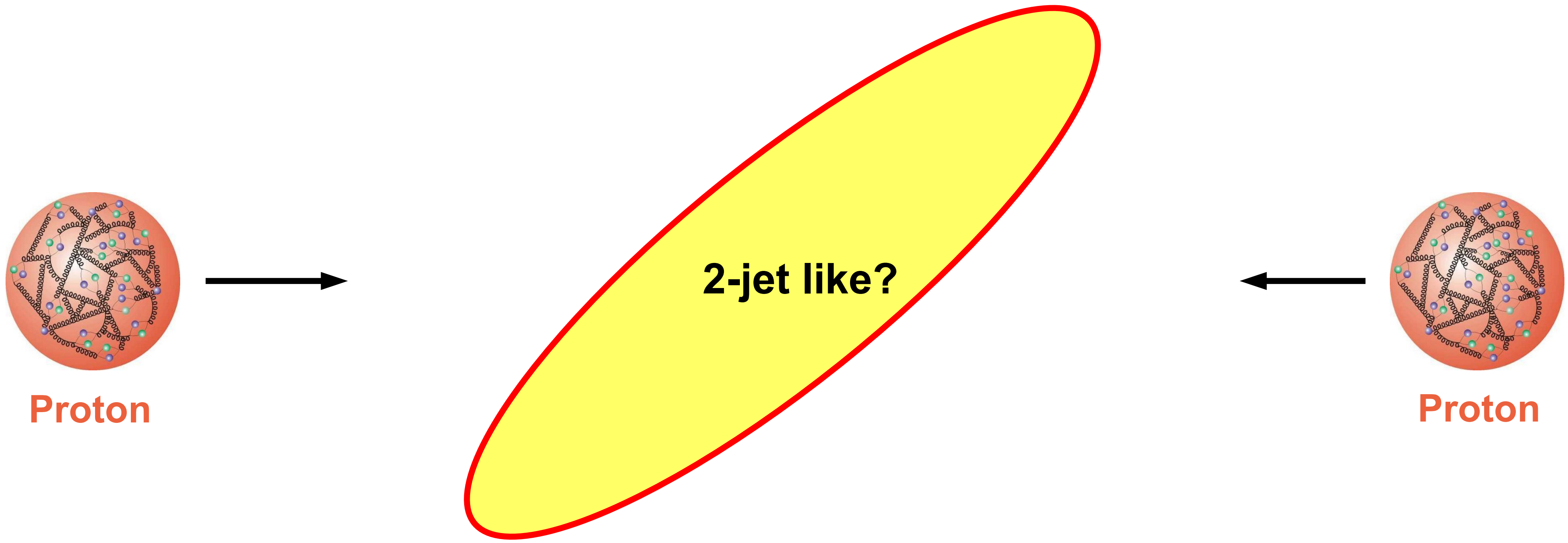
- Well not quarks (or gluons) thanks to confinement
- Searches for particles with non-integer charge unsuccessful
- Examine distribution of energy flow
  - event shapes: continuous measure of energy flow
  - jets: integer quantity counting the number of “peaks” in energy flow

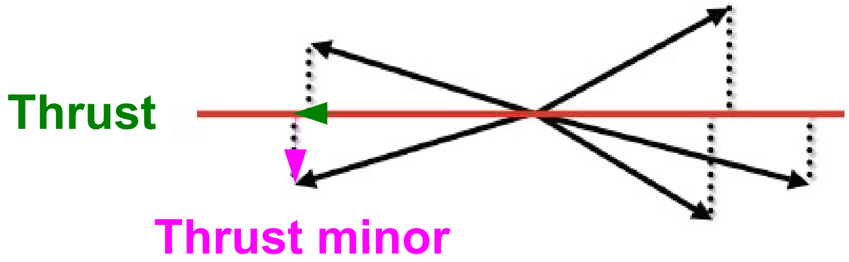
# So what do we expect to see?

- Well not quarks (or gluons) thanks to confinement
- Searches for particles with non-integer charge unsuccessful
- Examine distribution of energy flow
  - event shapes: continuous measure of energy flow
  - jets: integer quantity counting the number of “peaks” in energy flow
- Separation not strict:
  - can subdivide event shape distributions into say 2-jet and multi-jet region
  - can increase jet resolution (decrease jet radius) parameter to find value when event changes from having n jets to n+1 → continuous measure, e.g. $y_{23}$ in e+e-

# Shapes

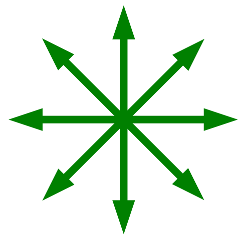
## Investigate the energy/momentum flow in an event






**Redefine to get:** $\tau \equiv 1 - T$

$$T = \max_{\vec{n}_T} \frac{\sum_i |\vec{p}_i \cdot \vec{n}_T|}{\sum_i |\vec{p}_i|}$$



**linear ~ dijet**

$\tau \to 0$

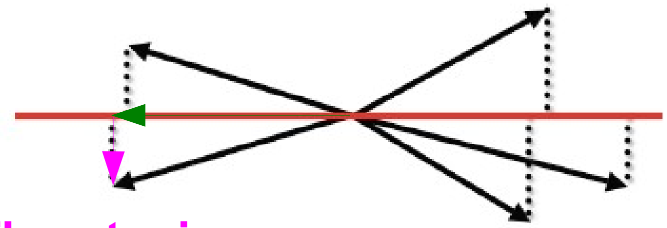
**spherical ~ multijet**

$\tau \to 1/2$

→ 0 in LO dijet case

**Originally:**
**Event Shapes in $e^+e^-$ (and ep)**
**Played a key role in the discovery of the gluon at DESY in 1979!**

**Old but still-used definition since collinear and infrared safe:**
**Thrust** S. Brandt et al., PL12 (1964), E. Farhi, PRL39 (1977).

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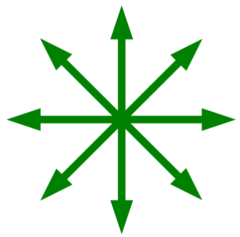

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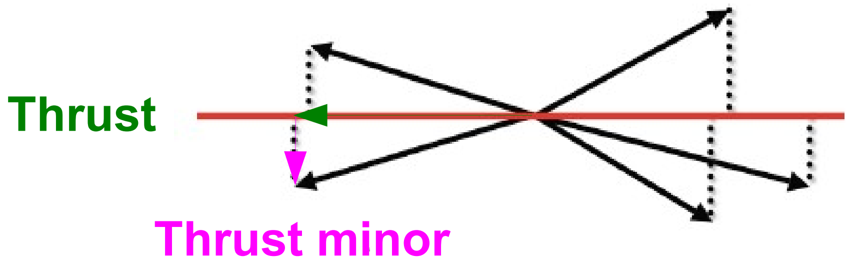
# Event Shapes

**Originally:**
**Event Shapes in e⁺e⁻ (and ep)**
**Played a key role in the discovery of the gluon at DESY in 1978!**




**Old but still-used definition since collinear and infrared safe:**
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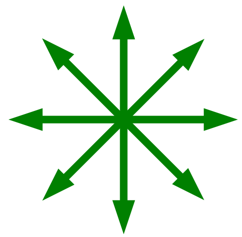
$$T_\perp = \max_{\vec{n}_T} \frac{\sum_i |\vec{p}_{\perp,i} \cdot \vec{n}_T|}{\sum_i p_{\perp,i}}$$

**At LHC: Transverse (to beam pipe) global thrust**
➔ **In praxis, need to restrict rapidity range: $|\eta| < \eta_{max}$ →**
**Transverse central thrust**



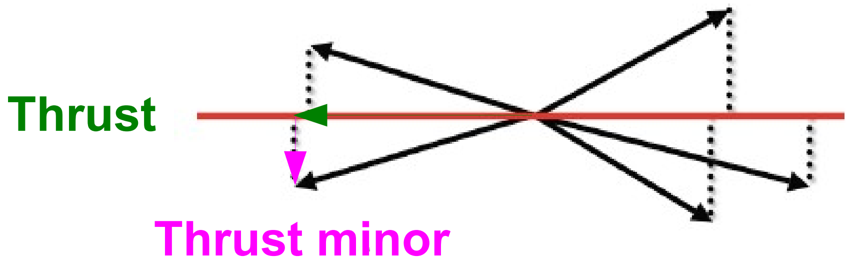
**linear ~ dijet** | **spherical ~ multijet**

$\tau \to 0$ | $\tau \to 2/\pi$

⟶ 0 in LO dijet case

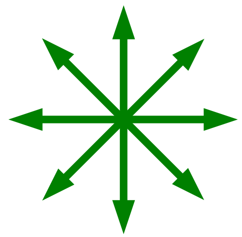
## At LHC:

**Transverse central thrust**




- Comparison to perturbative QCD (ME + resummation)
- Useful for MC tuning
- No luminosity uncertainty
- Reduced sensitivity to exp. effects
- Nonperturbative effects might be sizeable

$$T_\perp = \max_{\vec{n}_T} \frac{\sum_i |\vec{p}_{\perp,i} \cdot \vec{n}_T|}{\sum_i p_{\perp,i}}$$



linear ~ dijet

$\tau \to 0$

spherical ~ multijet

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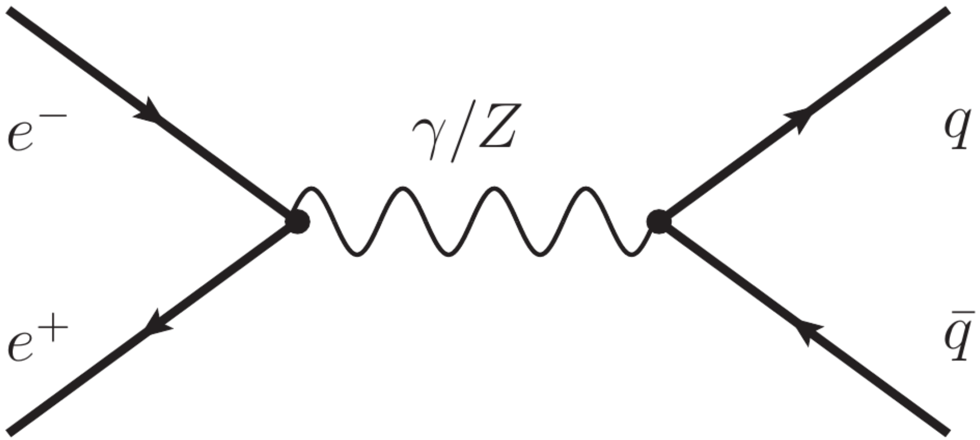
→ 0 in LO dijet case

See e.g. A. Banfi, G. Zanderighi et al., JHEP06, 2010

# So how to see gluons now?

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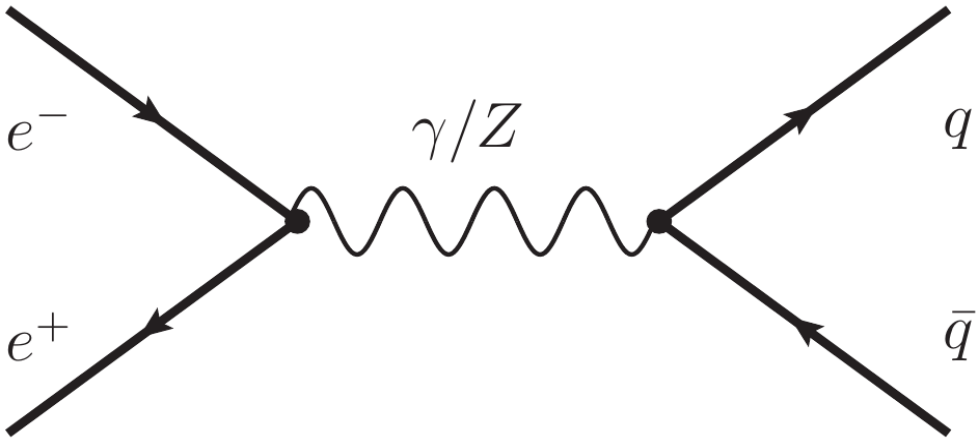


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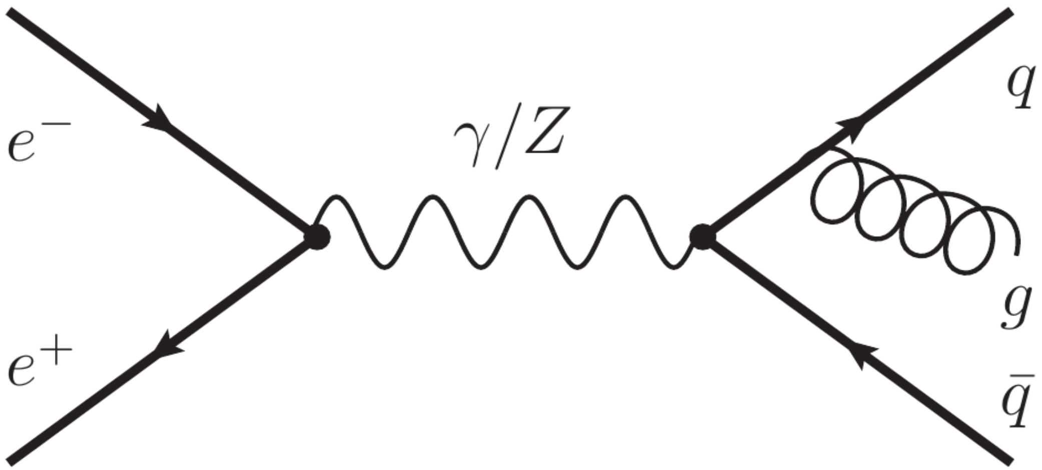
**Real correction:**
Additional gluon emission (bremsstrahlung)

**→ search for 3-"jet" events**




$$\frac{d^2\sigma_{q\bar{q}g}}{dx_1 dx_2} \propto \frac{\alpha_s}{2\pi} \frac{x_1^2 + x_2^2}{(1-x_1)(1-x_2)}$$

- **Singularities for**
  - **$x_1$ or $x_2$ → 1: collinear gluon $\theta_{gq} \to 0$**
  - **$x_1$ and $x_2$ → 1: soft gluon $E_g \to 0$**

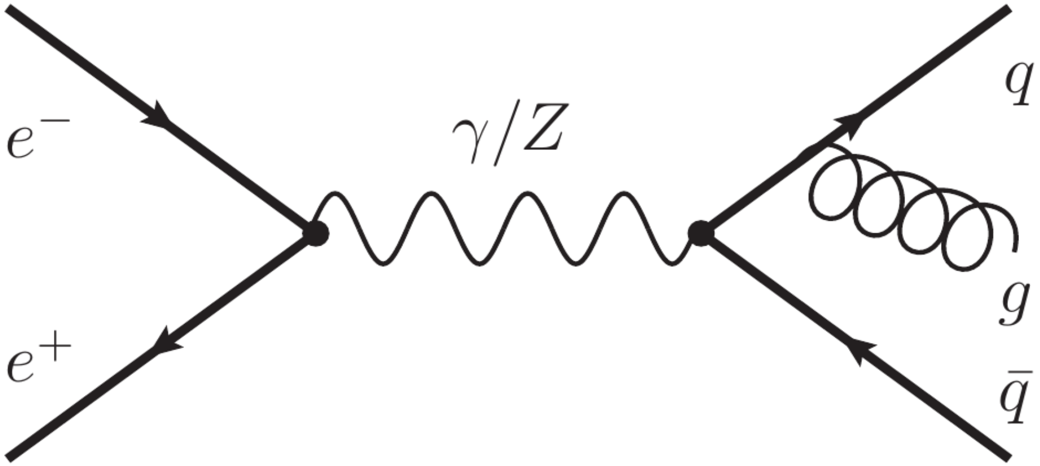
Ellis, Gaillard, Ross, Graham, Nucl. Phys. B 111 (1976) 253.

## Theory concerns:

- Small-angle and soft emissions → huge corrections
  - Real singularities must be cancelled against virtual corrections
- Large-angle emissions → moderate corrections, 3-jet events
- Partonic degrees of freedom: BAD, not observable
- Observables must be: insensitive to collinear or soft gluon emissions

**Real correction:**
Additional gluon emission (bremsstrahlung)

→ search for 3-"jet" events

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  - $x_1$ and $x_2 \to 1$: soft gluon $E_g \to 0$

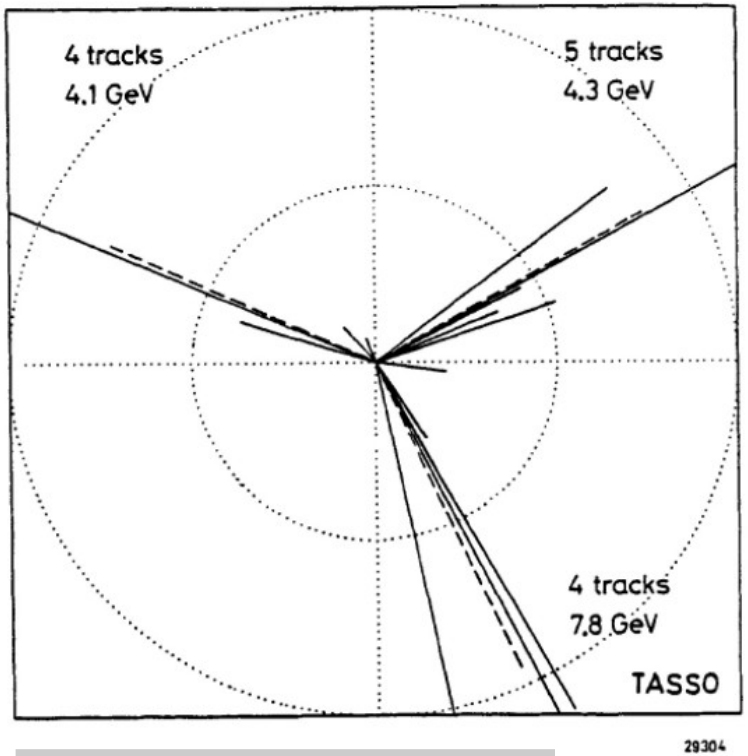
Ellis, Gaillard, Ross, Graham, Nucl. Phys. B 111 (1976) 253.

# Gluon discovery 1979

## Experimental concerns:

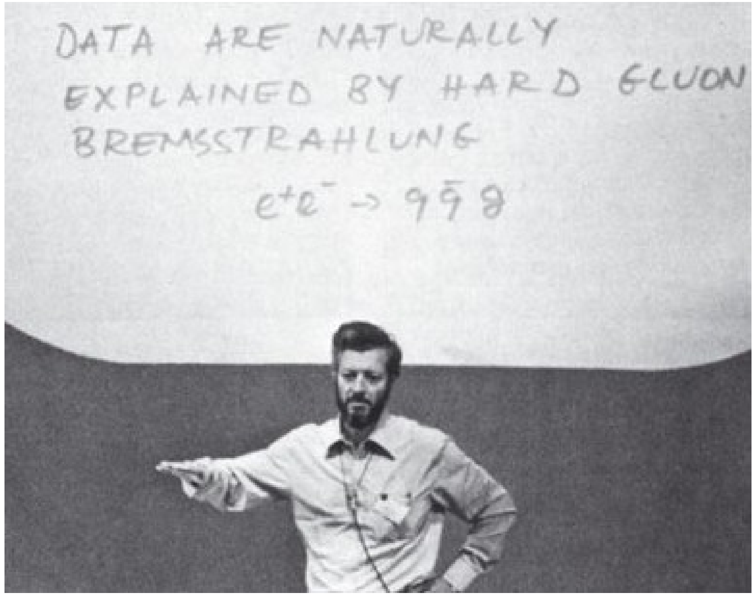
- Must have large enough cms energy: PETRA: 13, 17, 27.4 GeV
- Must have algorithm to identify candidate events

**First "3-jet" event from TASSO presented by B. Wiik at Neutrino '79 in Bergen**




50 years of QCD arXiv:2212.11107:

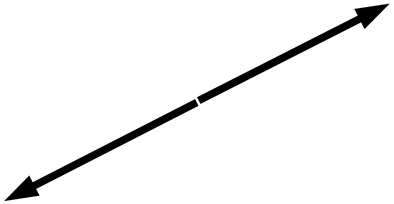
**Later confirmed by measurements also from JADE, Mark J and PLUTO i.a. using event shapes**



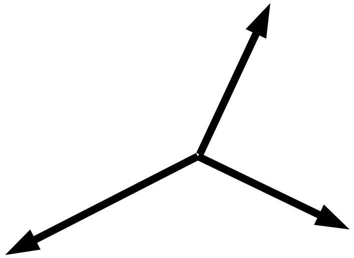

G. Wolf: talk at Lepton-Photon 1979

J. Ellis: CERN Courier Volume 49, Number 6, July-August 2009

# Jets in OPAL

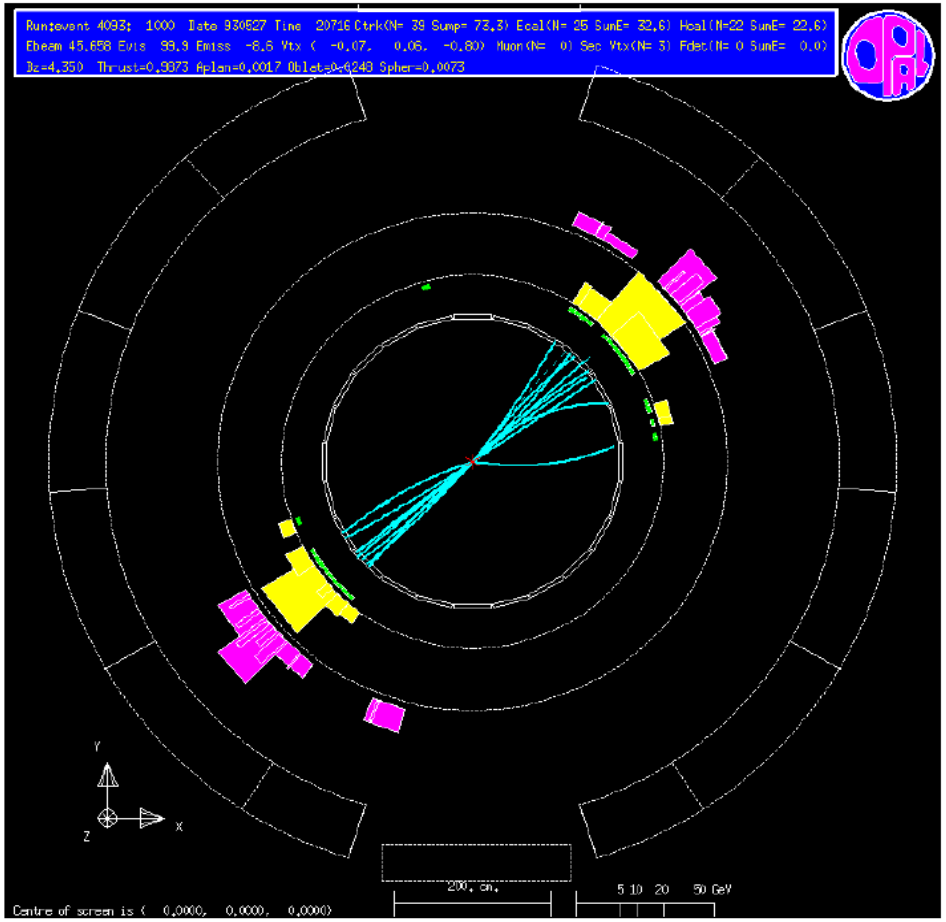


$$\sqrt{s} = 90\,\mathrm{GeV}$$

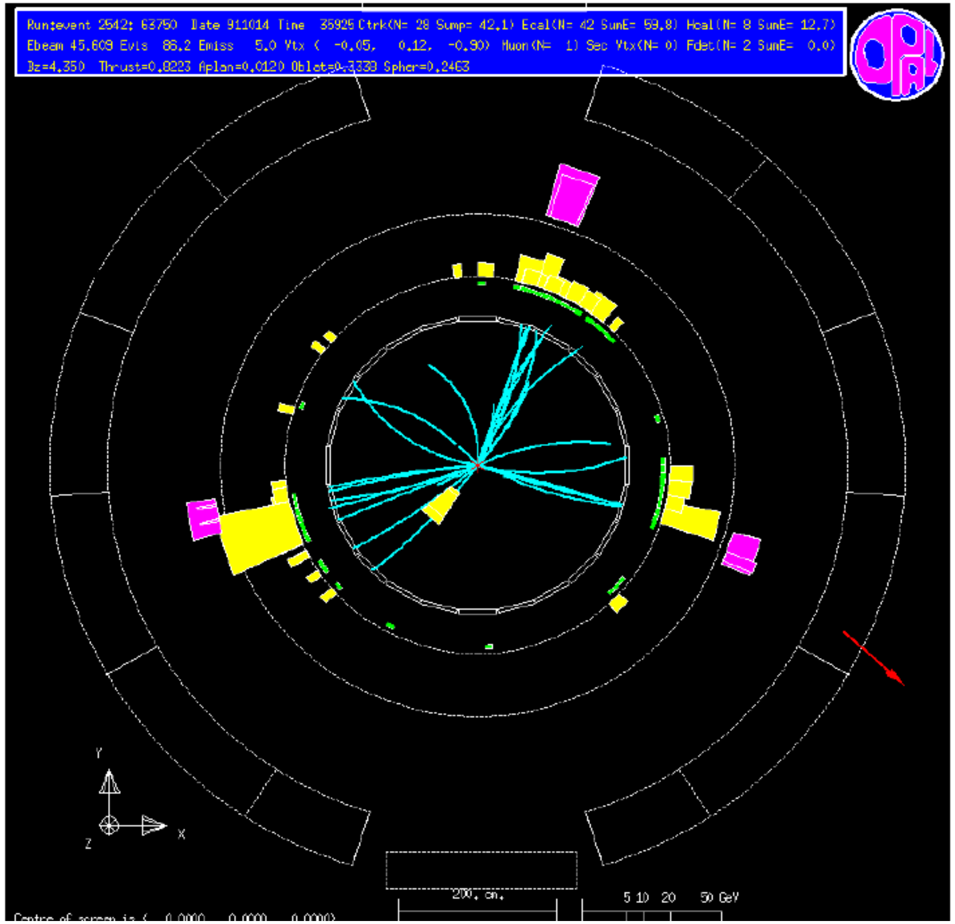


**qq 2-jet event**

**qqg 3-jet event?**




$= 90$

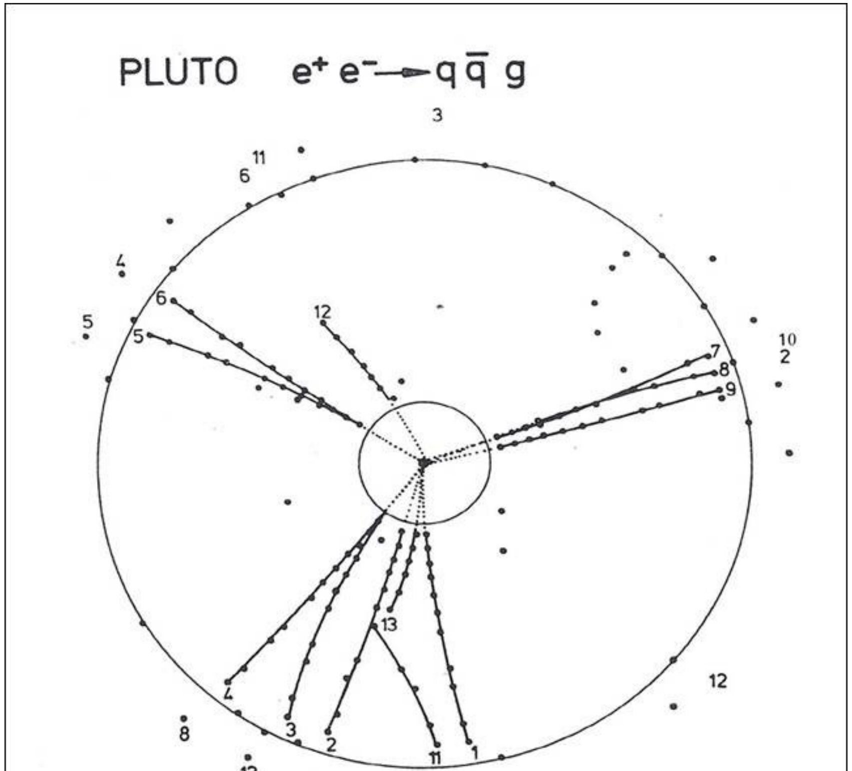


# 3-Jet events 1979 – 2010

**Jets even clearer visible … but what exactly is part of a jet?**

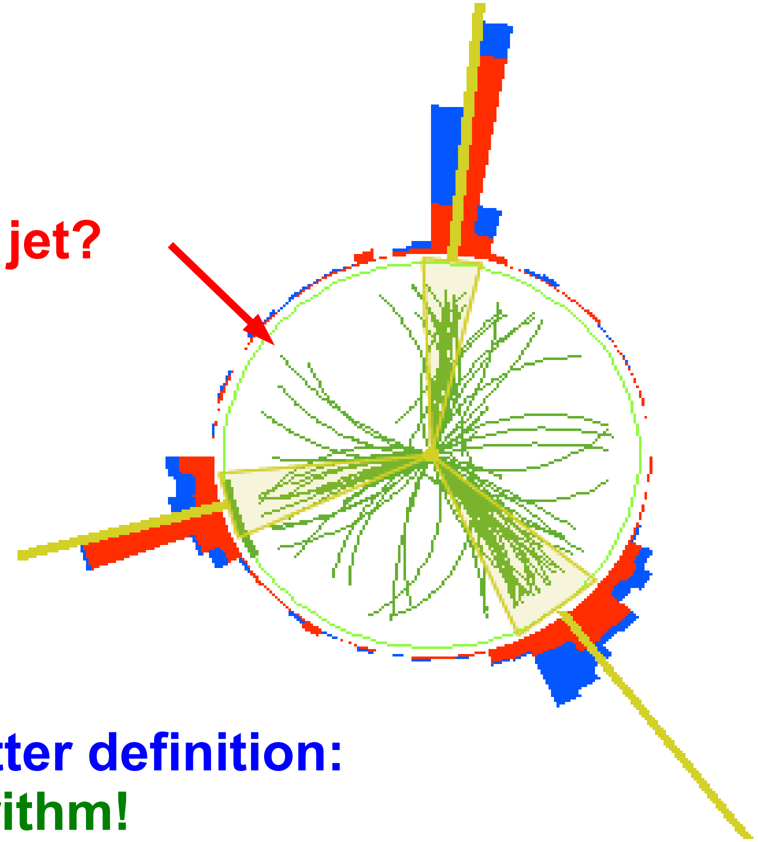
**PLUTO,1979**
**$e^+e^-$,$\sqrt{s}$ = 30 GeV**
**Multiplicity ~ 10**

**CMS, 2010**
**pp,$\sqrt{s}$ = 7000 GeV**
**Multiplicity ~ 100**




PLUTO(1979).

**Which jet?**

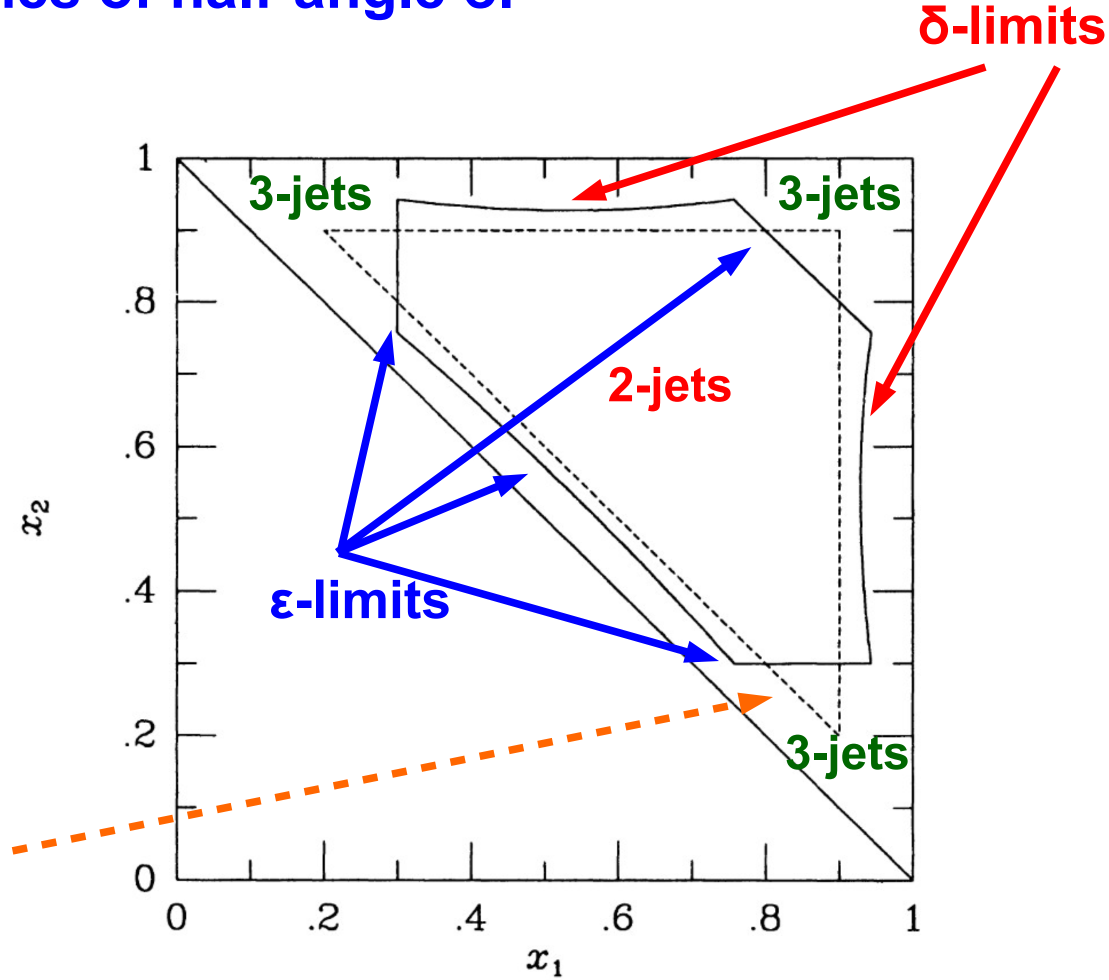


**Need better definition:**
**Jet algorithm!**

# First "jet" definition: $e^+e^-$

**Sterman and Weinberg 1977:**

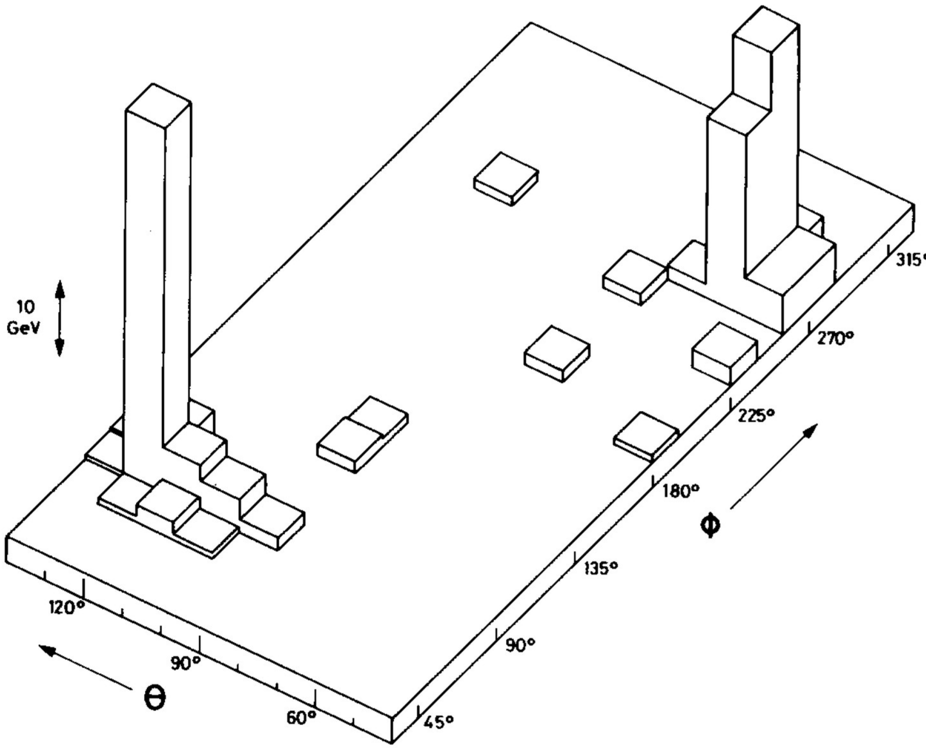
- **A final state is classified as a 2-jet event, if all but a fraction ε of the total energy is contained in a pair of cones of half-angle δ.**
  - **Minimal energy criterion: ε**
    **Jet cone size: δ**
- **Collinear and infrared safe**
- **Impractical for multi-jet states**
  - **JADE algorithm**




Ellis, Sterling, Webber

## Di-jet event with clearly separated energy depositions

'Jet algorithm' based on cell structure of the calorimeters (UA1 & UA2)
UA1 later also cone algorithm!



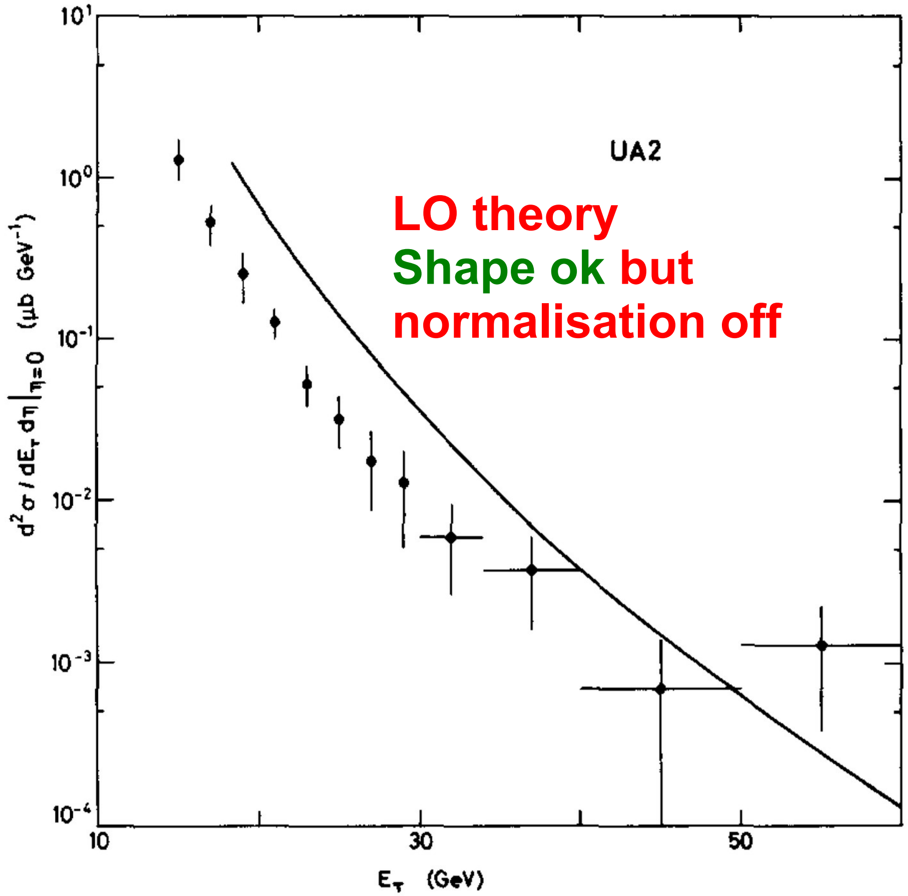
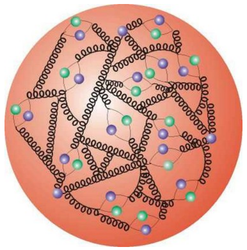



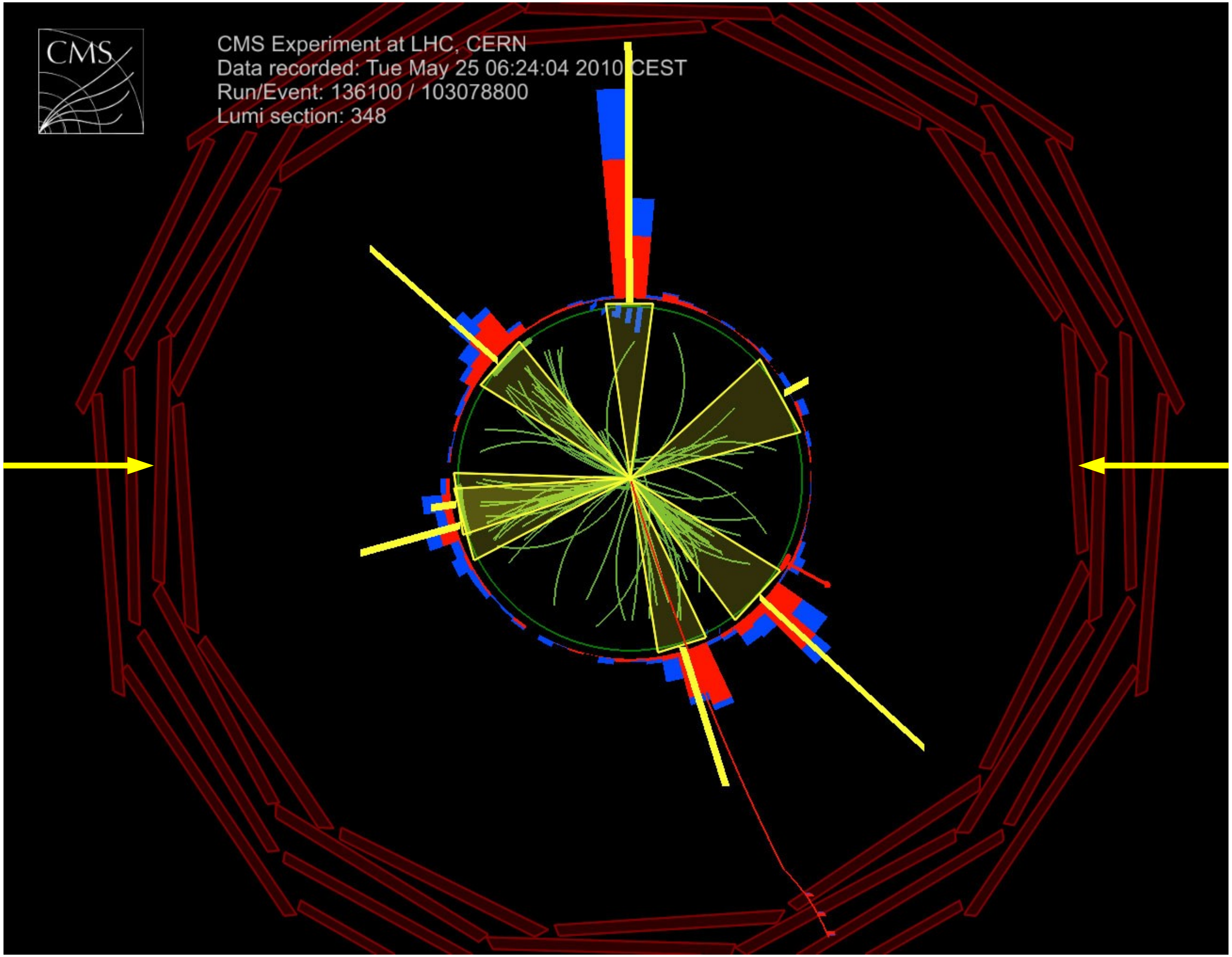

Fig. 6. Inclusive jet production cross section. The solid line (ref. [6]) uses $\Lambda$ = 0.5 GeV while $\Lambda$ = 0.15 GeV would bring the calculated rates in better agreement with the data. However various uncertainties preclude a determination of $\Lambda$ from the data [13].

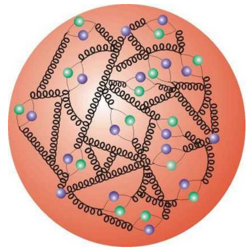
UA2, PLB 118 (1982).

# More bundles of particles



Proton



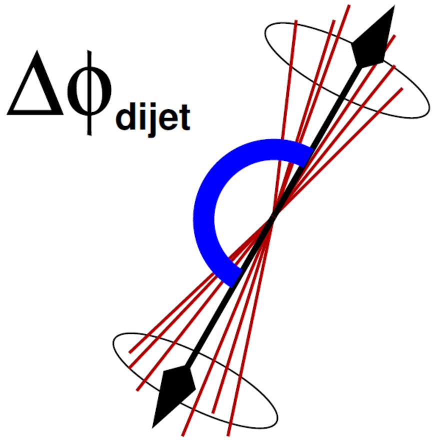



Proton

# $1^{st}$ cone algorithm

- **UA1 Collaboration at CERN SppS, PLB 123 (1982) 115:**
  - **Cluster algorithm around cells with more than 2.5 GeV energy ('seed')**
  - **Distance criterium in (pseudo-)rapidity and azimuthal angle wrt. cell (or jet) → cone in (Φ,η) space**
  - **4-vector addition to combine**
  - **Further criteria to add less energetic cells**

$$R = \sqrt{(\Delta\phi)^2 + (\Delta\eta)^2}$$




M. Wobisch

# JADE algorithm

- **JADE Collaboration, ZPhysC 33 (1986) 23:**
  - **Algorithm with sequential recombination**
  - **For $e^+e^-$ → no treatment of proton remainder**
  - **1. Define metric for distance between two objects i and j via their 4-vectors**
  - **2. Calculate the distances for all pairwise combinations i, j**
  - **3. Compare the smallest distance to a threshold $y_{cut}$**
  - **4. If smaller → combine both objects i, j to a new one → iterate step 2**
  - **5. If larger → stop algorithm and declare all remaining 4-vectors to jets!**

$$y_{ij}^{\mathrm{J}} = \frac{2E_iE_j(1-\cos(\theta_{ij}))}{E_{\mathrm{vis}}^2}$$

$$y_{i,j;\mathrm{min}} < y_{\mathrm{cut}}$$

**Not unsafe, but soft wide-angle radiation attributed to the same JADE jet!**
**Leads to:**
**- larger hadronisation corrections**
**- not resummable**

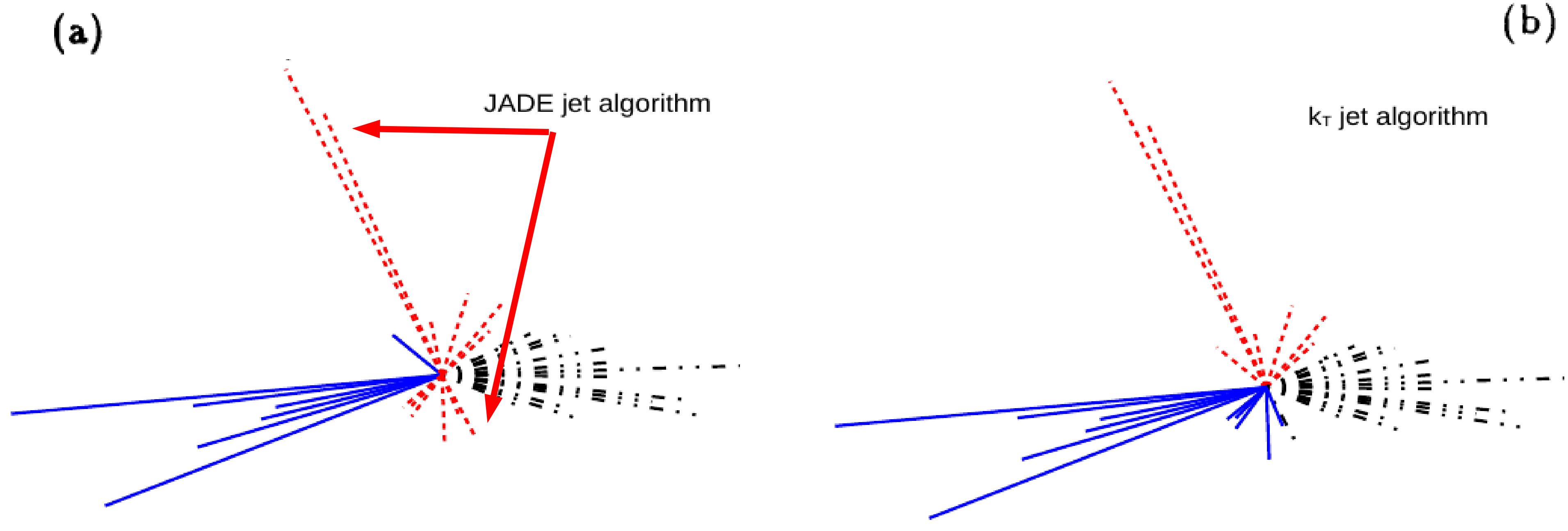


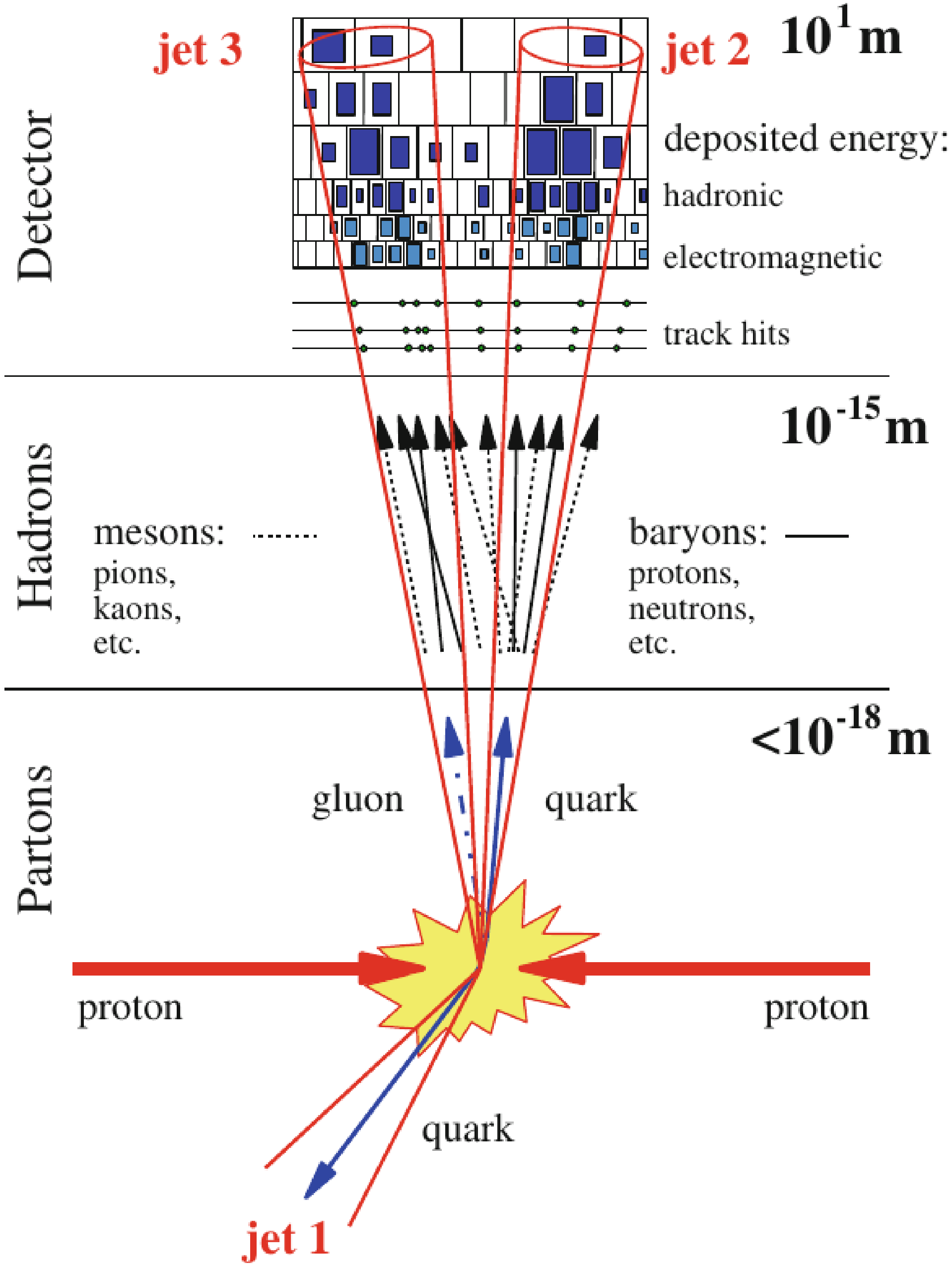

Figure 4: A three-jet final state and the assignment of particles to the first (solid), second (dotted) and third (dashed) jets according to the (a) JADE and (b) $k_{\perp}$ algorithms.

# $k_T$/Durham algorithm

- **Catani, Dokshitzer, Olsson, Turnock, Webber, PLB 269 (1991) 432:**
  - **Algorithm with sequential recombination**
  - **For $e^+e^-$ → no treatment of proton remainder**
  - **1. Define metric for distance between two objects i and j via their 4-vectors**
  - **2. Calculate the distances for all pairwise combinations i, j**
  - **3. Compare the smallest distance to a threshold $y_{cut}$**
  - **4. If smaller → combine both objects i, j to a new one → iterate step 2**
  - **5. If larger → stop algorithm and declare all remaining 4-vectors to jets!**

$$y_{ij}^{\mathrm{k_T}} = \frac{2\min(E_i^2, E_j^2)(1-\cos(\theta_{ij}))}{E_{\mathrm{vis}}^2} \qquad y_{i,j;\mathrm{min}} < y_{\mathrm{cut}}$$

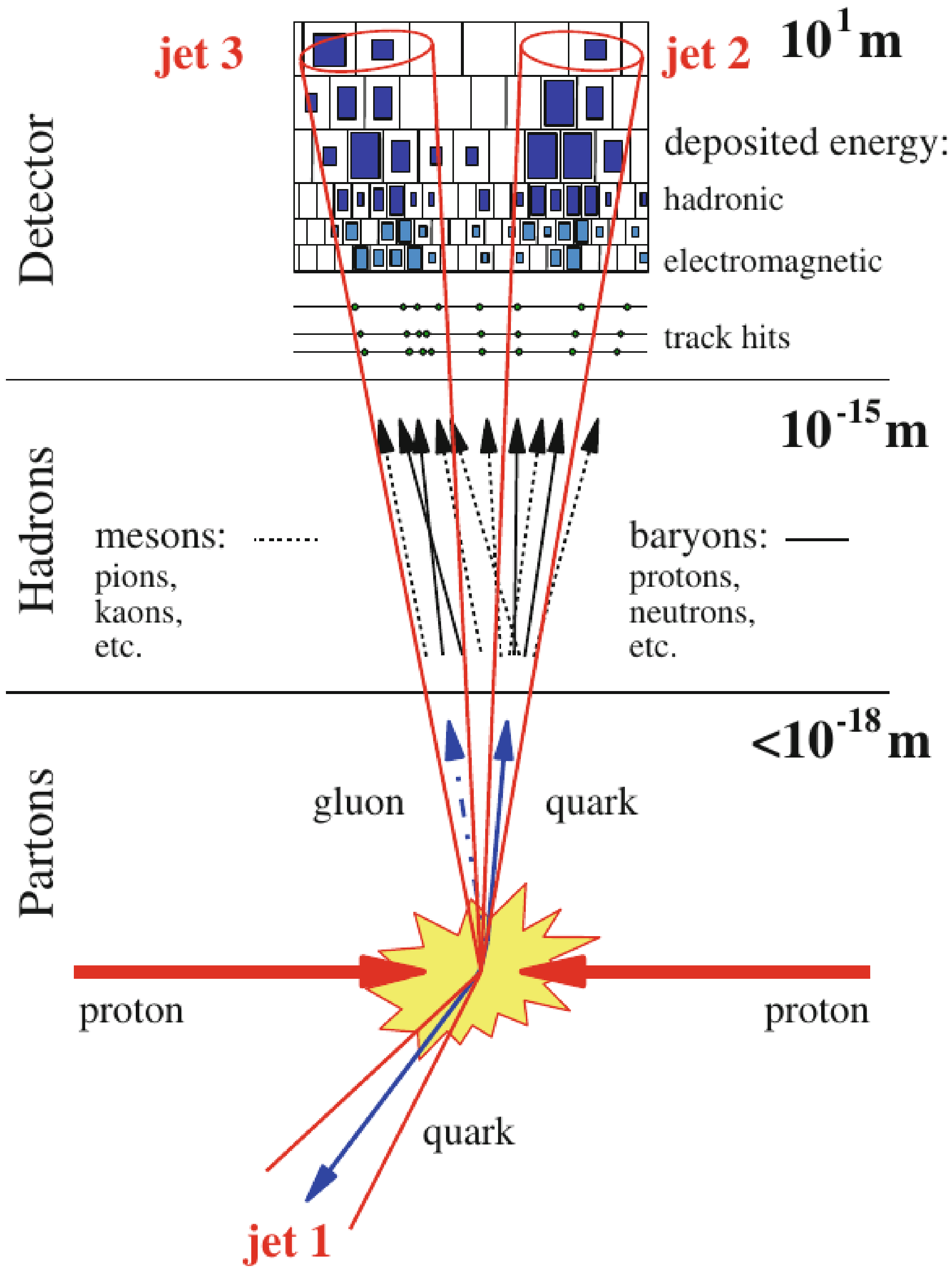
# Tools in particle physics: Jets



# Jet algorithms

## Primary goal:

**Good correspondence among:**
- Detector measurements
- Particles in final state and
- "hard" partons




**Two classes of algorithms:**

1. **Cone algorithms:** "Geometrical" attribution of objects to the direction of largest energy flow in an event (First choice at hadron colliders)
2. **Sequential recombination:** Iterated combination of closest neighbors among all pairs of objects (First choice at $e^+e^-$ & ep colliders)

# Desiderata – Theory

- **Jet Algorithm Desiderata (Theory):**
  - Infrared safety
  - Collinear safety
  - Longitudinal boost invariance
    (recombination scheme!)
  - Boundary stability
    (→ 4-vector addition, rapidity y)
  - Order independence
    (parton, particle, detector)
  - Ease of implementation
    (standardized public code?)

See also:
"Snowmass Accord", FNAL-C-90-249-E
Tevatron Run II Jet Physics, hep-ex/0005012
Les Houches 2007 Tools and Jets Summary , arXiv:0803.0678

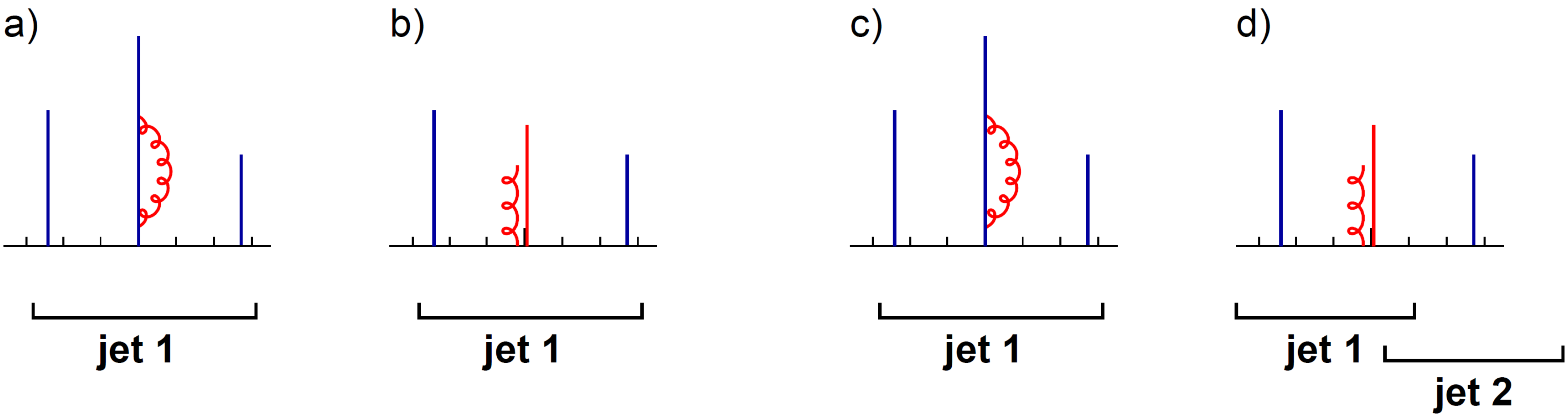
# Desiderata – Experiment

- **Jet Algorithm Desiderata (Experiment):**
  - **Computational efficiency and predictability**
    **(use in trigger?, reconstruction times?)**
  - **Maximal reconstruction efficiency**
    **(no dark jets)**
  - **Minimal resolution smearing and angular biasing**
  - **Insensitivity to pile-up**
    **(mult. collisions at high luminosity ...)**
  - **Ease of calibration**
  - **Detector independence**
  - **Fully specified**
    **(details?, code?)**
  - **Ease of implementation**
    **(standardized public code?)**

# Collinear safety

## Collinear safe jet alg.

## Collinear unsafe jet alg




$\alpha_s^n \times (-\infty)$ $\alpha_s^n \times (+\infty)$ $\alpha_s^n \times (-\infty)$ $\alpha_s^n \times (+\infty)$

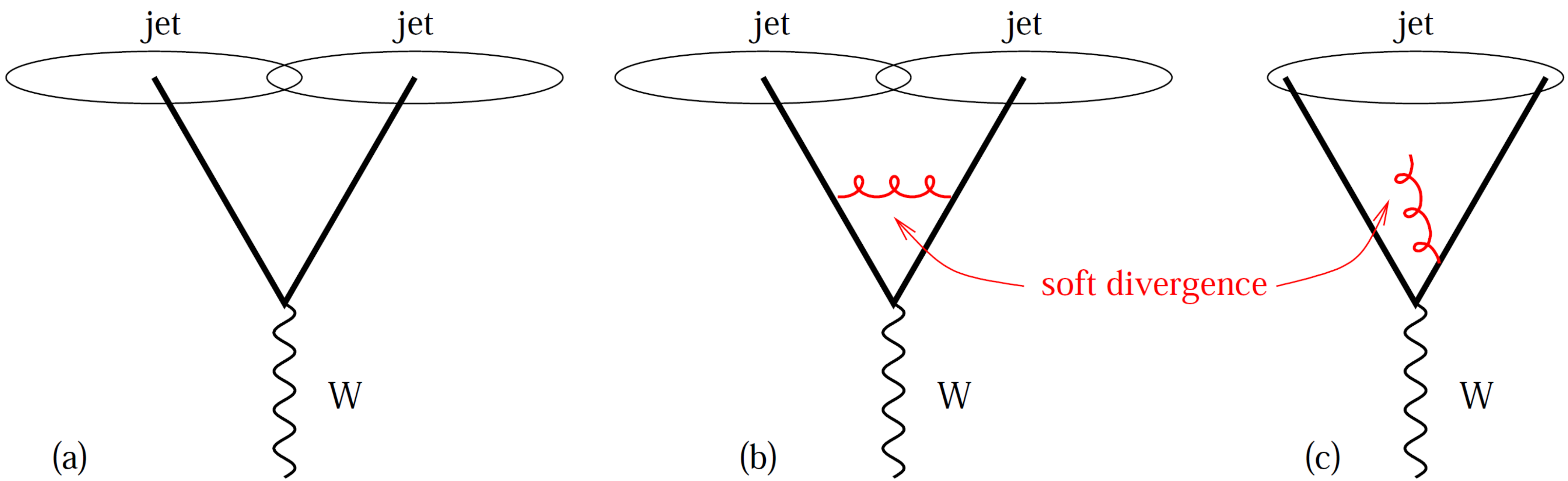
**Infinities cancel** **Infinities do not cancel**

Jetography, G. Salam, EPJC 67 (2010) 637.

# Infrared safety

**Iterative cone with Split/Merge:**

→ **not all objects end in jets, e.g. if no starting cone close by (dark Jets)**
→ **collinear unsafe because of minimal pT on cone seeds**
→ **infrared unsafe ...**




**Trial to fix issue: MidPoint Cone → Investigate add. all middle points between seeds**
→ **also unsafe, becomes apparent only for more complex topology**
**Discovered rather late: Real safe algorithm Seedless Infrared-Safe Cone (SISCone)**
→ **rarely used because of 2 orders of magnitude larger computing needs**

Jetography, G. Salam, EPJC 67 (2010) 637.