Soft-gluon resummations and NNNLO expansions

Nikolaos Kidonakis
(Kennesaw State University)

- **Higher-order two-loop corrections**
- **NNLL soft-gluon resummation**
- **Comparison of resummation methods**
- **NNLO and NNNLO expansions**
- **Top-pair and single-top production**
- W **production at large** p_T

Higher-order corrections

Soft-gluon corrections are important in many QCD processes

production of top-antitop, single top, W, jets, direct photons, etc.

Soft terms: $\left[\frac{\ln^k(s_4/M^2)}{s_4}\right]_+$ **with** $k \le 2n-1$, s_4 **distance from threshold**

Resum these soft corrections - factorization and RGE

Complete results at NNLL–two-loop soft anomalous dimensions

Approximate NNLO cross section from expansion of resummed cross section

Calculation is for partonic threshold at the double differential cross section level using the standard moment-space resummation in pQCD

Recent results for:

top-pair production, N. Kidonakis, 1304.7775 [hep-ph]

single-top production, N. Kidonakis, 1306.3592 [hep-ph]

W-production, N. Kidonakis and R.J. Gonsalves, Phys. Rev. D 87, 014001 (2013)

Factorization and Resummation

Resummation follows from factorization properties of the cross section - performed in moment space

$\sigma = (\prod \psi)\, H_{IL}\, S_{LI}\, (\prod J)$ H**: hard function** S**: soft-gluon function**

Use RGE to evolve soft-gluon function

$$\left(\mu \frac{\partial}{\partial \mu} + \beta(g_s)\frac{\partial}{\partial g_s}\right) S_{LI} = -(\Gamma_S^\dagger)_{LB} S_{BI} - S_{LA} (\Gamma_S)_{AI}$$

Γ_S **is the soft anomalous dimension - a matrix in color space and a function of kinematical invariants s, t, u**

Resummed cross section

$$\hat{\sigma}^{res}(N) = \exp\left[\sum_i E_i(N_i)\right] \exp\left[\sum_j E'_j(N')\right] \exp\left[\sum_{i=1,2} 2\int_{\mu_F}^{\sqrt{s}} \frac{d\mu}{\mu}\, \gamma_{i/i}\left(\tilde{N}_i, \alpha_s(\mu)\right)\right]$$
$$\times\, \mathrm{tr}\left\{ H\left(\alpha_s\right) \exp\left[\int_{\sqrt{s}}^{\sqrt{s}/\tilde{N}'} \frac{d\mu}{\mu}\, \Gamma_S^\dagger\left(\alpha_s(\mu)\right)\right] S\left(\alpha_s\left(\frac{\sqrt{s}}{\tilde{N}'}\right)\right) \exp\left[\int_{\sqrt{s}}^{\sqrt{s}/\tilde{N}'} \frac{d\mu}{\mu}\, \Gamma_S\left(\alpha_s(\mu)\right)\right]\right\}$$

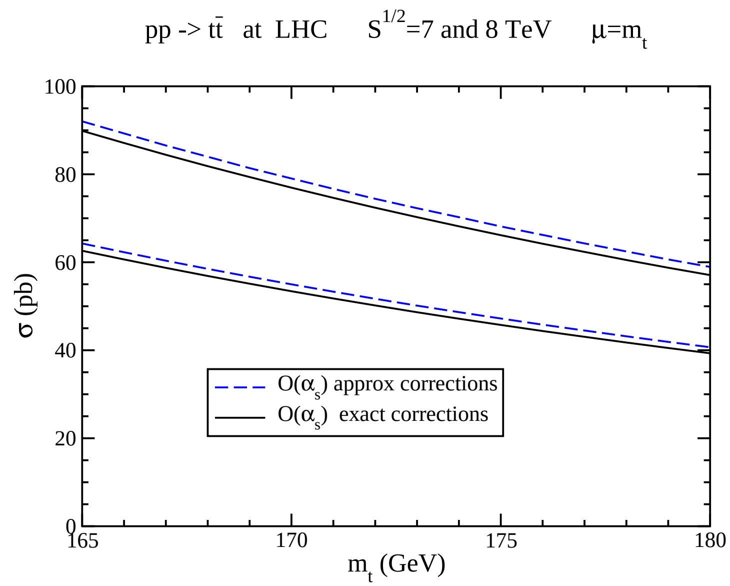
determine Γ_S from ultraviolet poles in dimensionally regularized eikonal diagrams

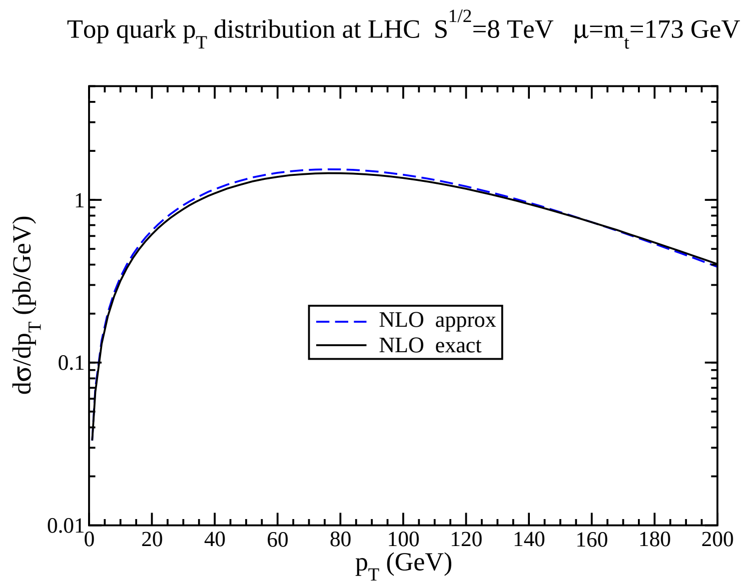
Γ_S **is process-dependent; calculated at two loops**

We are resumming $\ln^k N$ - we can expand to fixed order and invert to get $\ln^k(s_4/m_t^2)/s_4$

Threshold approximation

Approximation works very well for LHC and Tevatron energies





excellent approximation:
$\sim$1% difference between NLO approximate and exact cross sections;
and also for differential distributions;
also true at NNLO for total cross sections

For best prediction for differential distributions add NNLO approximate corrections to exact NLO result

Differences between various resummation/NNLO approx approaches

Total vs differential cross section
moment-space pQCD vs SCET

Name	Observable	Soft limit
single-particle-inclusive (1PI)	$d\sigma/dp_T dy$	$s_4 = s + t_1 + u_1 \to 0$
pair-invariant-mass (PIM)	$d\sigma/dM_{t\bar{t}} d\theta$	$(1-z) = 1 - M_{t\bar{t}}^2/s \to 0$
production threshold	σ	$\beta = \sqrt{1 - 4m_t^2/s} \to 0$

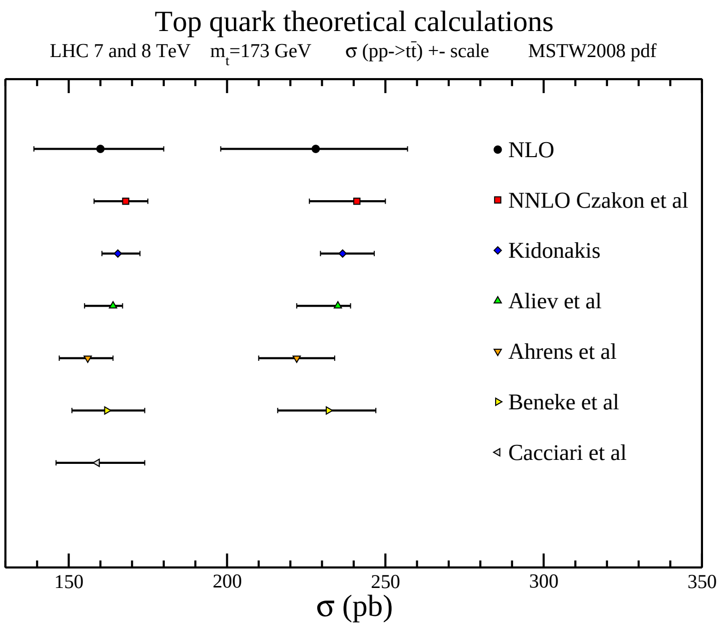
The more general approach is double-differential
$\to p_T$ and rapidity distributions

total-only approaches are limit/special case (absolute vs partonic threshold)

For differential calculations, further differences arise from how the relation $s + t_1 + u_1 = 0$ is used in the plus-distribution coefficients, how subleading terms are treated, damping factors, etc.

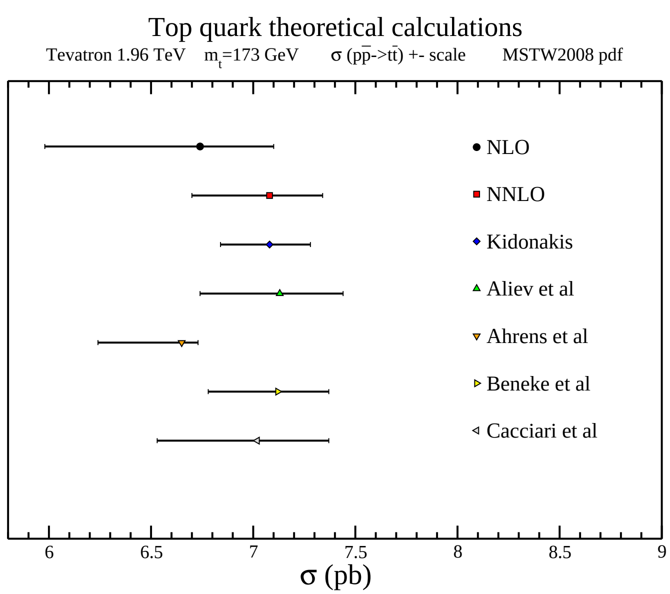
see N. Kidonakis and B.D. Pecjak, Eur. Phys. J C 72, 2084 (2012)

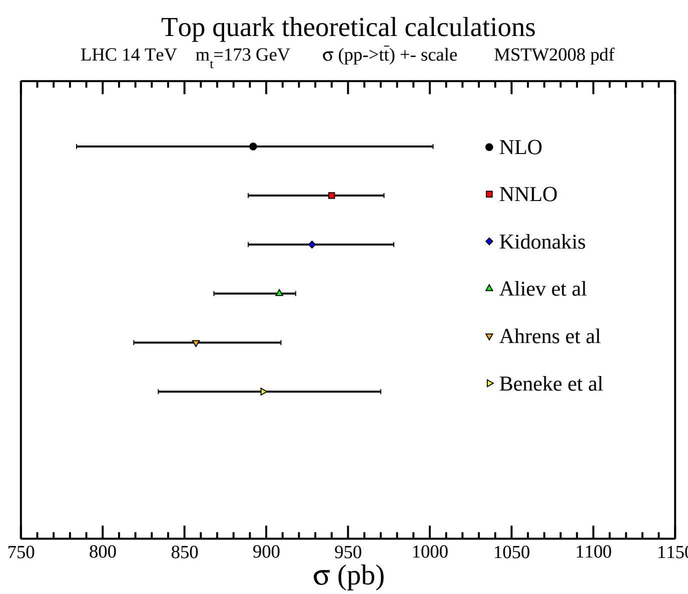
for details and review



Comparison of various NNLO approx approaches
all with the same choice of parameters

Kidonakis, PRD 82, 114030 (2010) differential-pQCD
Aliev *et al*, CPC 182, 1034 (2011) total-pQCD
Ahrens *et al*, PLB 703, 135 (2011) differential -SCET
Beneke *et al*, NPB 855, 695 (2012) total-SCET
Cacciari *et al*, PLB 710, 612 (2012) total-pQCD





The result from my formalism is very close to the exact NNLO:
both the central values and the scale uncertainty are nearly the same
true for all collider energies and top quark masses

This was expected from comparison to NLO, and analytical/numerical study of NNLO corrections in different kinematics

(PRD 68, N. Kidonakis & R. Vogt; see also discussion in PRD78 and PRD82)

$\sim$1% difference between approximate and exact cross sections at both NLO and NNLO

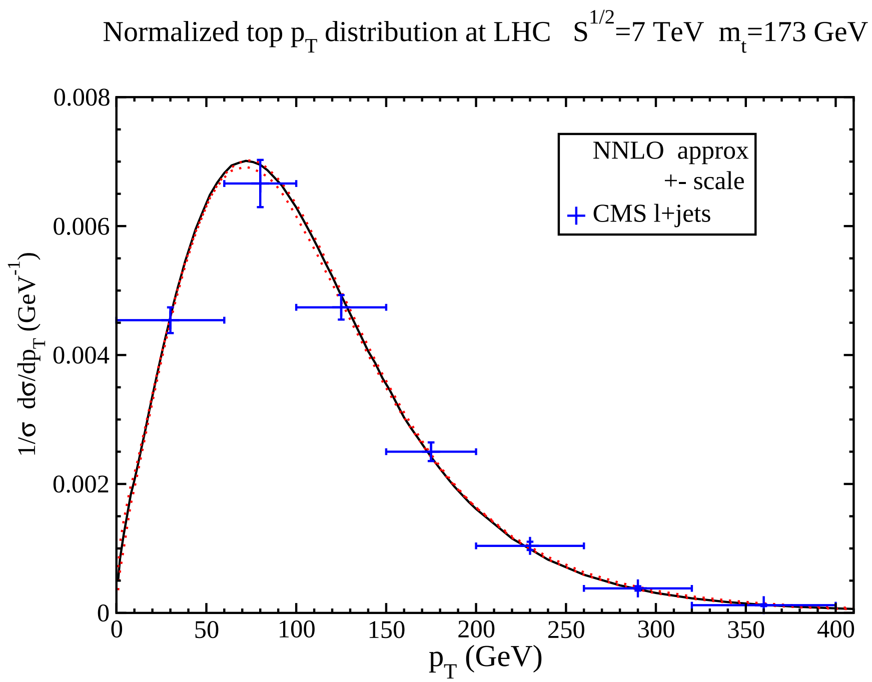
working on approximate NNNLO (see N. Kidonakis PRD 73,034001 (2006) for early NNNLO results for top-pair; small NNNLO corrections for $q\bar{q} \to t\bar{t}$; also past work NNNLO for single-top at NLL)

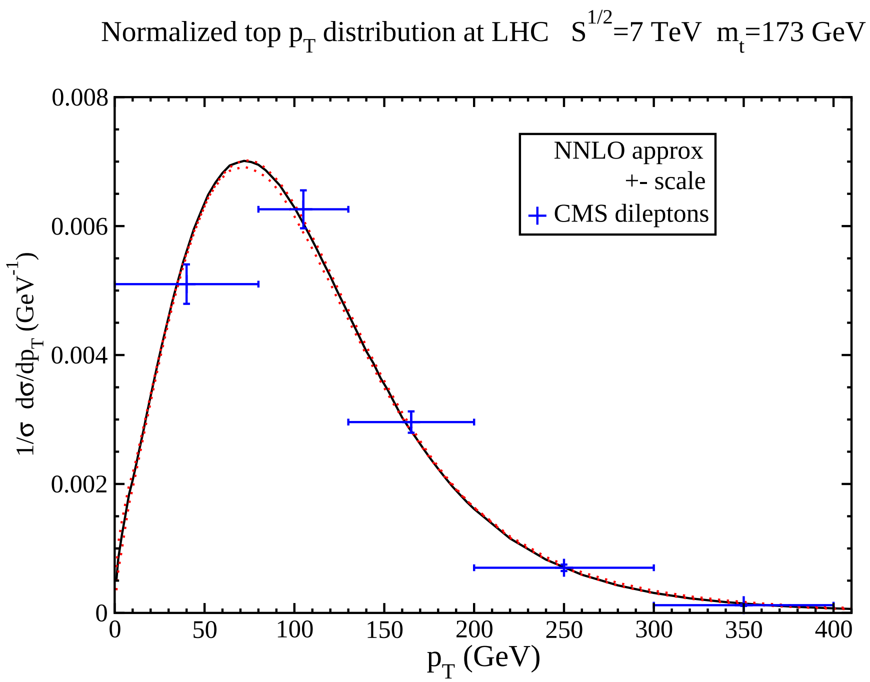
stability of the theoretical NNLO approx result over the past decade

the reliability of the NNLO approximate result and near-identical value to exact NNLO is very important for several reasons

- provides confidence of application to other processes (single-top, W, etc)
- results used as background for many analyses (Higgs, etc)
- means that we have near-exact NNLO p_T and rapidity distributions

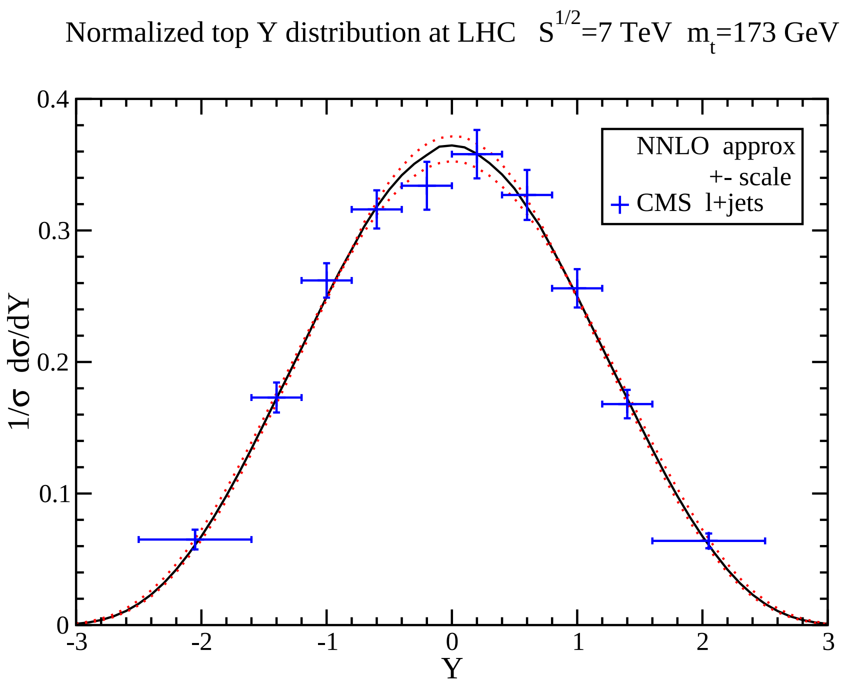
Normalized top quark p_T distribution at the LHC

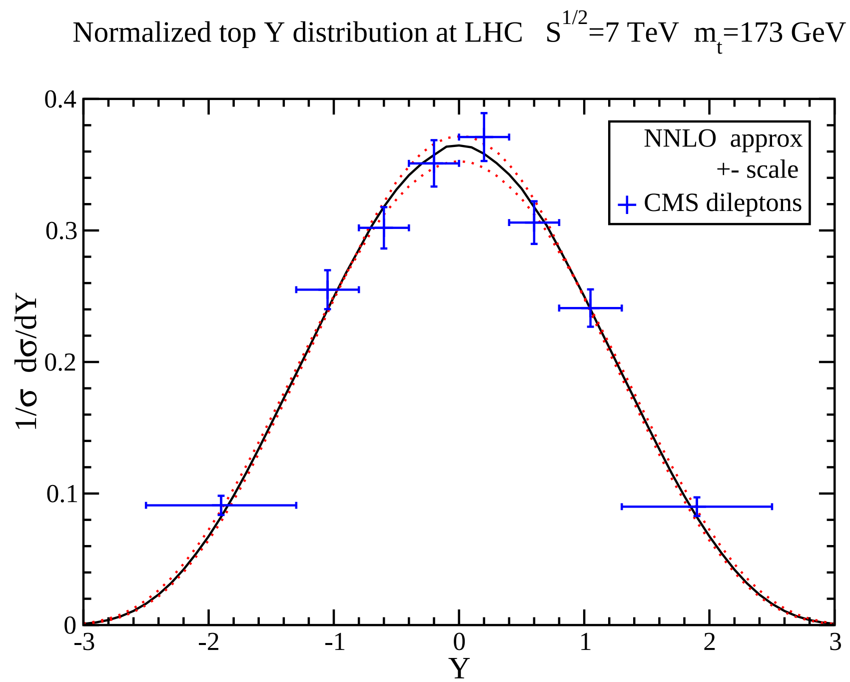




Excellent agreement with CMS data at 7 TeV; also at 8 TeV

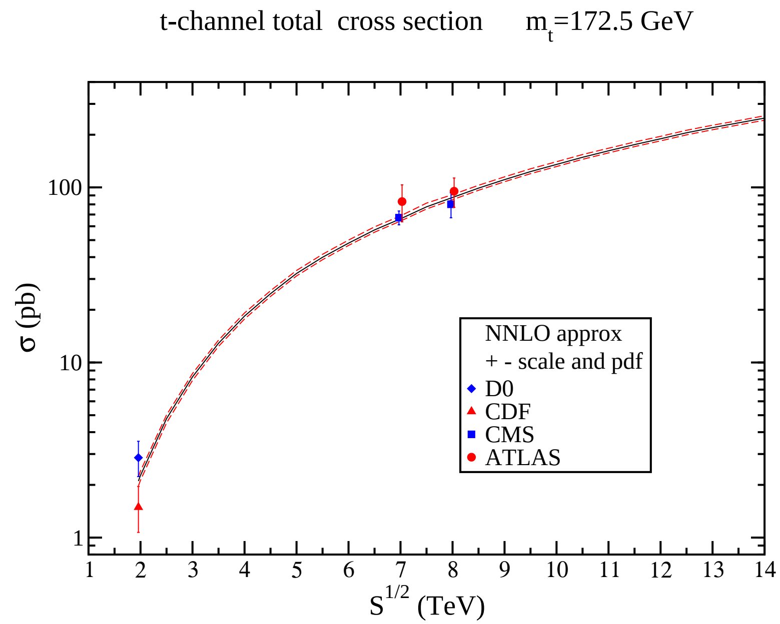
Normalized top quark rapidity distribution at LHC

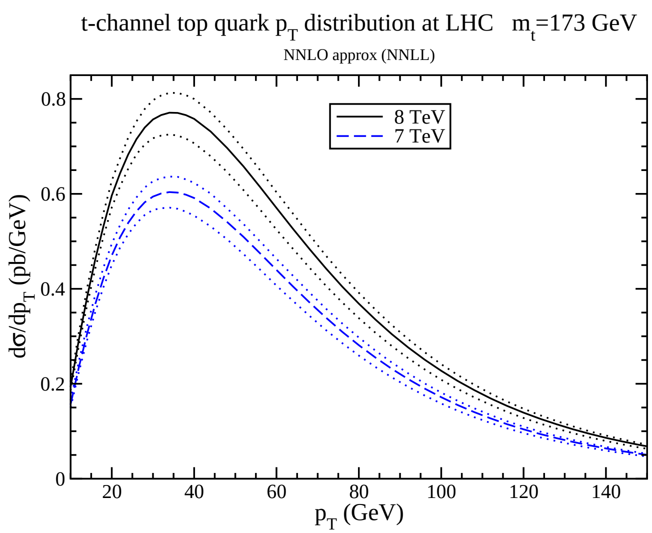


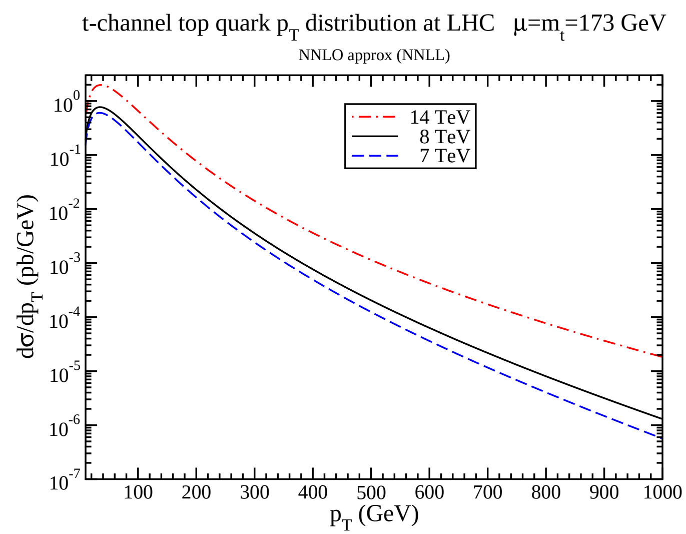


Excellent agreement with CMS data at 7 TeV; also at 8 TeV

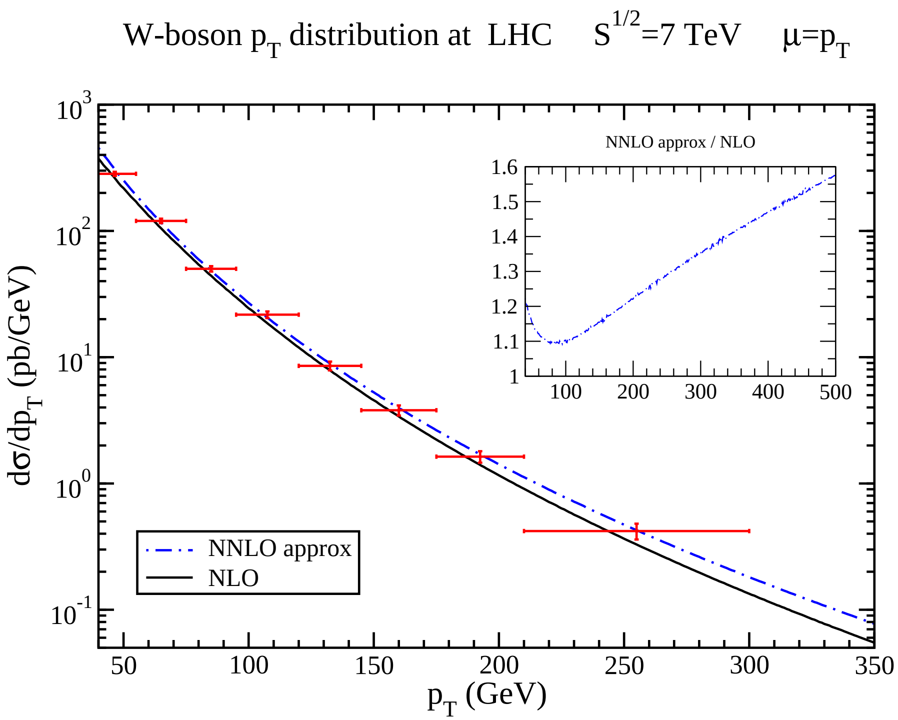
t-channel single-top production at the LHC

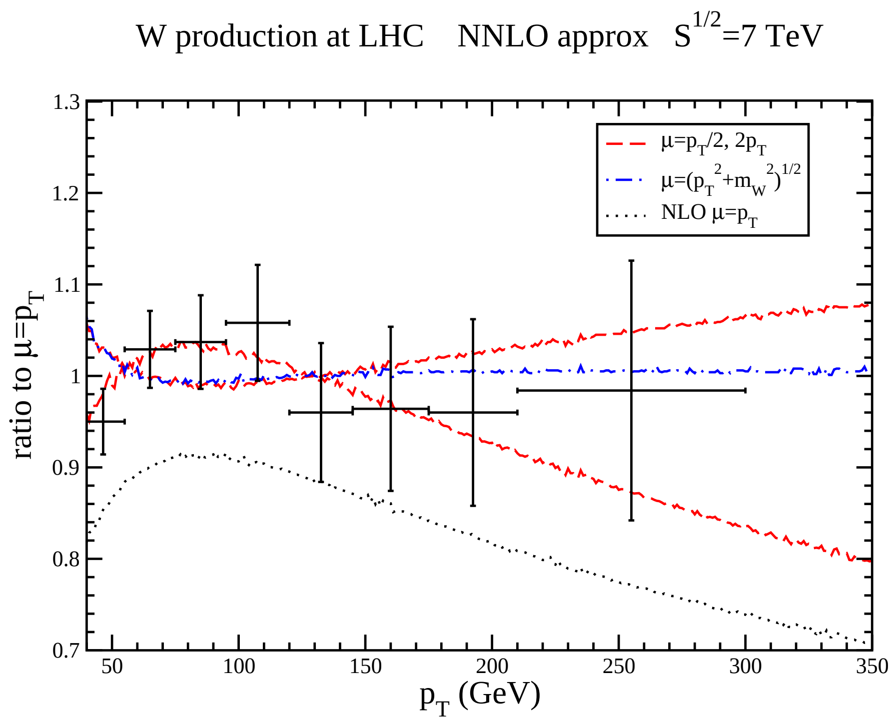






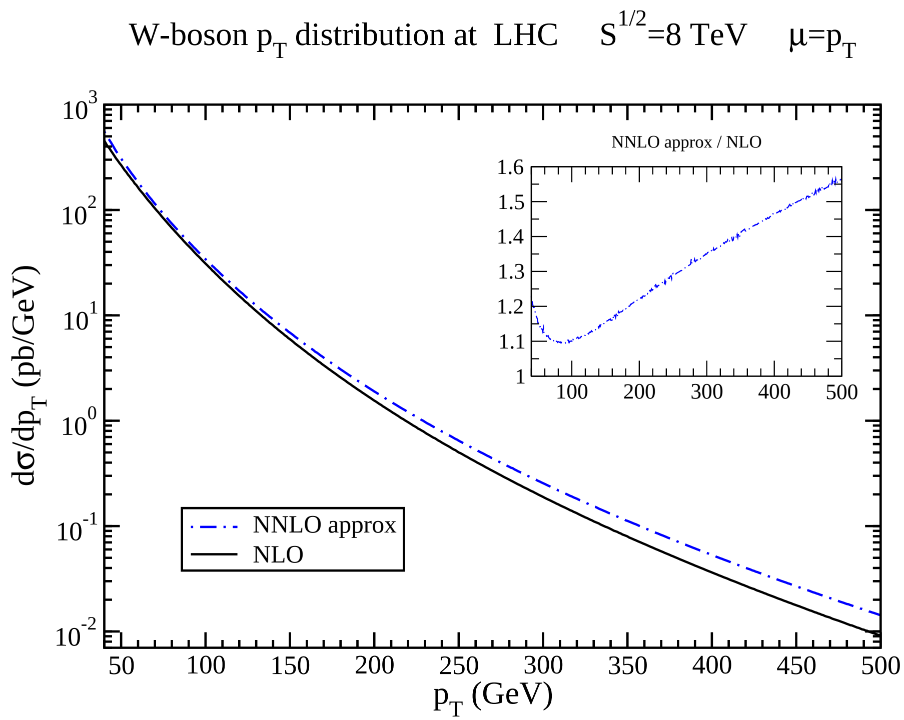
W production at large p_T at the LHC - 7 TeV

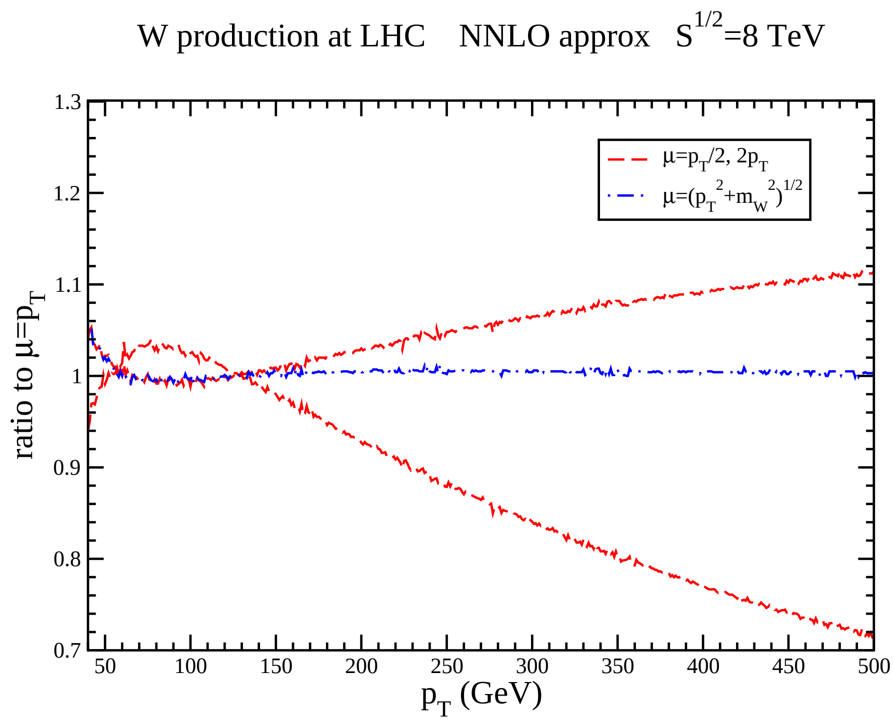




N. Kidonakis and R.J. Gonsalves, Phys. Rev. D 87, 014001 (2013)

W production at large p_T at the LHC - 8 TeV





N. Kidonakis and R.J. Gonsalves, Phys. Rev. D 87, 014001 (2013)

Summary

- **Soft-gluon resummations at NNLL**
- **NNLO and NNNLO expansions**
- **applications to top-quark and W-boson production**
- **NNLO approx corrections are significant at the LHC and the Tevatron**
- **excellent agreement with collider data**