α_S from F_π and Renormalization Group Optimized Perturbation

Jean-Loïc Kneur (LCC Montpellier, France)

in collaboration with André Neveu

based on arXiv:1305.6910

EPS-HEP 2013, Stockholm

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1. Introduction/Motivations

For $m_{quarks} \to 0$, $\alpha_S(\mu)$ [equiv. $\Lambda_{\overline{\text{MS}}} \sim \mu\, e^{-\frac{1}{\beta_0 \alpha_S}}$ (higher orders)] is the only fundamental QCD parameter.

$\alpha_S(m_Z)$ known with impressive accuracy:
PDG 2012 World average: $\alpha_S(m_Z) = .1184 \pm .0007$ from

many different determinations (jets, DIS, Z, τ-decay [previous talks], $e^+e^- \to hadrons$, lattice,...)

Still, worth to get $\Lambda_{\overline{\text{MS}}}$ ($\alpha_S(Q^2)$) from further independent analyses/methods, specially for $n_f = 2$ and/or in infrared range not perturbatively extrapolable from high scale

Our more general goal: get approximations (of reasonable accuracy?) to chiral sym. breaking order parameters from a different (optimized) use of perturbation...(α_S a by-product)

Chiral Symmetry Breaking (χSB) Order parameters

Usually considered hopeless from standard perturbation:

1. $\langle \bar{q}q \rangle^{1/3}$, F_π,.. $\sim \mathcal{O}(\Lambda_{QCD}) \simeq$ 100–300 MeV
$\rightarrow \alpha_S$ (a priori) large $\rightarrow$ invalidates pert. expansion

2. $\langle \bar{q}q \rangle$, F_π,.. perturbative series $\sim (m_q)^d \sum_{n,p} \alpha_s^n \ln^p(m_q)$
vanish for $m_q \rightarrow 0$ at any pert. order (trivial chiral limit)

3. More sophisticated arguments e.g. (infrared) renormalon ambiguities (signature of (factorially) divergent pert. expansion)

All seems to tell that χSB parameters are intrinsically NP

•Optimized pert. (OPT): appear to circumvent at least 1., 2., and may give more clues to pert./NP bridge

2. (Variationally) Optimized Perturbation (OPT)

$$\mathcal{L}_{QCD}(g, m_q) \to \mathcal{L}_{QCD}(\delta^{\frac{1}{2}} g, m(1-\delta)) \quad (\alpha_S \equiv g^2/(4\pi))$$

δ interpolate between $\mathcal{L}_{free}$ and $\mathcal{L}_{int}$

(quark) mass $m \to$ arbitrary trial parameter

- Take any standard (renormalized) pert. series, expand in δ *after:*

$$m \to m\,(1-\delta); \quad \alpha_S \to \delta\,\alpha_S$$

then take $\delta \to 1$ (to recover original massless theory):

BUT a m-dependence remains at any finite δ^k-order:

fixed typically by optimization (OPT):

$$\frac{\partial}{\partial m}(\text{physical quantity}) = 0 \text{ for } m = m_{opt}(\alpha_S) \neq 0$$

Expect increasingly flatter m-dependence at increasing δ orders... empirically seen to be the case in various models

But does this ’cheap trick’ always work? and why?

Simpler model's support + properties

•Convergence proof of this procedure for $D = 1$ $\lambda\phi^4$ oscillator (cancels large pert. order factorial divergences!) Guida et al '95

particular case of 'order-dependent mapping' Seznec+Zinn-Justin '79

(exponentially fast convergence for ground state energy $E_0 = const.\lambda^{1/3}$; good to % level at 2d δ-order)

•In renormalizable QFT, first order consistent with Hartree-Fock (or large N) approximation, + results beyond

•+ also produces factorial damping at large perturbative orders (JLK, Reynaud '2002)

('delay' infrared renormalon behaviour to higher orders)

•Flexible, Renormalization-compatible, gauge-invariant applications also at finite temperature (phase transitions beyond mean field approx in 2D, 3D models, QCD...)

3. RG improved OPT (RGOPT)

Our main new ingredient (JLK + A. Neveu PRD 81, 125012): Consider a physical quantity (perturbatively RG invariant), e.g. pole mass M:

in addition to OPT Eq: $\frac{\partial}{\partial m} M^{(k)}(m, g, \delta = 1)|_{m \equiv \tilde{m}} \equiv 0$

Require (δ-modified!) series at order δ^k to satisfy a standard perturbative RG equation:

$$\mathrm{RG}\left(M^{(k)}(m, g, \delta = 1)\right) = 0$$

with standard RG operator:

$$\text{RG} \equiv \mu \frac{d}{d\,\mu} = \mu \frac{\partial}{\partial \mu} + \beta(g) \frac{\partial}{\partial g} - \gamma_m(g)\, m \frac{\partial}{\partial m}$$

$[\beta(g) \equiv -2b_0 g^2 - 2b_1 g^3 + \cdots\,, \quad \gamma_m(g) \equiv \gamma_0 g + \gamma_1 g^2 + \cdots\ ]$

$\rightarrow$ Combined with OPT, RG Eq. takes a reduced form:

$$\left[\mu\frac{\partial}{\partial\mu}+\beta(g)\frac{\partial}{\partial g}\right] M^{(k)}(m,g,\delta=1)=0$$

Note: OPT+RG completely fix $m\equiv\tilde{m}$ and $g\equiv\tilde{g}$ (two constraints for two parameters).

• Now $\Lambda_{\overline{\rm MS}}(g)$ satisfies by def. $\left[\mu\frac{\partial}{\partial\mu}+\beta(g)\frac{\partial}{\partial g}\right]\ \Lambda_{\overline{\rm MS}}\equiv 0$ consistently at a given pert. order for $\beta(g)$.

Thus equivalent to:

$$\frac{\partial}{\partial\, m}\left(\frac{M^k(m,g,\delta=1)}{\Lambda_{\overline{\rm MS}}(g)}\right)=0\ ;\quad \frac{\partial}{\partial\, g}\left(\frac{M^k(m,g,\delta=1)}{\Lambda_{\overline{\rm MS}}(g)}\right)=0$$

Pre-QCD guidance: Gross Neveu model

- $D = 2$ $O(2N)$ GN model shares many properties with $D = 4$ QCD (asymptotic freedom, chiral symmetry, mass gap,...)
- Mass gap known exactly (for any N): $\frac{M_{exact}(N)}{\Lambda_{\overline{\text{MS}}}} = \frac{(4e)^{\frac{1}{2N-2}}}{\Gamma[1-\frac{1}{2N-2}]}$

(Using $D = 2$ integrability: Bethe Ansatz) Forgacs et al '91

Now consider (large N) *massive* case:

$M(m,g) \equiv m(1 + 2b_0 g \ln \frac{M}{\mu})^{-\frac{\gamma_0}{2b_0}}$ (generic RG resummed)

$= m(1 - g \ln \frac{m}{\mu} + g^2(\ln \frac{m}{\mu} + \ln^2 \frac{m}{\mu}) + \cdots)$ (pert. re-expanded)

Fully summed $M(m,g)$ gives right result: $M(m \to 0) = \Lambda_{\overline{\text{MS}}}$, never seen from standard perturbation ($M_{pert}(m \to 0) \to 0$)

- But RGOPT gives $M = \Lambda_{\overline{\text{MS}}}$ at *first* (and any) δ-order
- At δ^2-order (2-loop), RGOPT $\sim 1 - 2\%$ from M_{exact}(any$N$)

4. QCD Application: Pion decay constant F_π

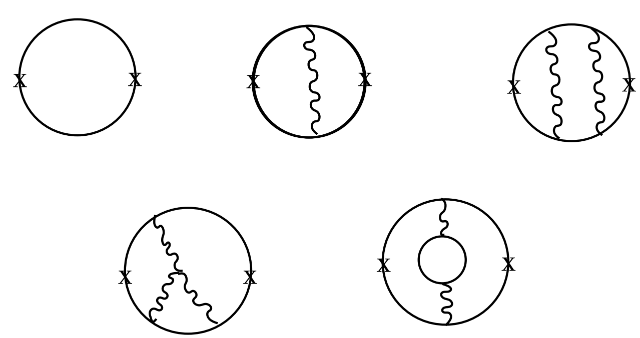
Consider $SU(n_f)_L \times SU(n_f)_R \to SU(n_f)_{L+R}$ for n_f massless quarks. ($n_f = 2$, $n_f = 3$)
Define/calculate pion decay constant F_π from

$$i\langle 0|T A^i_\mu(p) A^j_\nu(0)|0\rangle \equiv \delta^{ij} g_{\mu\nu} F^2_\pi + \mathcal{O}(p_\mu p_\nu)$$

where quark axial current: $A^i_\mu \equiv \bar{q}\gamma_\mu\gamma_5 \frac{\tau_i}{2}\, q$
$F_\pi \neq 0 \to$ Chiral symmetry breaking order parameter

Advantage: Perturbative expression known to high loops

(3-loop Chetyrkin et al ’95; 4-loop Maier et al ’08 ’09, +Maier, Marquard private comm.)



(Standard) perturbative available information

$$F_\pi^2(pert) = N_c \frac{m^2}{2\pi^2}\left[-L + \frac{\alpha_S}{4\pi}(8L^2 + \frac{4}{3}L + \frac{1}{6})\right.$$
$$\left. + (\frac{\alpha_S}{4\pi})^2[f_{30}(n_f)L^3 + f_{31}(n_f)L + f_{32}(n_f)L + f_{33}(n_f)] + \mathcal{O}(\alpha_S^3)\right]$$

$(L \equiv \ln\frac{m}{\mu})$, $n_f = 2(3)$

Note, finite part (after mass + coupling renormalization) not separately RG-inv: (i.e. F_π^2 as defined mixes with the $m^2\,1$ operator)

$\rightarrow$ (extra) renormalization by subtraction of the form:

$S(m, \alpha_S) = m^2(s_0/\alpha_S + s_1 + s_2\alpha_S + ...)$ where s_i fixed requiring RG-inv order by order: $s_0 = \frac{3}{16\pi^3(b_0-\gamma_0)}$, $s_1 = ...$

But to fix s_k needs knowing order $k+1$ (L or $1/\epsilon$ coefficient)
At $\mathcal{O}(\alpha_S^2)$ (3-loop) s_3 can be fixed unambiguously from 4-loop

OPT + RG main features

•OPT: (too) much freedom in the interpolating Lagrangian?:

$$m \to m\,(1-\delta)^{a}$$

in most previous works: linear case $a = 1$ for simplicity...

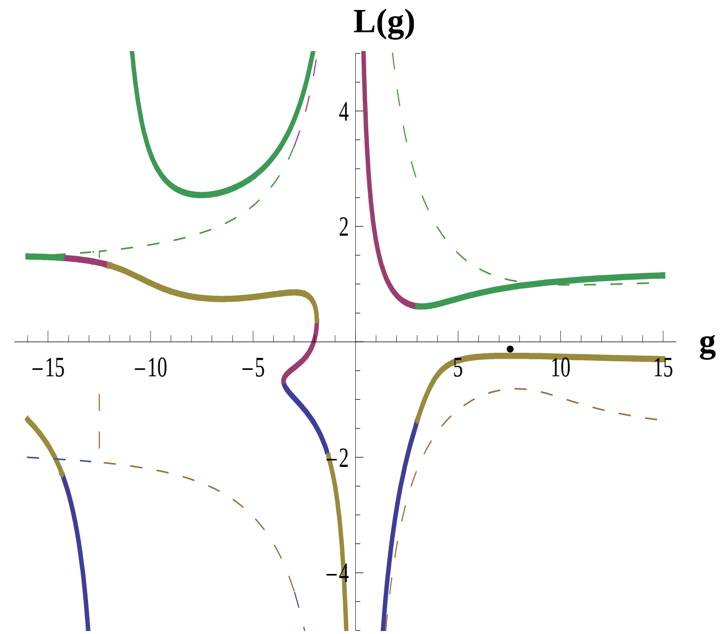
•OPT, RG Eqs. polynomial in $(L \equiv \ln\frac{m}{\mu}, \alpha_S)$:
serious drawback: polynomial Eqs of order $k \to$ (too) many solutions, mostly complex, at increasing δ-orders

•compelling solution: require asymptotic freedom (AF) compatible solutions (for $\alpha_S \to 0, |L| \to \infty$): $\alpha_S \sim -\frac{1}{2b_0 L} + \cdots$

$\to$ at arbitrary RG order, AF-compatible RG + OPT branches *only* appear for a specific universal a value:

$$m \to m\,(1-\delta)^{\frac{\gamma_0}{2b_0}};$$

Removes spurious solutions of wrong (non AF) behaviour!



All branches of RG (thick) and OPT(dashed) solutions $Re[L \equiv \ln \frac{m}{\mu}(g)]$ to the δ-modified 3rd order (4-loop) perturbation ($g = 4\pi\alpha_S$). Unique AF compatible sol. = dot

•However beyond lowest order, AF-compatibility and reality of solutions appear mutually exclusive...
complex solutions: artefact of solving exactly polynomial Eqs., no physical meaning a priori

Warm-up example: pure RG approximation

neglect non-RG (non-logarithmic) terms:

$$F_\pi^2(\text{RG-1}, \mathcal{O}(g)) = 3\frac{m^2}{2\pi^2}\left[-L + \frac{\alpha_S}{4\pi}(8L^2 + \frac{4}{3}L) - (\frac{1}{8\pi(b_0-\gamma_0)\,\alpha_S} - \frac{5}{12})\right]$$

$$\to F_\pi^2(m \to m(1-\delta)^{\gamma_0/(2b_0)}, \alpha_S \to \delta\alpha_S, \mathcal{O}(\delta))|_{\delta\to 1} =$$

$$3\frac{m^2}{2\pi^2}\left[-\frac{102\pi}{841\,\alpha_S} + \frac{169}{348} - \frac{5}{29}L + \frac{\alpha_S}{4\pi}(8L^2 + \frac{4}{3}L)\right]$$

RG+OPT Eqs. have a unique AF-compatible, real solution:

$$\tilde{L} \equiv \ln\frac{\tilde{m}}{\mu} = -\frac{\gamma_0}{2b_0}\;;\quad \tilde{\alpha}_S = \frac{\pi}{2}$$

$$\to F_\pi(\mathcal{O}(\delta))(\tilde{m}, \tilde{\alpha}_S) = (\frac{5}{8\pi^2})^{1/2}\tilde{m} \simeq 0.25\Lambda_{\overline{\text{MS}}}$$

•Higher orders +non-RG terms: $\tilde{m}_{opt}$ consistently $\mathcal{O}(\Lambda_{\overline{\text{MS}}})$ (rather than $m \sim 0$): plays the role of a mass gap, supporting why (modifed) series is more stable:

$F_\pi^{opt} \sim m_{opt} \times$ (perturbation) $\sim \Lambda_{\overline{\text{MS}}} \times$ (perturbation)

And OPT *stabilizes* $\alpha_S^{opt} \simeq .5$ to more perturbative values

Recovering real AF-compatible solutions

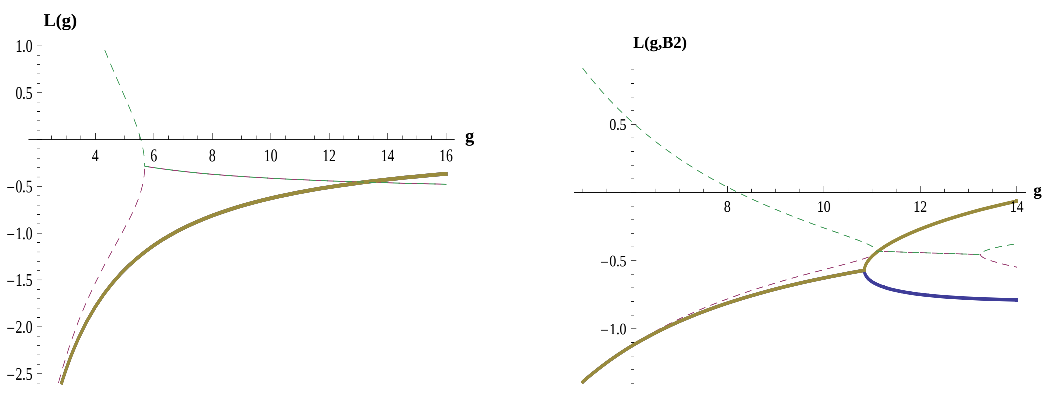
Perturbative 'deformation' consistent with RG?:

Ren. scheme change (RSC)!

$$m \to m'(1 + B_1 g + B_2 g^2 + \cdots),\ g \to g'(1 + A_1 g + A_2 g^2 + \cdots)$$

Require *contact* solution (thus closest to $\overline{\text{MS}}$):

$$\frac{\partial}{\partial g}\text{RG}(g, L, B_i)\frac{\partial}{\partial L}\text{OPT}(g, L, B_i) - \frac{\partial}{\partial L}\text{RG}\,\frac{\partial}{\partial g}\text{OPT} \equiv 0$$



RSC affects pert. coefficients but property:

$$F_\pi^{\overline{MS}}(\overline{m}, g; \overline{f}_{ij}) = F'_\pi(m', g; f'_{ij}(B_i)) + g^{k+1} r(B_i)$$

$\to$ differences should decrease with pert. order

Theoretical uncertainties of the method

Beside recovering real solution, RSC offers natural, reasonably convincing uncertainty estimates:

non-unique RSC prescriptions

$\rightarrow$ differences between them taken as uncertainty

$n_f = 2$: $\frac{F}{\overline{\Lambda}}(\delta^2) = 0.213 - 0.269$ ($\tilde{\alpha}_S = 0.46 - 0.64$)

$\frac{F}{\overline{\Lambda}}(\delta^3) = 0.2224 - 0.2495$ ($\tilde{\alpha}_S = 0.35 - 0.42$)

$n_f = 3$: $\frac{F_0}{\overline{\Lambda}}(\delta^2) = 0.236 - 0.255$ ($\tilde{\alpha}_S = 0.51 - 0.57$)

$\frac{F_0}{\overline{\Lambda}}(\delta^3) = 0.2409 - 0.2546$ ($\tilde{\alpha}_S = 0.37 - 0.42$)

+ empirical stability/convergence seen, with $-2b_0 g L \simeq 1$ (cf RG 1rst order) and $\tilde{m}_{opt} \simeq \overline{\Lambda}$

Explicit symmetry breaking

•Subtract effect from explicit chiral symmetry breaking from genuine quark masses $m_u, m_d, m_s \neq 0$:

$\frac{F_\pi}{F} \sim 1.073 \pm 0.015$ [robust, $n_f = 2$ ChPT + Lattice]

$\frac{F_\pi}{F_0} \sim 1.172(3)(43)$ (Lattice MILC collaboration '10 using NNLO ChPT fits)

But quite different values found by other groups
+ hinted slower convergence of $n_f = 3$ ChPT

(Alternative?: try to implement explicit sym. break. within OPT? (to be fully independent of ChPT+lattice results): promising but rather non trivial, work under progress...)

Combined results with theoretical uncertainties:

Average different RSC +average δ^2 and δ^3 results:

$\overline{\Lambda}_4^{n_f=2} \simeq 359^{+38}_{-26} \pm 5$ MeV

$\overline{\Lambda}_4^{n_f=3} \simeq 317^{+14}_{-7} \pm 13$ MeV

To be compared to recent lattice results, e.g.:

•'Schrödinger functional scheme' (ALPHA coll. Della Morte et al '12):

$\Lambda_{\overline{\text{MS}}}(n_f = 2) = 310 \pm 30$ MeV

•Wilson fermions (Göckeler et al '05)

$\Lambda_{\overline{\text{MS}}}(n_f = 2) = 261 \pm 17(\text{stat}) \pm 26(\text{syst})$ MeV

•Twisted fermions (+NP power corrections) (Blossier et al '10):

$\Lambda_{\overline{\text{MS}}}(n_f = 2) = 330 \pm 23 \pm 22_{-33}$ MeV

•static potential (Jansen et al '12): $\Lambda_{\overline{\text{MS}}}(n_f = 2) = 315 \pm 30$ MeV

NB lattice result differences seems to come mainly from quark mass effects and different chiral extrapolations

Extrapolation to α_S at high (perturbative) q^2

From $n_f = 2$ to $n_f = 3$ i.e. 'crossing' m_s threshold: deeply NP, can't trust perturbative extrapolation.

But we can use directly $\Lambda_{\overline{\mathrm{MS}}}(n_f = 3)$, more trustable

•Standard perturbative extrapolation (3,4-loop with m_c, m_b threshold etc):

$$\alpha_S^{n_f+1}(\mu) = \alpha_S^{n_f}(\mu)\left(1 - \frac{11}{72}\left(\frac{\alpha_S}{\pi}\right)^2 + (-0.972057 + .0846515 n_f)\left(\frac{\alpha_S}{\pi}\right)^3\right)$$

$$\overline{\alpha}_S(m_Z) = 0.1174^{+.0010}_{-.0005} \pm .0010 \pm .0005_{evol}$$

$$\overline{\alpha}_S^{n_f=3}(m_\tau) = 0.308^{+.007}_{-.004} \pm .007 \pm .002_{evol}$$

Alternatively using world average: $\alpha_S(m_Z) = .1184 \pm .0007$ as input, predicts $\frac{F_\pi}{F_0} \simeq 1.12^{+.05}_{-.025}(\text{th,rgopt}) \pm .03_{\alpha_S^{w.a.}} \pm .02_{evol}$

5. Summary and Outlook

•OPT gives a simple procedure to go beyond "large N" in many models, using only perturbative information.

•Our RGOPT version includes 2 major differences w.r.t. most previous OPT approaches:
1) OPT+ RG minimizations fix optimized mass $\tilde{m}$ *and* coupling $\tilde{g} = 4\pi\tilde{\alpha}_S$
2) requiring AF-compatible solutions fixes the basic interpolation $m \to m(1-\delta)^{\gamma_0/(2b_0)}$, discarding spurious solutions, and accelerating convergence.

$\to \mathcal{O}(10\%)$ accuracy on $F_\pi/\Lambda_{\overline{\mathrm{MS}}}$ using only 2-loop order, empirical stability exhibited at 3-loop

Our $\Lambda_{\overline{\mathrm{MS}}}$, α_S values and theoretical accuracies compare reasonably well with (some) recent other determinations.
•Outlook: implement *explicit* chiral sym. breaking in OPT framework, specially for important $m_s \neq 0$ effects for $n_f = 3$