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A non-conventional description of quark matter

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Abstract
We point out that in a simple model with multiplicative noise non-Boltzmann–Gibbs distributions with power-law tail arise as stationary solutions. We compare it to hadronic transverse momentum distributions at RHIC energies and consider average properties of a non-conventional quark matter with Tsallis distribution at the instant of hadronization.

The search for a thermal equilibrium state in heavy-ion collisions, as well as the doubt about its findings, has a long history. Exponential transverse mass spectra of particles were interpreted as a signature of a constant temperature determining the inverse logarithmic slope. Deviations from this shape, in particular a power-law tail, have been considered as of dynamical origin, described by perturbative QCD and jet physics.

In this paper we point out that a power-law distribution, $(1 + m_t/E_c)^{-v}$, can also be obtained as a stationary distribution, resembling the essential properties of a canonical state for the single-particle distribution. Neglecting collective flow effects, the measured transverse mass, $m_t = \sqrt{m^2 + p_t^2}$, can be identified with the energy of a single particle at zero rapidity. Corrections due to a possible transverse flow will be considered at the end, when we compare transverse spectra of different mass hadrons. Here the m_t-scaling, i.e. the description of all spectra by a single-universal curve up to normalization, is violated at very low p_t in PHENIX data. This violation, however, is hard to see in logarithmic spectral plots, as has been considered in figure 2 of reference [1]. We consider the numerical derivative of data on a linear scale and there realize some flow effect.

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1. Additive and multiplicative noise

In order to understand how a non-Boltzmann–Gibbs stationary distribution may occur, let us consider the following simple model. The momentum-distribution, and eventually the energy-distribution of a free particle with mass m under the influence of stochastic forces may be obtained from the solution of a Langevin equation. We generalize the classical Langevin equation with a constant damping coefficient to one, where this coefficient is also fluctuating. This is a special case for multiplicative noise [3]. We consider the following equation of motion for the observed particle:

$$\dot{p} = -\gamma p + \eta \tag{1}$$

with p being its (transverse) momentum, η is the additive and γ is the multiplicative noise. We consider a non-vanishing expectation value of the damping constant, $\langle\gamma\rangle = G$, and uncorrelated Markovian (white) noise of both variables,

$$\langle\gamma(t)\gamma(t')\rangle = 2C\delta(t-t'), \qquad \langle\eta(t)\eta(t')\rangle = 2D\delta(t-t'). \tag{2}$$

The equivalent Fokker–Planck equation (using the Ito discretization scheme) is given by

$$\frac{\partial}{\partial t}f = \frac{\partial}{\partial p}(K_1(p)f) + \frac{\partial^2}{\partial p^2}(K_2(p)f). \tag{3}$$

The drive and diffusion terms are related to the noise properties. In our case we obtain $K_1(p) = -Gp$ and $K_2(p) = D + Cp^2$. The stationary solution becomes

$$f_s(p) = A\left(1 + \frac{Cp^2}{D}\right)^{-1-G/2C}. \tag{4}$$

Using the free particle energy, $E = p^2/2m$, this is the known Tsallis distribution with the same temperature parameter as the classical Brownian motion, $T = D/mG$. A further parameter, the Tsallis index, $q = 1 + 2C/G$ describes the canonical distribution in the framework of extended thermodynamics which uses an altered definition of entropy. This distribution, $f(E) \sim (1 + E/E_c)^{-v}$, approximates an exponential, $\exp(-E/T)$ for low energy $E \ll E_c$ and a power law, E^{-v} for large energy $E \gg E_c$. There is furthermore a definite relation between the inclination point, E_c, the power v and the temperature (low-energy slope) T:

$$v = 1 + E_c/T. \tag{5}$$

This simple relation can be experimentally tested. Considering on other hand the power law and the exponential part as stemming from distinct hard and soft parton physics, there would be no reason for such a relation.

2. Particle spectra

To obtain the canonical distribution in heavy-ion collisions is not an easy enterprise. Actually, even if a collective state in an approximately stationary form may be suspected, the finally observed one-particle spectra of hadrons probably stem from a strongly interacting stage. Whether one measures an unconventional thermodynamics or simply just a particular quasi-particle spectrum due to some interaction cannot be disentangled from the end-state observations. It is, however, desirable to consider possibilities beyond the thermal equilibrium picture.

The conventional picture, interpreting the power law and the exponential part of hadron spectra as stemming from different components and formed by different physical mechanisms was very much supported by the assumption that a stationary state of a canonical distribution

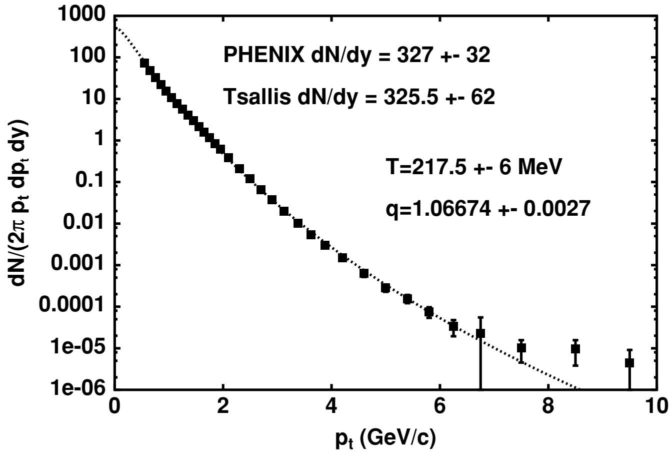

Figure 1. Neutral pion transverse momentum spectrum for the 5% most central events at RHIC in Au–Au collisions. PHENIX data and a Tsallis fit are compared. The Tsallis parameters T and q are indicated in the figure as well as the integrated dN/dy values.

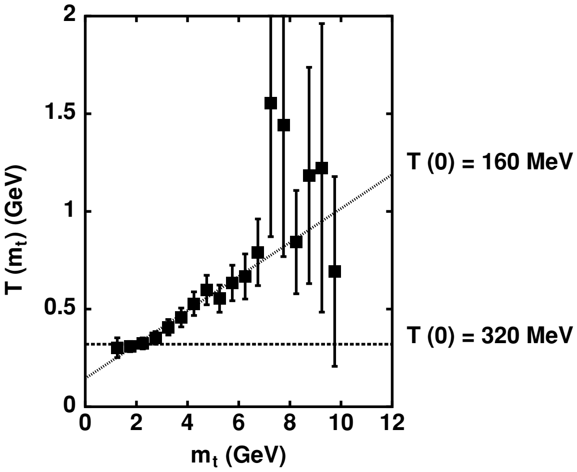

Figure 2. The inverse logarithmic slope information extracted from the neutral pion transverse spectrum at RHIC in Au–Au collisons (PHENIX data). A constant and a linear slope fit gives quite divergent estimates for zero transverse mass.

can only be of Boltzmann–Gibbs type. This is not the only possibility: generalized canonical equilibria may result in non-exponential one-particle spectra, as we also have demonstrated above.

With this view in mind we analysed different transverse momentum spectra of high-energy particles stemming from heavy-ion collisions. In particular, the data measured and published by the PHENIX group at RHIC [2] were used for our present demonstrative purposes. We mention here that theoretical considerations can also lead to a closely Tsallis particle distribution of heavy quarks in quark matter, as it has been shown by Walton and Rafelski [4].

Figure 1 shows the traditional transverse momentum spectrum for neutral pions and a fit with Tsallis distribution. This fit is in good agreement with the integral dN/dy measured

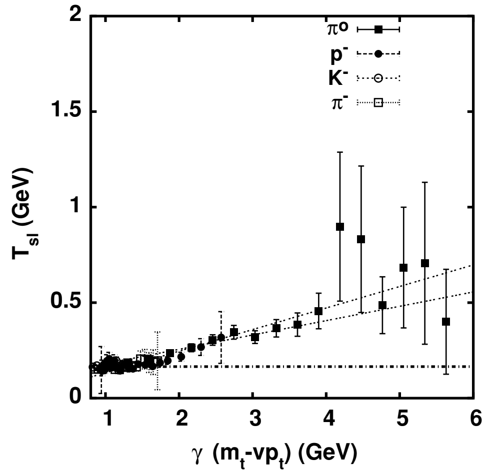

Figure 3. Neutral and charged pion, kaon and antiproton transverse spectra show m_t-scaling [1] in the inverse logarithmic slopes after transverse flow correction. The constant slope, belonging to a Boltzmann–Gibbs distribution locally is not supported over the whole range of measured data.

independently. For deciding between the Boltzmann and Tsallis (or another) distribution, the local inverse logarithmic m_t-slope, $T(m_t)$ is interesting: it is plotted in figure 2. Here the constant can be excluded. Figure 3 presents these slopes for different particles, neutral and charged pions, kaons and antiprotons. Universality (scaling) is achieved after considering a transverse flow of $v = 0.5$ in a simple blast wave picture, where the independent variable is replaced by a Lorentz-blue-shifted combination, $\gamma(m_t - vp_t)$ with $\gamma = 1/\sqrt{1 - v^2}$ being the Lorentz factor. By deriving the slope, T, the corresponding Jacobian was taken into account.

Assuming now a Tsallis distribution for the non-perturbative, effectively recombining, massive quarks at hadronization, the energy per particle, E/N, can be obtained by integrals over the transverse momentum. The result for massless particles in d dimensions, $E/N = dT/(1 - d(q-1))$, deviates from the Boltzmann–Gibbs value given by $q = 1$: $E/N_{\mathrm{BG}} = dT$. A difference also remains for the massive case, which can be integrated analytically only in even dimensions (and without blue shift corrections). We conclude that a non-conventional quark matter, with quarks in a non-Boltzmann–Gibbs canonical state, may already offer a high E/N value found phenomenologically on the hadronic side by massive resonance gas fits [5].

Acknowledgments

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References

[1] Schaffner-Bielich J, Kharzeev D, McLerran L and Venugopalan R 2002 Generalized scaling of the transverse mass spectrum at the relativistic heavy-ion collider *Nucl. Phys.* A **705** 494

[2] PHENIX Collaboration 2003 *Acta Phys. Hung. Heavy Ion Phys.* **15** 291
PHENIX Collaboration 2003 *Preprint* nucl-ex/0307022
PHENIX Collaboration 2003 *Preprint* nucl-ex/0307010
PHENIX Collaboration 2003 *Preprint* nucl-ex/0304022
PHENIX Collaboration 2003 *Preprint* nucl-ex/0212014

Adler S S *et al* 2004 *Phys. Rev.* C **69** 034909, 1034910
Reygers K *et al* 2004 *Nucl. Phys.* A **734** 74
[3] Anteneodo C and Tsallis C 2003 Multiplicative noise: a mechanism leading to nonextensive statistical mechanics *Physica* A **324** 89 (*Preprint* cond-mat/0205314)
Biró T S and Jakovác A 2004 Power-law tails from multiplicative noise *Preprint* hep-ph/0405202
[4] Walton D and Rafelski J 2000 *Phys. Rev. Lett.* **84** 31
[5] Redlich K 2005 *J. Phys. G: Nucl. Part. Phys.* **31** S943 (these proceedings)