

$$\mathcal{L} = \frac{1}{4g^2} G_{\mu\nu}^a G_{\mu\nu}^a + \sum_j \bar{q}_j (i\gamma^\mu D_\mu + m_j) q_j$$

where  $G_{\mu\nu}^a \equiv \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + if_{bc}^a A_\mu^b A_\nu^c$

and  $D_\mu \equiv \partial_\mu + it^a A_\mu^a$

## PDG $\alpha_s(m_Z)$ discussion

J. Huston, K. Rabbertz, (G. Zanderighi)

Les Houches 2023

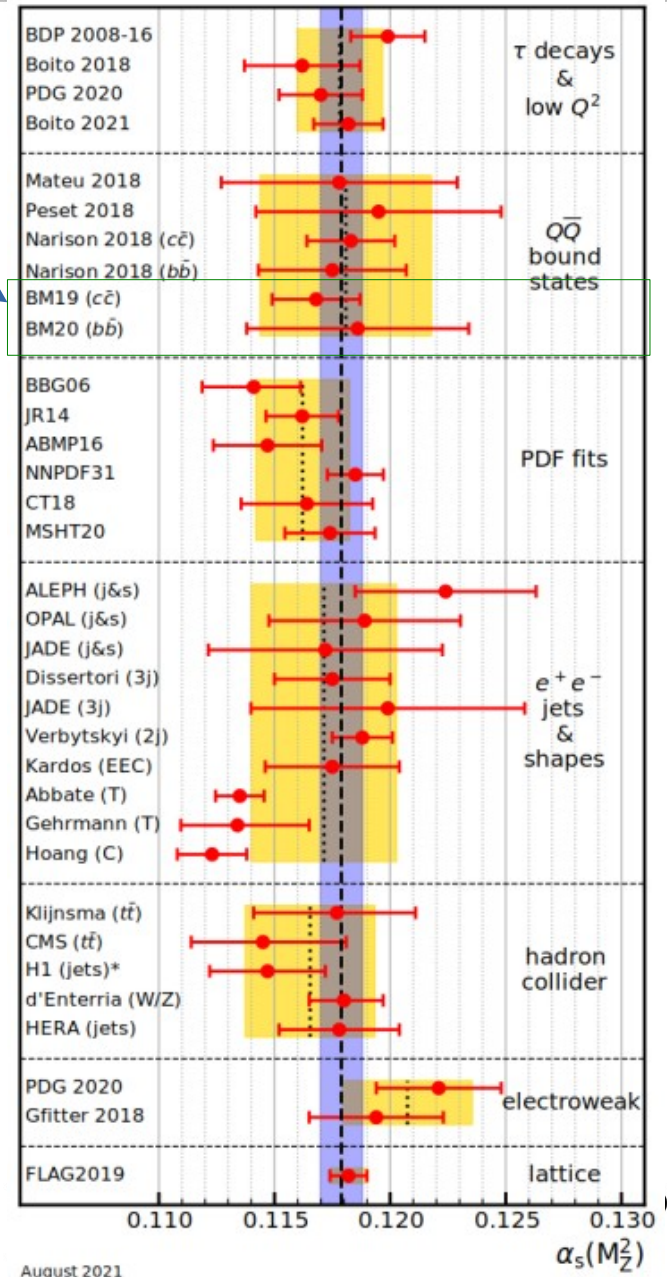
# PDG averaging for $\alpha_s(m_Z)$

- Every two years, the QCD section in the Particle Data Book is updated; part of that update is a review of the world average of  $\alpha_s(m_Z)$ , revising it to include the impact of new measurements and calculations
- The last revision was in 2021 (minor update in 2022)
- The selection of results to include in the  $\alpha_s$  averaging are restricted by the following considerations:
  - published in a peer-reviewed paper at the time of the report (or is based on a summary of results that have been published in a peer-reviewed journal, such as the FLAG report)
  - based on the most complete perturbative predictions of at least NNLO accuracy, accompanied by reliable estimates of all experimental and theoretical uncertainties

average of 3 results that are not totally independent

2022 update

- The world average value of  $\alpha_s(m_Z)$  was determined using results from 7 sub-fields
  - hadronic  $\tau$  decays and low  $Q^2$  continuum
  - heavy quarkonia decays
  - deep-inelastic scattering and global PDF fits
  - hadronic final states of  $e^+e^-$  annihilations
  - hadron collider results
  - electroweak precision fits
- The 7<sup>th</sup> sub-field is just the 2019 FLAG result
- To be used, each result must be based on the most complete perturbative predictions ( $\geq$ NNLO), have a reliable estimate of the uncertainty, and non-perturbative effects under control

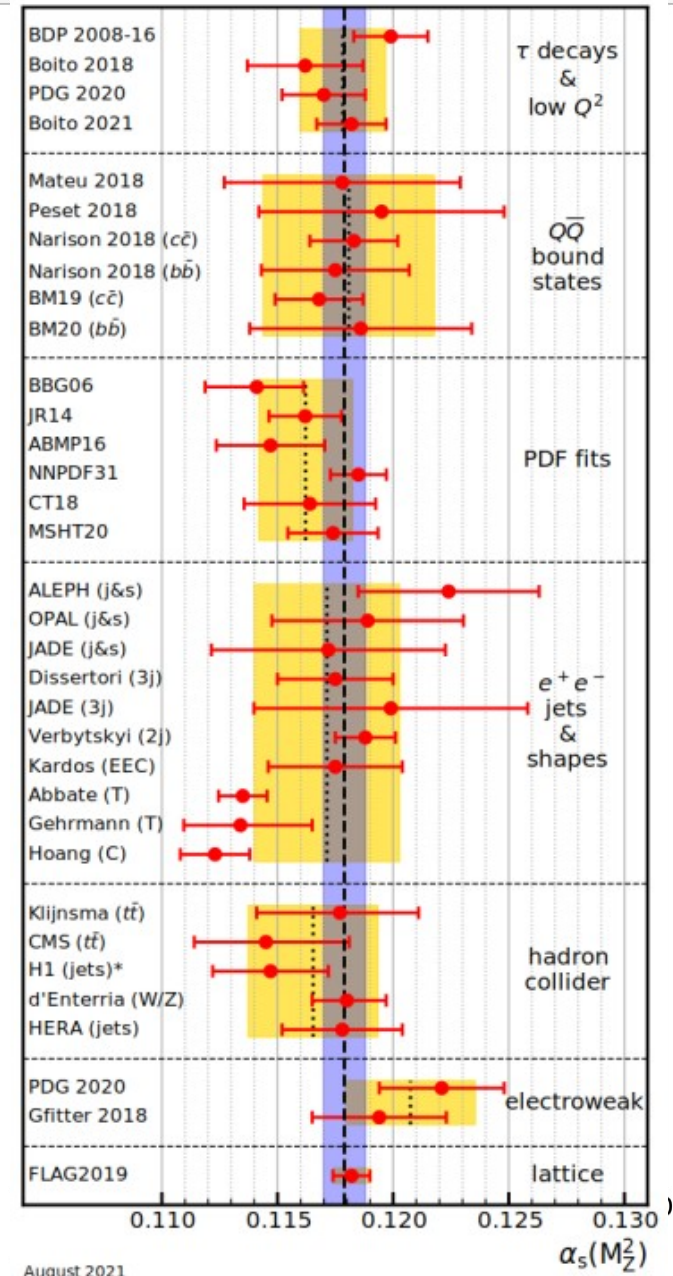


subdivision into the categories shown has been in place for some time

**Figure 9.2:** Summary of determinations of  $\alpha_s(M_Z^2)$  from the seven sub-fields discussed in the text. The yellow (light shaded) bands and dotted lines indicate the pre-average values of each sub-field. The dashed line and blue (dark shaded) band represent the final world average value of  $\alpha_s(M_Z^2)$ . The “\*” symbol within the “hadron colliders” sub-field indicates a determination including a simultaneous fit of PDFs.

# Non-perturbative effects

- Some inclusive quantities such as the  $e^+e^-$  cross sections to hadrons have small non-perturbative corrections ( $\sim \Lambda^4/Q^4$ ), while others such as event-shape distributions, can have corrections that go as  $\Lambda/Q$
- Analyses of the  $\tau$  hadronic decay width and spectral functions are performed with  $N^3$ LO predictions, but low  $Q$  ( $m_\tau$ ) results in non-negligible non-perturbative corrections, whose treatment differs among the different calculations
- Collider measurements access the highest values of  $Q$  where non-perturbative effects are expected to be less important
- Both collider and DIS/DY data go into global PDF fits, which themselves are dependent on non-perturbative forms



August 2021

**Figure 9.2:** Summary of determinations of  $\alpha_s(M_Z^2)$  from the seven sub-fields in the text. The yellow (light shaded) bands and dotted lines indicate the pre-average for each sub-field. The dashed line and blue (dark shaded) band represent the final world average and its uncertainty for  $\alpha_s(M_Z^2)$ . The “\*” symbol within the “hadron colliders” sub-field indicates a determination from a simultaneous fit of PDFs.

# Significant advance

## Fits of $\alpha_s$ using power corrections in the three-jet region

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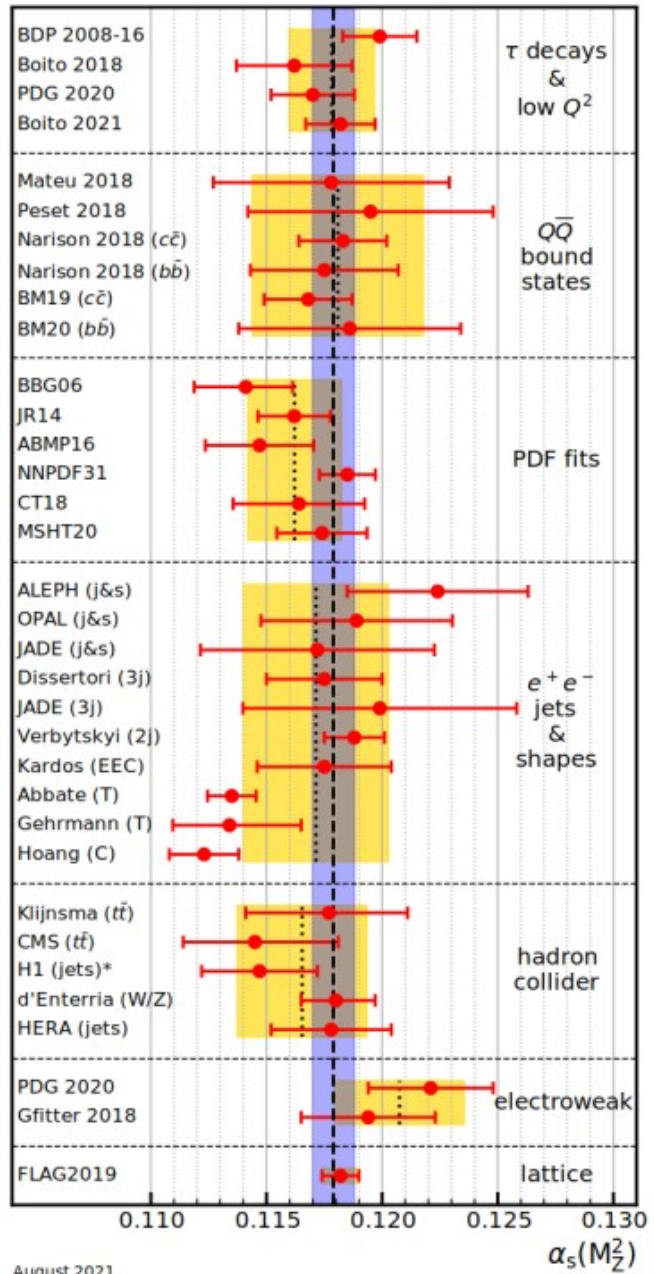
E-mail: [paolo.nason@mib.infn.it](mailto:paolo.nason@mib.infn.it), [zanderi@mpp.mpg.de](mailto:zanderi@mpp.mpg.de)

ABSTRACT: In this work we study the impact of recent findings regarding non-perturbative corrections in the three-jet region to  $e^+e^-$  hadronic observables, by performing a simultaneous fit of the strong coupling constant  $\alpha_s$  and the non-perturbative parameter  $\alpha_0$ . We extend the calculation of these power corrections, already known for thrust and C-parameter, to other  $e^+e^-$  hadronic observables. We find that for some observables the non-perturbative corrections are reasonably well behaved in the two-jet limit, while for others they have a more problematic behaviour. If one limits the fit to the three-jet region and to the well-behaved observables, one finds in general very good results, with the extracted value of  $\alpha_s$  agreeing well with the world average. This is the case in particular for the thrust and C-parameter for which notably small values of  $\alpha_s$  have been reported when non-perturbative corrections have been computed using analytic methods. Furthermore, the more problematic variables are also well described provided one stays far enough from the two-jet limit, while in this same region they cannot be described using the traditional implementation of power-corrections based on two-jet kinematics.

KEYWORDS: Perturbative QCD, QCD Phenomenology, electron-positron scattering

PDF fits often do not have explicit estimate of theory uncertainty

issue of whether necessary to do simultaneous PDF fit



August 2021

$$\alpha_s(M_Z^2) = 0.1177 \pm 0.0019,$$

$$\alpha_s(M_Z^2) = 0.1181 \pm 0.0037$$

unweighted averages of central value and uncertainties within sub-fields

$$\alpha_s(M_Z^2) = 0.1162 \pm 0.0020,$$

$$\alpha_s(M_Z^2) = 0.1171 \pm 0.0031$$

$$\alpha_s(M_Z^2) = 0.1147 \pm 0.0025$$

if use only H1, which used PDF fit

$$\alpha_s(M_Z^2) = 0.1165 \pm 0.0028$$

$$\alpha_s(M_Z^2) = 0.1208 \pm 0.0028$$

combination of first 6 pre-averages using  $\chi^2$  averaging gives

$$0.1175 \pm 0.0010$$

**Figure 9.2:** Summary of determinations of  $\alpha_s(M_Z^2)$  from the seven sub-fields discussed in the text. The yellow (light shaded) bands and dotted lines indicate the pre-average values of each sub-field. The dashed line and blue (dark shaded) band represent the final world average value of  $\alpha_s(M_Z^2)$ . The “\*” symbol within the “hadron colliders” sub-field indicates a determination including a simultaneous fit of PDFs.

- Results within sub-fields 1-6 were pre-averaged (using unweighted average)
- FLAG19 result itself is an average and is taken  $\alpha_s(M_Z^2) = 0.1182 \pm 0.0008$ .

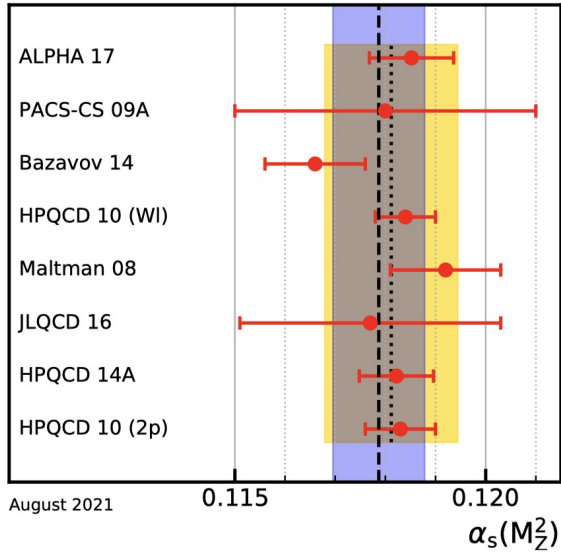


Figure 9.5: Lattice determinations that enter the FLAG2019 average. The yellow (light shaded) band and dotted line indicates the average value for this sub-field. The dashed line and blue (dark shaded) band represent the final world average value of  $\alpha_s(M_Z^2)$ .

- FLAG21 result (too late to be used in previous combination)

$$\alpha_{\overline{\text{MS}}}^{(5)}(M_Z) = 0.1184(8)$$

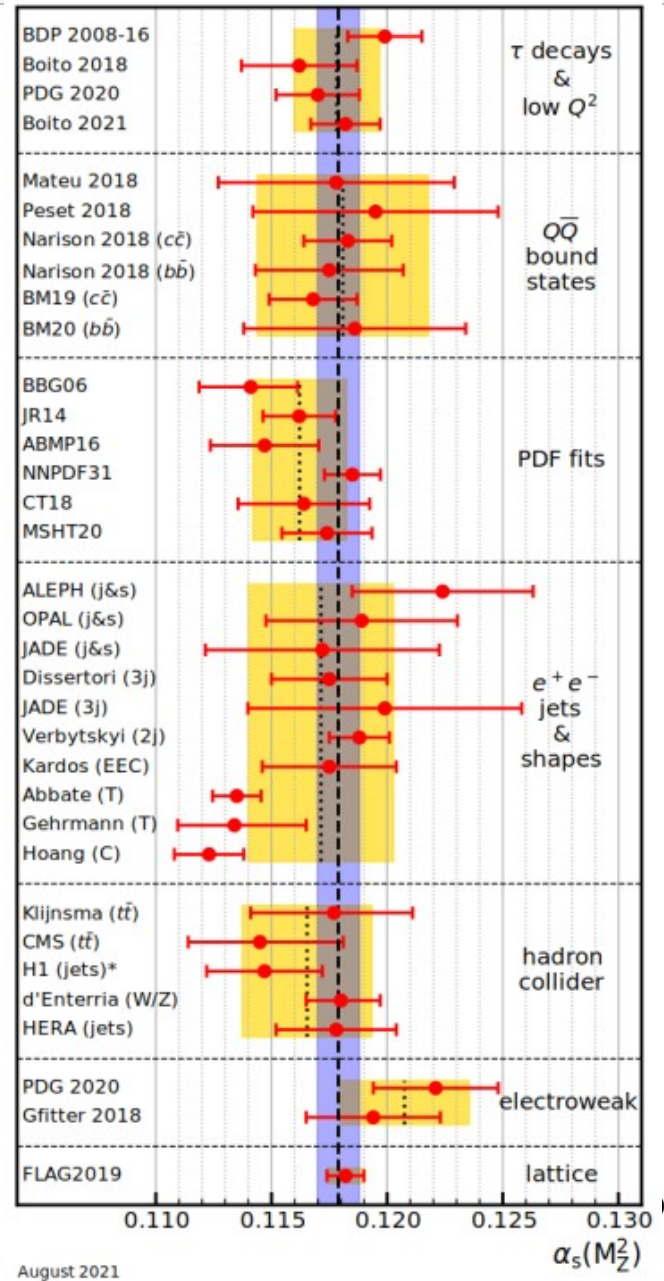


Figure 9.2: Summary of determinations of  $\alpha_s(M_Z^2)$  from the seven sub-fields discussed in the text. The yellow (light shaded) bands and dotted lines indicate the pre-average values of each sub-field. The dashed line and blue (dark shaded) band represent the final world average value of  $\alpha_s(M_Z^2)$ . The “\*” symbol within the “hadron colliders” sub-field indicates a determination including a simultaneous fit of PDFs.

- A non-lattice result was determined from sub-fields 1-6 using a  $\chi^2$ -averaging method

$$\alpha_s(M_Z^2) = 0.1176 \pm 0.0010, \quad (\text{without lattice})$$

- FLAG result itself is an average and is taken as is

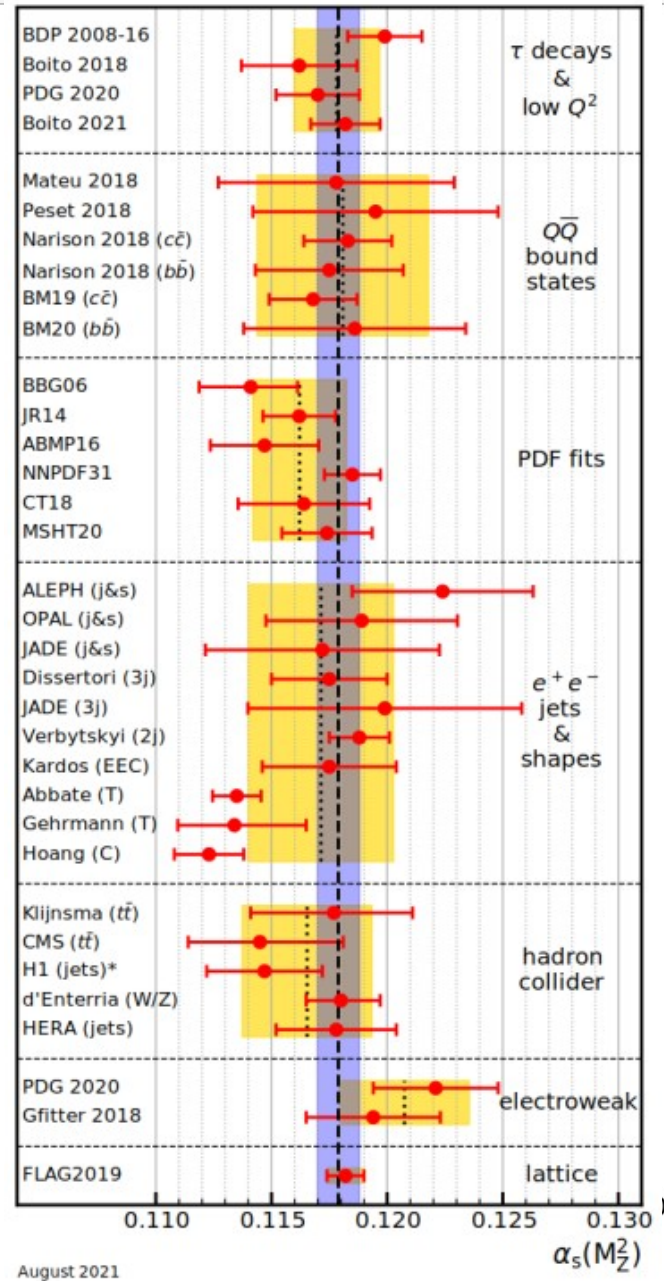
$$\alpha_s(M_Z^2) = 0.1182 \pm 0.0008, \quad (\text{lattice})$$

- Combine two numbers in un-weighted average, and take uncertainty as an average of the two uncertainties (conservative)

$$\alpha_s(M_Z^2) = 0.1179 \pm 0.0009$$

- A weighted average of all 7 categories would give

$$\alpha_s(M_Z^2) = 0.1180 \pm 0.0006.$$

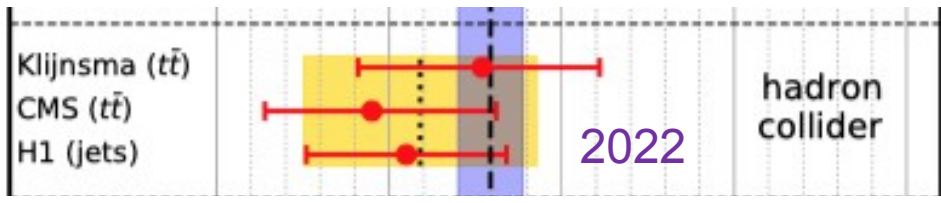


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**Figure 9.2:** Summary of determinations of  $\alpha_s(M_Z^2)$  from the seven sub-fields discussed in the text. The yellow (light shaded) bands and dotted lines indicate the pre-average values of each sub-field. The dashed line and blue (dark shaded) band represent the final world average value of  $\alpha_s(M_Z^2)$ . The "\*" symbol within the "hadron colliders" sub-field indicates a determination including a simultaneous fit of PDFs.



# Collider measurements of $\alpha_s$

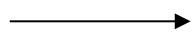
- As the number of NNLO calculations has increased, there have been a growing number of determinations of  $\alpha_s(m_Z)$  at that order (or higher) from the LHC experiments that have nominal uncertainties that rival the full world average uncertainty



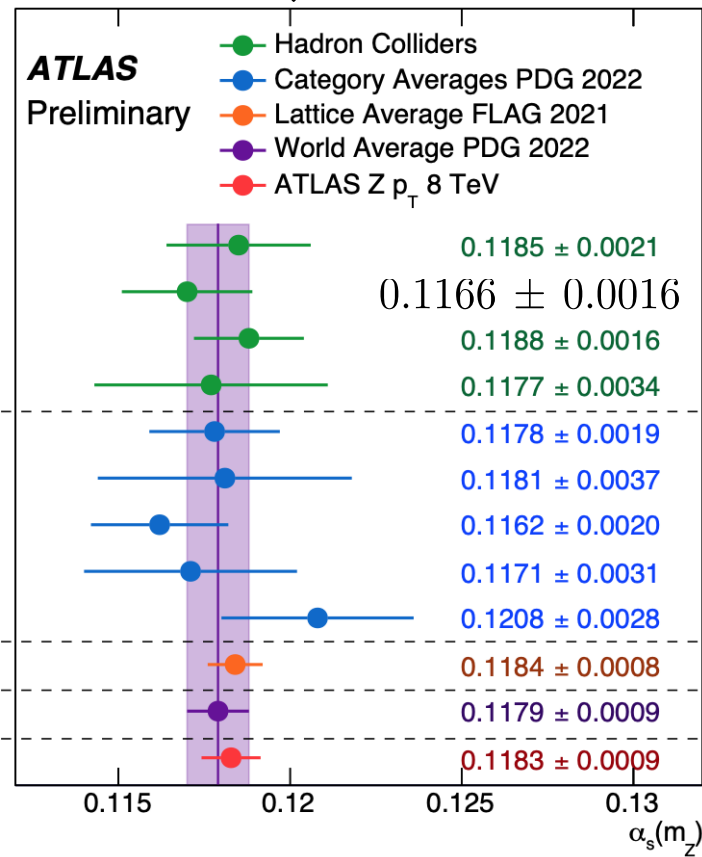
- $Z p_T$
- event shapes

- It would be nice to understand those uncertainties better, especially if PDF uncertainties are taken into account

$N^3LL+N^3LO$



ATLAS ATEEC	
CMS jets	
W, Z inclusive	
tt inclusive	
tau decays	
QQ bound states	
PDF fits	
e+e- jets and shapes	
Electroweak fit	
Lattice	
World average	
ATLAS Z p <sub>T</sub> 8 TeV	





# New LHC results

Exp.	$\sqrt{s}$ / TeV	Lumi / $\text{fb}^{-1}$	Theory	Obs.	$\alpha_s(M_Z)$	$\Delta\alpha_s$ exp	$\Delta\alpha_s$ oth	$\Delta\alpha_s$ scl	Ref.
CMS	13	33.5	NNLO	Jet pT	0.1166	14 (NP)	7	4	JHEP12 (2022) 035
ATLAS	13	139	NNLO	TEEC	0.1175	6	12	+32 -11	2301.09 351
ATLAS	13	139	NNLO	ATEEC	0.1185	9	11	+22 -2	2301.09 351
CMS	13	36.3	NNLO	2D $m_{jj}$	0.1201	12 (NP)	9	8	SMP-21-008
CMS	13	36.3	NNLO	3D $m_{jj}$	0.1201	10 (NP)	10	5	SMP-21-008
ATLAS	8	20.2	N4LLa+ N3LO	Z pT	0.1183	4	6	4	CONF-2023-015

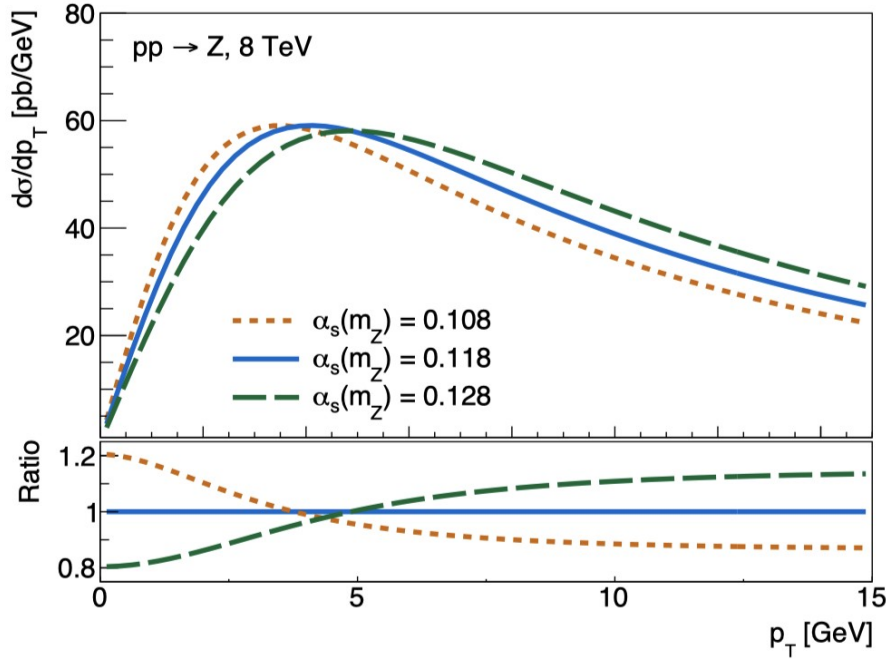


Figure 2: Transverse-momentum distribution of Z bosons predicted with DYTURBO [31] at different values of  $\alpha_s(m_Z)$ , using the MSHT20 PDF set [32].

The statistical analysis for the determination of  $\alpha_s(m_Z)$  is performed with the xFitter framework [60]. The value of  $\alpha_s(m_Z)$  is determined by minimising a  $\chi^2$  function which includes both the experimental uncertainties and the theoretical uncertainties arising from PDF variations:

$$\begin{aligned}
 \chi^2(\beta_{\text{exp}}, \beta_{\text{th}}) = & \\
 & \sum_{i=1}^{N_{\text{data}}} \frac{\left( \sigma_i^{\text{exp}} + \sum_j \Gamma_{ij}^{\text{exp}} \beta_{j,\text{exp}} - \sigma_i^{\text{th}} - \sum_k \Gamma_{ik}^{\text{th}} \beta_{k,\text{th}} \right)^2}{\Delta_i^2} \\
 & + \sum_j \beta_{j,\text{exp}}^2 + \sum_k \beta_{k,\text{th}}^2.
 \end{aligned} \tag{1}$$

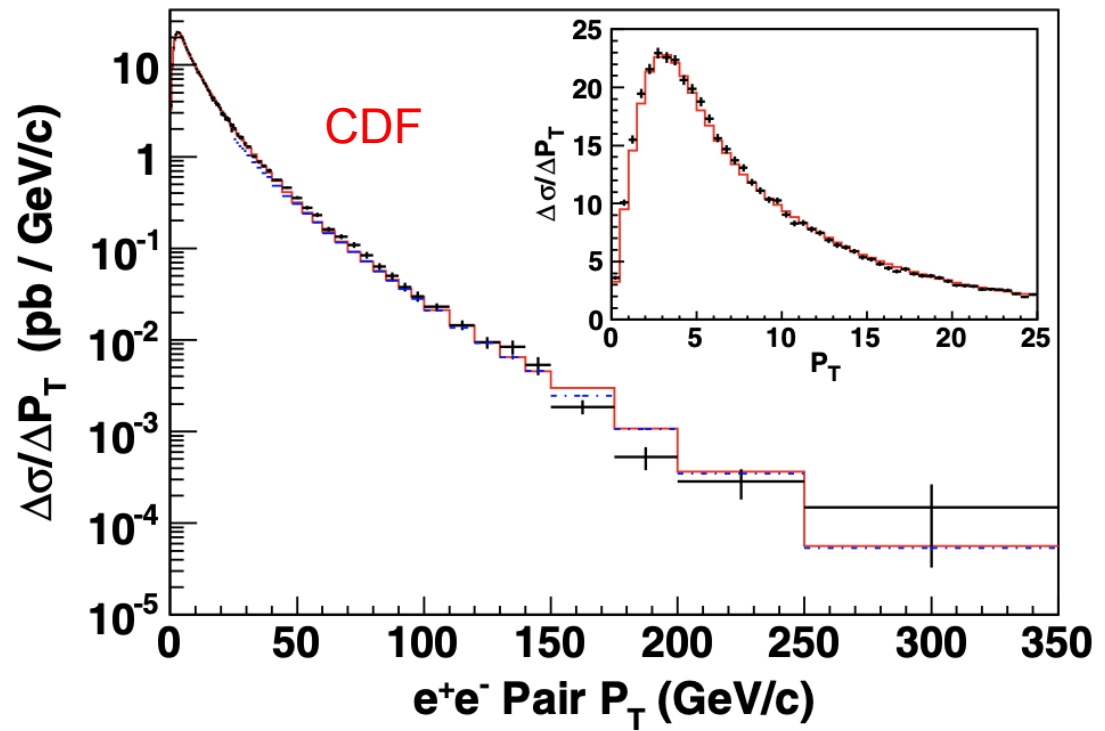


FIG. 7 (color online). The  $\Delta\sigma/\Delta P_T$  cross section versus  $P_T$ . Cross-section values are plotted at the bin center. The horizontal bars represent the bin extent and the vertical bars are the cross-section uncertainties. The solid (black) crosses are the data and all uncertainties except the integrated luminosity uncertainty are combined and plotted. The solid (red) histogram is the RESBOS calculation. The dash-dotted (blue) bars of the  $P_T > 25$  GeV/ $c$  region are the FEWZ2 calculation. For the calculations, only numerical uncertainties are included but they are too small to be visible. The inset is the  $P_T < 25$  GeV/ $c$  region with a linear ordinate scale.

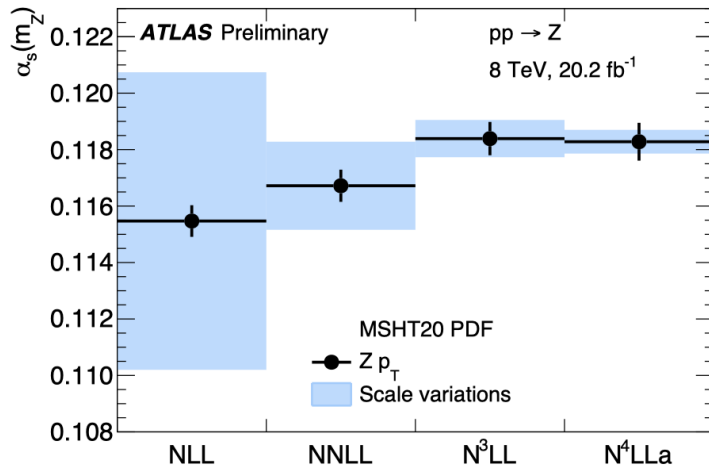


Figure 3: Determination of  $\alpha_s(m_Z)$  at various different orders in the QCD perturbative expansion, using the MSHT20 PDF set. The filled area represents missing higher order uncertainties estimated through scale variations, the vertical error bars include experimental and PDF uncertainties.

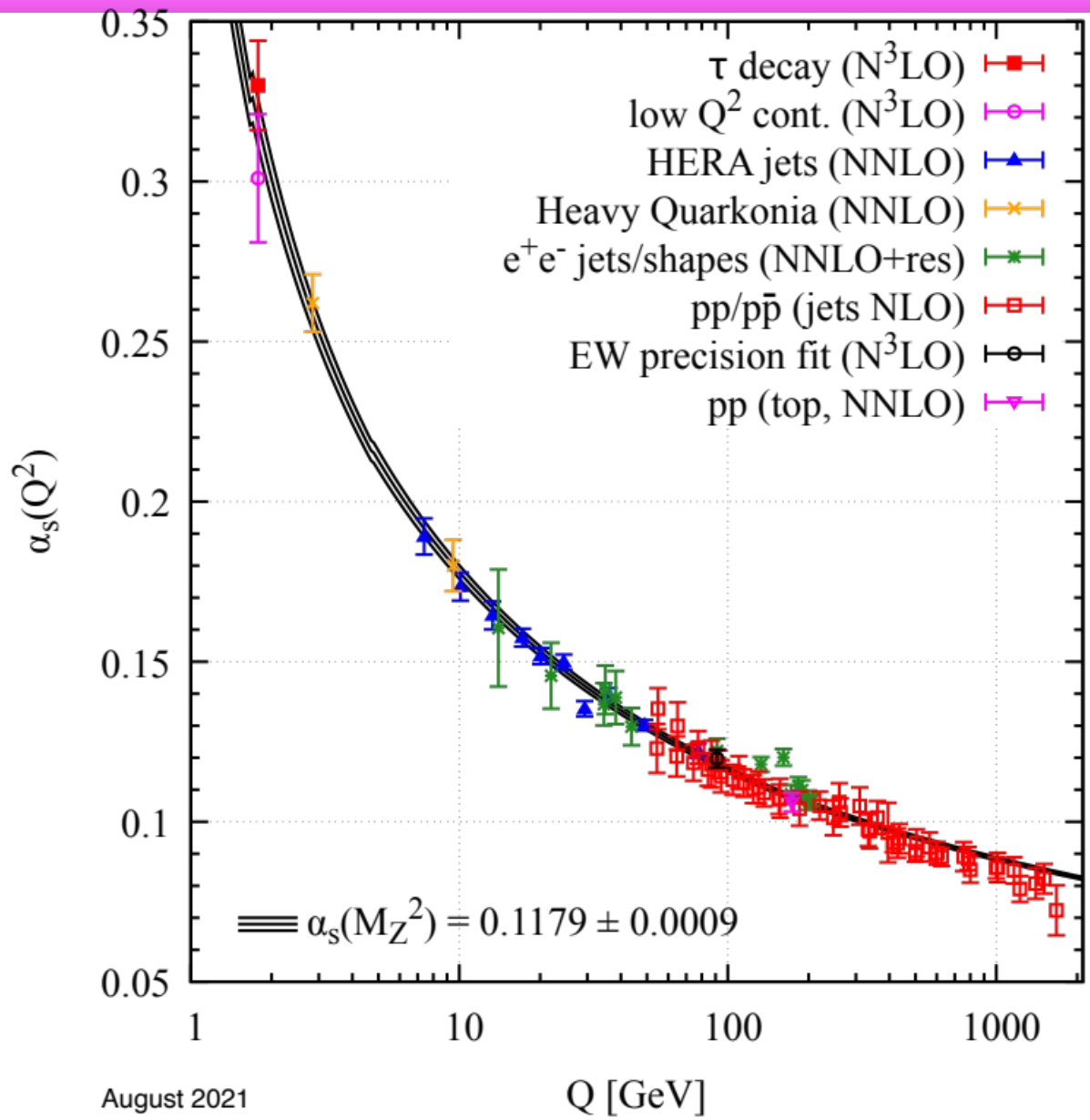
Table 1: Summary of the uncertainties for the determination of  $\alpha_s(m_Z)$ .

Experimental uncertainty	+0.00044	-0.00044
PDF uncertainty	+0.00051	-0.00051
Scale variations uncertainties	+0.00042	-0.00042
Matching to fixed order	0	-0.00008
Non-perturbative model	+0.00012	-0.00020
Flavour model	+0.00021	-0.00029
QED ISR	+0.00014	-0.00014
N4LL approximation	+0.00004	-0.00004
Total	+0.00084	-0.00088

Table 2: Summary of  $N^3LL$  fits with NNLO PDFs.

PDF set	$\alpha_s(m_Z)$	PDF uncertainty	$g$ [GeV <sup>2</sup> ]	$q$ [GeV <sup>4</sup> ]	$\chi^2/\text{dof}$
MSHT20 [32]	0.11839	0.00040	0.44	-0.07	96.0 /69
NNPDF40 [78]	0.11779	0.00024	0.50	-0.08	116.0/69
CT18A [79]	0.11982	0.00050	0.36	-0.03	97.7 /69
HERAPDF20 [63]	0.11890	0.00027	0.40	-0.04	132.3/69

# Running $\alpha_s$



# Looking forward

---

- We have been considering lattice QCD determinations of  $\alpha_s$  independently of experimental/phenomenological determinations
- In the future, it may be useful to group lattice QCD determinations with experimental determinations of  $\alpha_s$  that have systematics of similar origin
  - del Debbio, Ramos; arXiv:2101.04672

# Extras

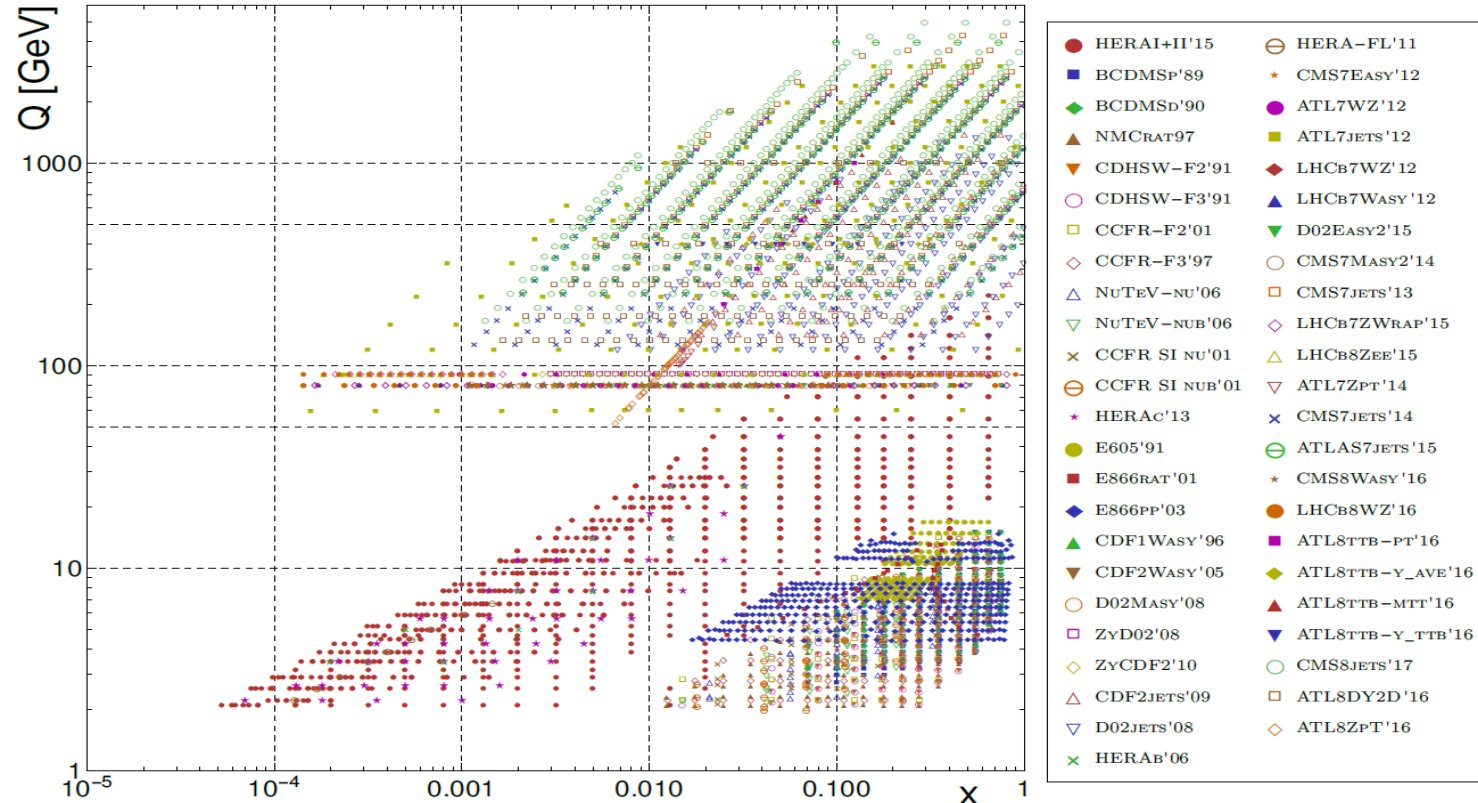
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# Global PDF fits

- There is a wide variety of data in modern global PDF analyses, over 3500 data points for CT18
- The data includes DIS, DY (including precision W/Z), jet production, top production
- All predictions at NNLO, all depending on  $\alpha_s$

Experimental data in CT18 PDF analysis



...but the power of  $\alpha_s$  depends on the process

Born for DY is  $\alpha_s^0$ ; Born for dijet/top production is  $\alpha_s^2$

# $\alpha_s$ and gluon (Lagrange multiplier studies)

- Also, all of the experiments in the global fit do not speak with a unified voice, further weakening the discrimination power
- We end up with a fairly parabolic  $\chi^2$  dependence of  $\alpha_s(m_Z)$ , but it's clear that different experiments have different preferences
- At 68% CL,  $\alpha_s(m_Z)=0.1166\pm 0.0018$  (for CT18 at NNLO)

