# Application of fastNLO to NNLO calculations

fastNLO

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# Motivation

**Interpretation of experimental data relies on**

- Availability of reasonably fast theory calculations
- Often needed: Repeated computation of (almost) same cross sections
  - Observables, binning, phase space given by experimental data

**Examples for a specific analysis:**

- Use of various PDFs (CT, MSTW, NNPDF, …) for data/theory comparison
- Determine PDF uncertainties
- Derivation of scale uncertainties
- Use data set in fit of PDFs and/or $\alpha_s(M_Z)$

**Sometimes NLO predictions can be computed fast**
**But some are very slow**

e.g. jet cross sections, VB+jets, Drell-Yan, …

**Need procedure for fast repeated computations of NLO cross sections**

- Use fastNLO (in use by most PDF fitting groups)

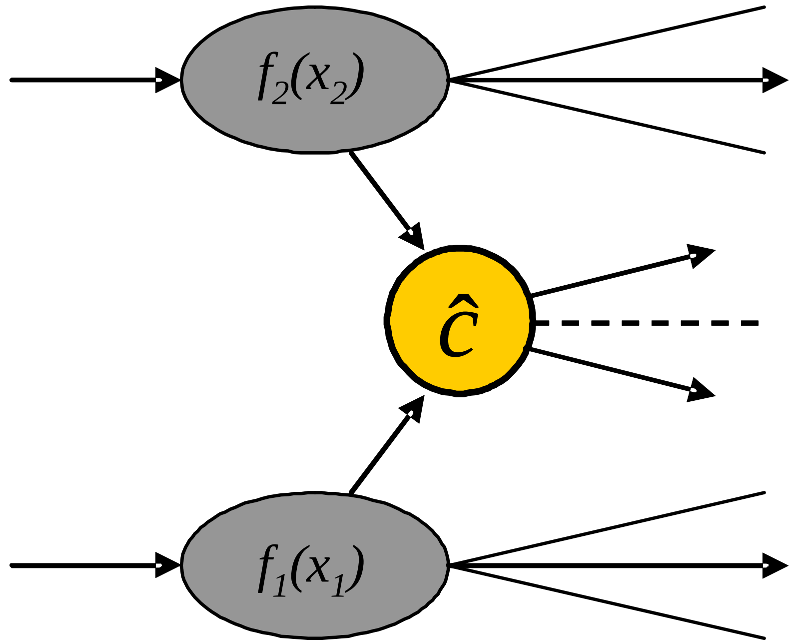
# Basic working principle

**Cross section in hadron-hadron collisions in pQCD**

- Many cross section calculations are time consuming (e.g. jets)

$$\sigma = \sum_{a,b,n} \int_0^1 dx_1 \int_0^1 dx_2 \alpha_s^n(\mu_r) \cdot c_{a,b,n}(x_1, x_2, \mu_r, \mu_f) \cdot f_{1,a}(x_1, \mu_f) f_{2,b}(x_2, \mu_f)$$

- strong coupling $\alpha_s$ in order $n$
- PDFs of two hadrons $f_1, f_2$
- Parton flavors $a, b$
- perturbative coefficent $c_{a,b,n}$
- renormalization and factorization scales $\mu_r, \mu_f$
- momentum fractions $x_1, x_2$



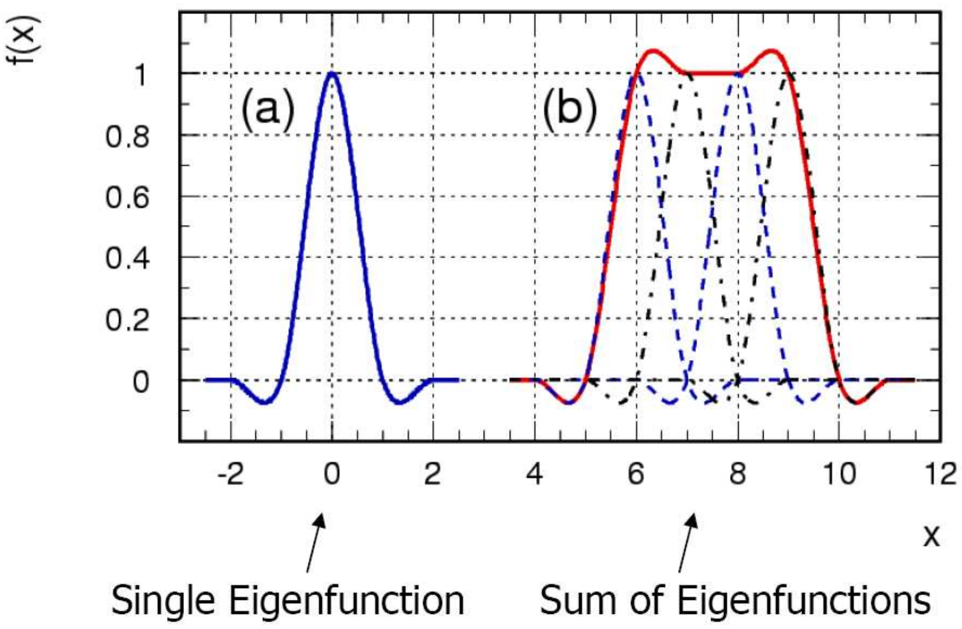

**PDF and $\alpha_s$ are external input**
**Perturbative coefficients are independent from PDF and $\alpha_s$**

Idea: factorize PDF, $\alpha_s$ and scale dependence

# The fastNLO concept

**Introduce interpolation kernel**

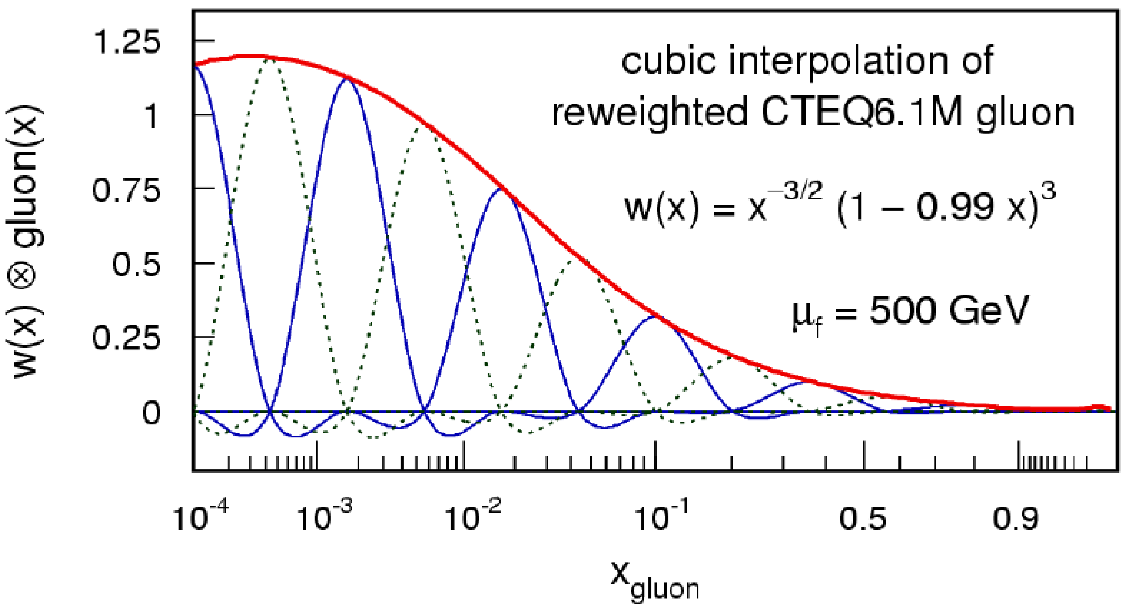
- Set of n discrete $x$-nodes $x_i$'s being equidistant in a function $f(x)$
- Take set of Eigenfunctions $E_i(x)$ around nodes $x_i$ -> interpolation kernels




**Single PDF is replaced by a linear combination of interpolation kernels**

Improve interpolation by reweighting PDF

$$f_a(x) \cong \sum_i f_a(x_i) \cdot E^{(i)}(x)$$




**Scale dependence**

- Similar interpolation procedure also for scales
- Interpolation nodes in x and scales are stored together in look-up table
- Two different observables can be chosen as scale in one table

Convolution integrals become discrete sums
=> Values of perturbative coefficents can be stored in a table

# Calculations with fastNLO in NNLO

**Problem**

- Scale variations become more difficult in NNLO than in NLO

**Current available implementations for NLO calculations**

**Renormalization scale variations**

- Scale variations applying RGE
  - Use LO matrix elements times $n\beta_0 \ln(c_r)$ *[fastNLO, APPLgrid (EPJ C (2010) 66: 503)]*
- Flexible-scale implementation
  - Store scale-independent weights: $\omega(\mu_R, \mu_F) = \omega_0 + \log(\mu_R)\omega_R + \log(\mu_F)\omega_F$ *[fastNLO]*

**Factorization scale variations**

- Calculate LO DGLAP splitting functions using HOPPET *[APPLgrid]*
- Store coefficients for desired scale factors *[fastNLO]*
- Flexible-scale implementation *[fastNLO]*

**Scale variations for NNLO calculations**

- a-posteriori renormalization scale variations become more complicated
- NLO splitting functions are needed for factorization scale variations
  - Calculations become slow again => Not desired for fast repeated calculations

# Flexible-scale implementation in NNLO

**Storage of scale-independent weights enable full scale flexibility also in NNLO**

- Additional logs in NNLO

$$\omega(\mu_R,\mu_F) = \underbrace{\omega_0 + \log(\mu_R^2)\omega_R + \log(\mu_F^2)\omega_F}_{\text{log's for NLO}} + \underbrace{\log^2(\mu_R^2)\omega_{RR} + \log^2(\mu_F^2)\omega_{FF} + \log(\mu_R^2)\log(\mu_F^2)\omega_{RF}}_{\text{additional log's in NNLO}}$$

- Store weights: $w_0$, $w_R$, $w_F$, $w_{RR}$, $w_{FF}$, $w_{RF}$ for order $\alpha_s^{n+2}$ contributions

**Advantages**

- Renormalization and factorization scale can be varied *independently* and by *any* factor
  - No time-consuming ‘re-calculation’ of splitting functions in NLO necessary
- Only small increase in amount of stored coefficients

**fastNLO implementation**

- *Two* different observables can be used for the scales
  - e.g.: $H_T$ and $p_{T,max}$
  - or e.g.: $p_T$ and $|y|$
  - …
- *Any function* of those *two observables* can be used for calculating scales

‘Flexible-scale concept‘: Best choice for performant NNLO calculations

# Application to differential ttbar cross sections in approx. NNLO

**Application of flexible-scale concept to NNLO calculations**

**Interface to DiffTop code**

- DiffTop
  - Code for calculation of heavy quark production within threshold resummation formalism in Mellin space
  - See talk by M.Guzzi
- Differential ttbar cross sections in approx. NNLO
  - $d\sigma/dp_T$
  - $d\sigma/dy$

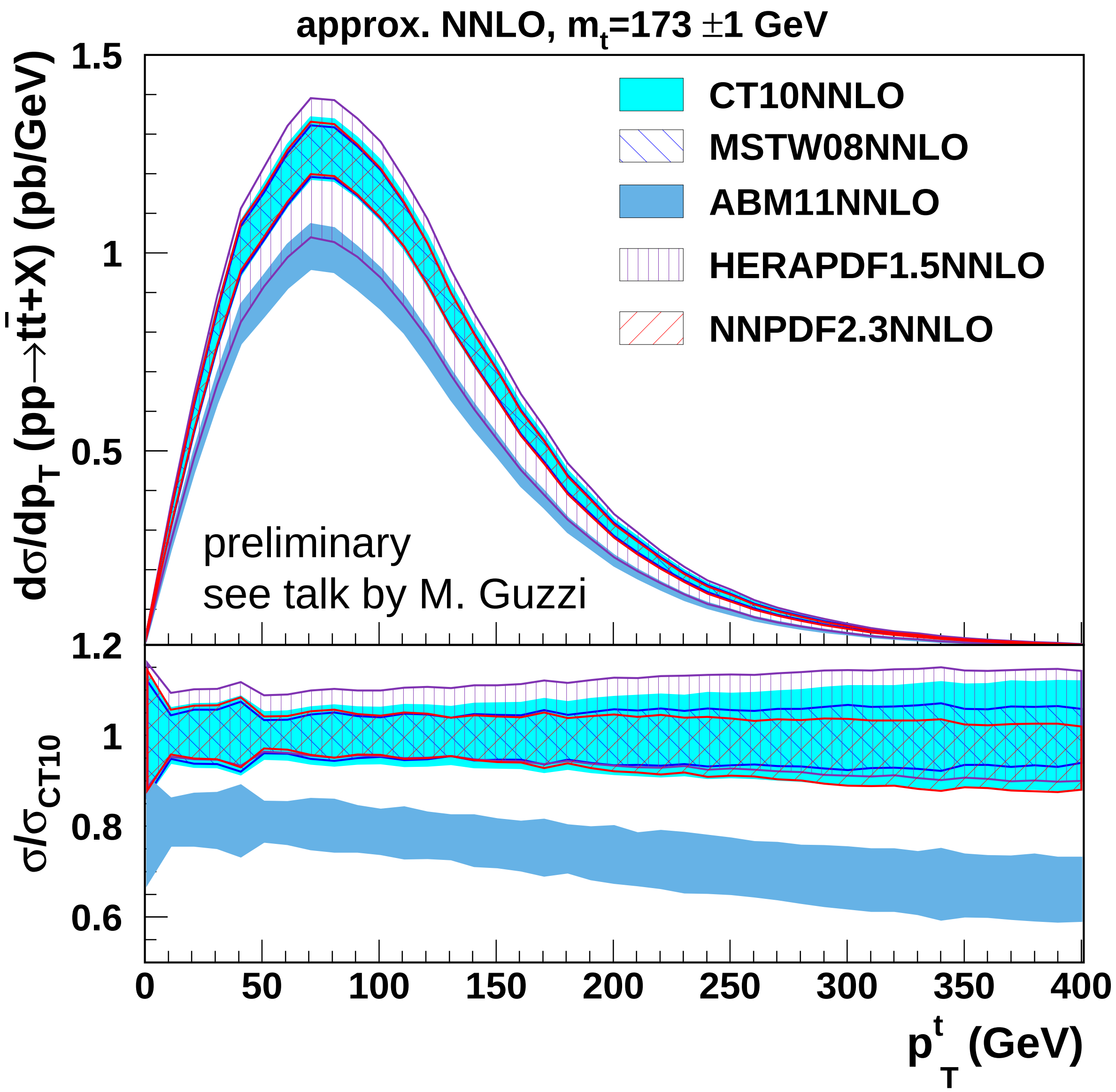
**Benefit in speed**

- NNLO calculation O(days-weeks)
- fastNLO calculation O(<1s)

**fastNLO facilitates study of PDF dependence**

Particularly including PDF uncertainties

- 262 re-calculations are required




**786 repeated calculations needed including variation of $m_t$**

# Application to differential ttbar cross sections in approx. NNLO

**Without recalculating the coefficients**

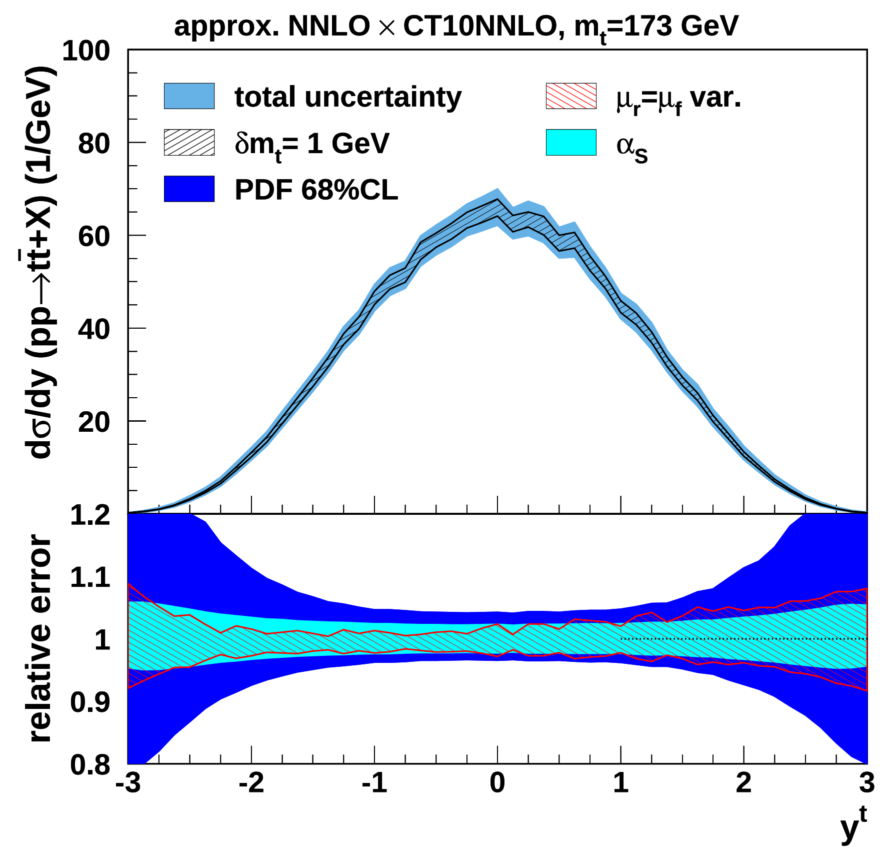
- Variation of the scales within milliseconds
- Variation of $\alpha_s$
- Determination of PDF uncertainties
- Also choice of the scales possible here: $\mu_{r/f} = f(p_T, y, m_t)$

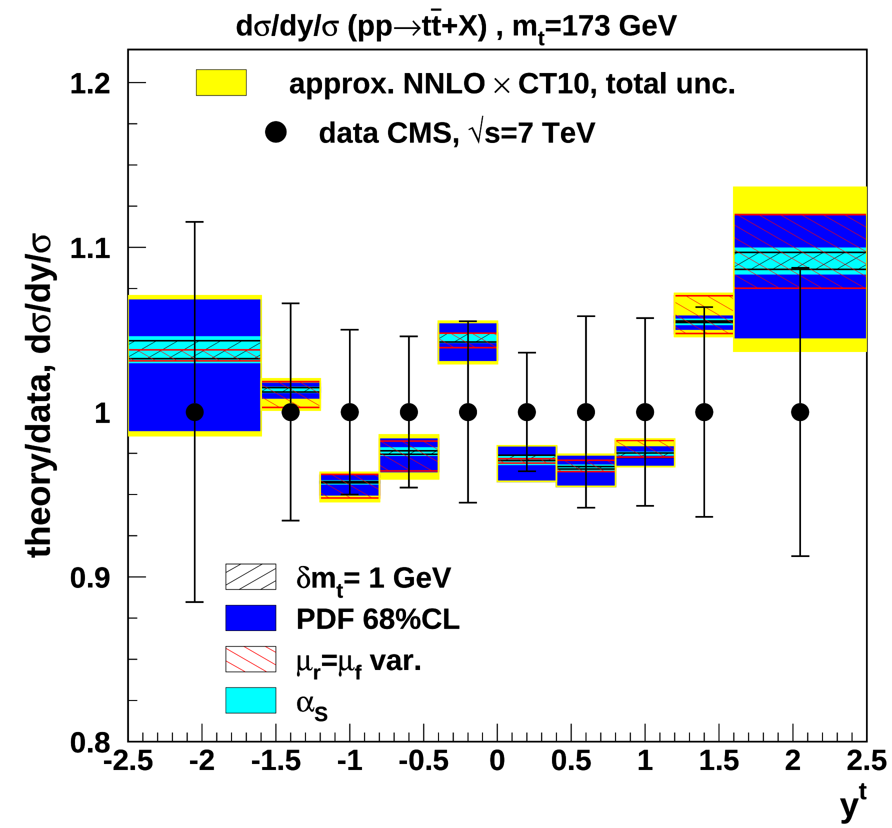
**Variation of $m_t$**

- $m_t$ is a third hard scale in this process
- $m_t$ is not factorized in the current approach
  - Separate fastNLO tables have been computed for different values of $m_t$

**Fast recomputation of cross sections for a given measurement enables application of time-consuming (N)NLO calculations to PDF and/or $\alpha_s$-fits**

- Large gluon uncertainties at high-x can be reduced using ttbar cross sections



# Accuracy of fastNLO interpolation in NNLO

**Compare cross sections calcuated with DiffTop standalone compared to fastNLO**

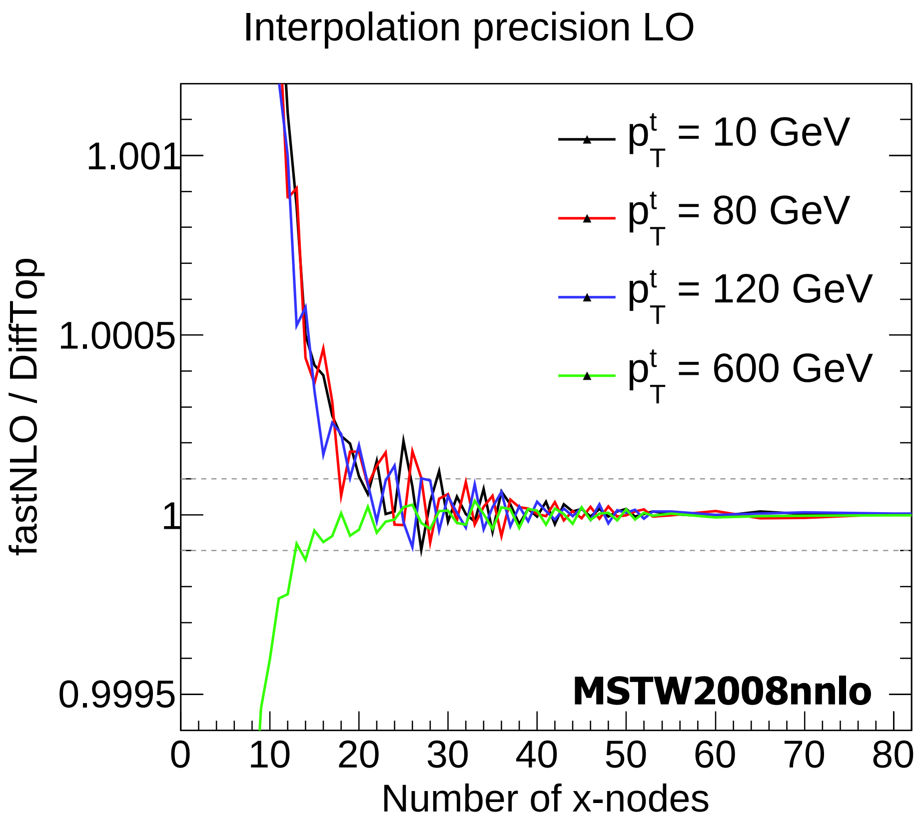
- Interpolation accuracy depends on number of nodes and on chosen interpolation kernel
- Bicubic interpolation kernels used
- Compare contribution order by order separately
- Data probe x-range of $2 \cdot 10^{-3} < x < 1$
  - High x-range has more distinct PDF shapes

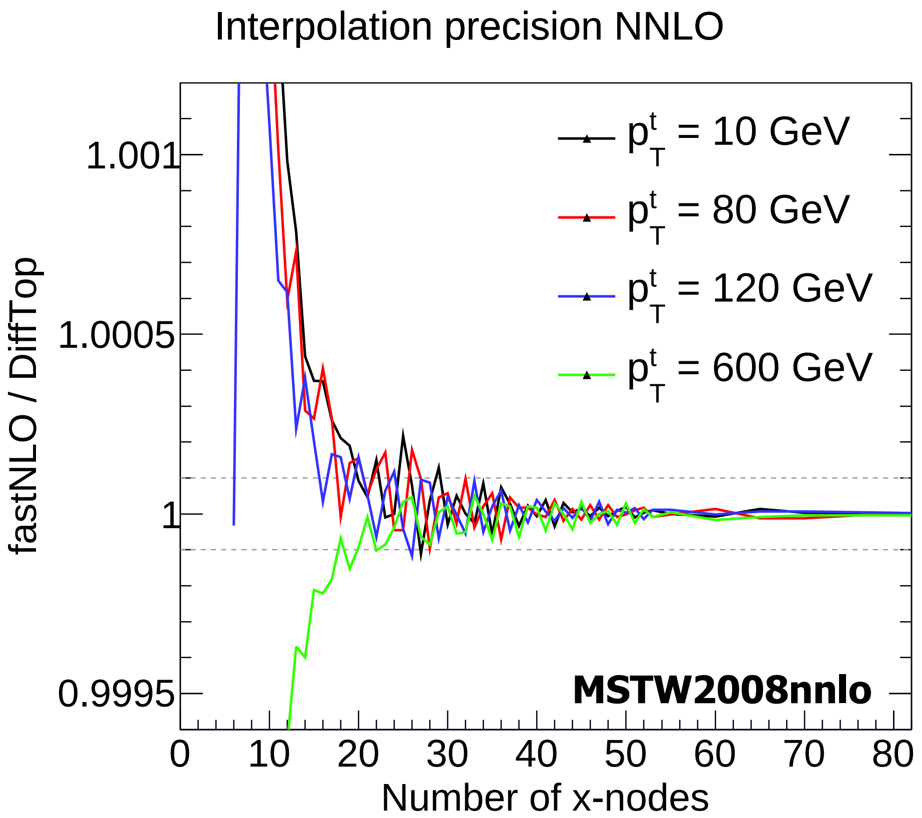
**fastNLO/DiffTop**

- Perfect agreement within numerical precision reachable ($O(10^{-6})$)
- NNLO has same accuracy as LO
- With 18 nodes agreement better than $2 \cdot 10^{-4}$
- Accidental 'interference effects' with PDF grid may cause small fluctuations $O(2 \cdot 10^{-4})$

  -> Numerical uncertainty of PDF grids

fastNLO interpolation does not introduce numerical biases



# New tool: fastNLO toolkit

**What about application of fastNLO to other processes/programs ?**

Hardly any theoretical limitation of fastNLO concept to pQCD or EW calculations

**Why not used more frequently?**

**Interface of fastNLO to theory programs often very complicated…**

- Theory codes are not optimized (at all) for fastNLO
- Technical difficulties are mostly limiting factor in usage

**Goal:**
**Provide simple and flexible code to interface fastNLO to any kind of (N)NLO program**

**Newly developed tool: *fast*NLO Toolkit**

# Application procedure of new ‘fastNLO Toolkit’

**fastNLO needs access to**

- Matrix elements before convolution with PDFs
- x-values where PDFs are evaluated
- Observables
- Scale definitions

**Various NLO and NNLO programs have very different software architecture**

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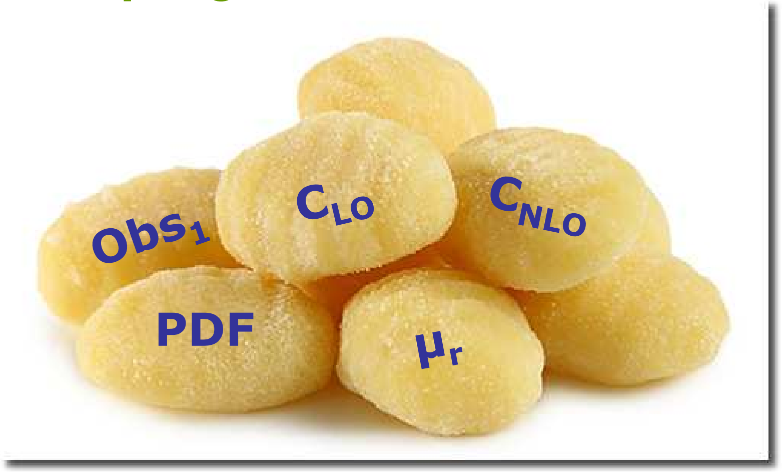
NLO program A



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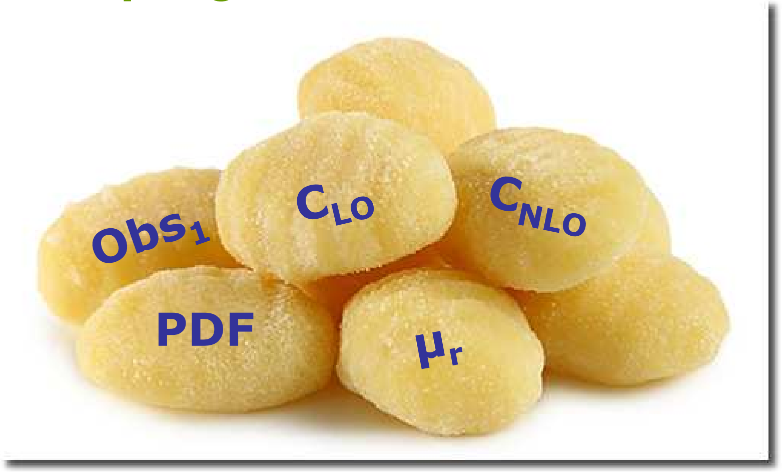
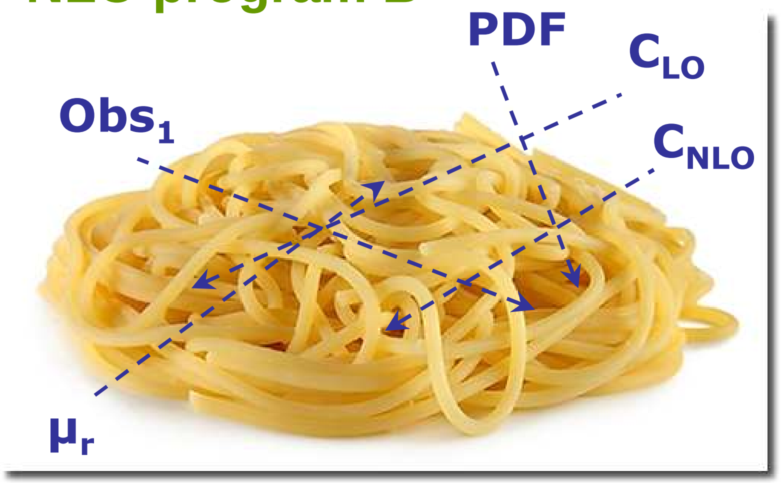



NLO program B



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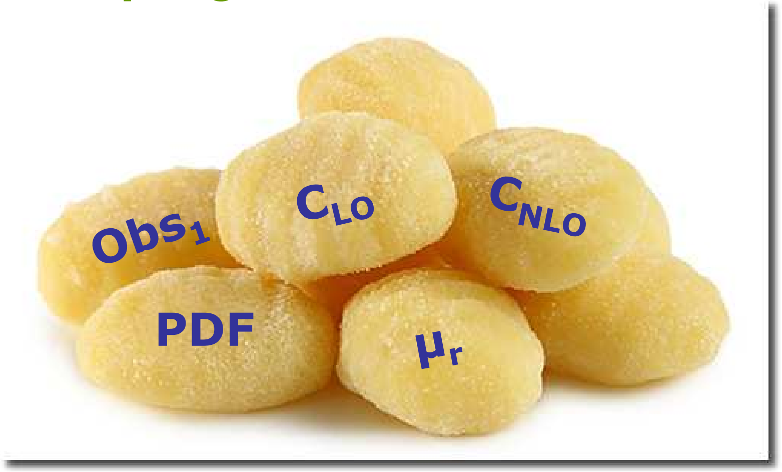
**Various NLO and NNLO programs have very different software architecture**

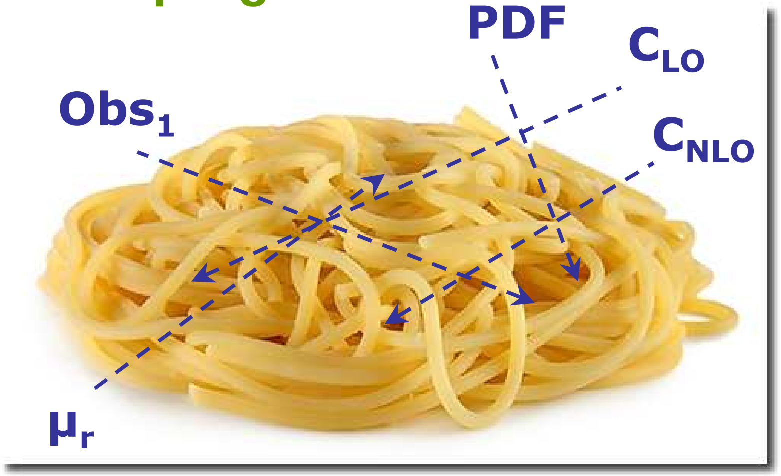
**Reasons**
- Optimized for efficient calculation
- Straight implementation of math. formulae
- Historically grown codes
- Usage of well established algorithms (e.g. vegas)

**(N)NLO programs often look to non-authors like different kind of pasta**

NLO program A

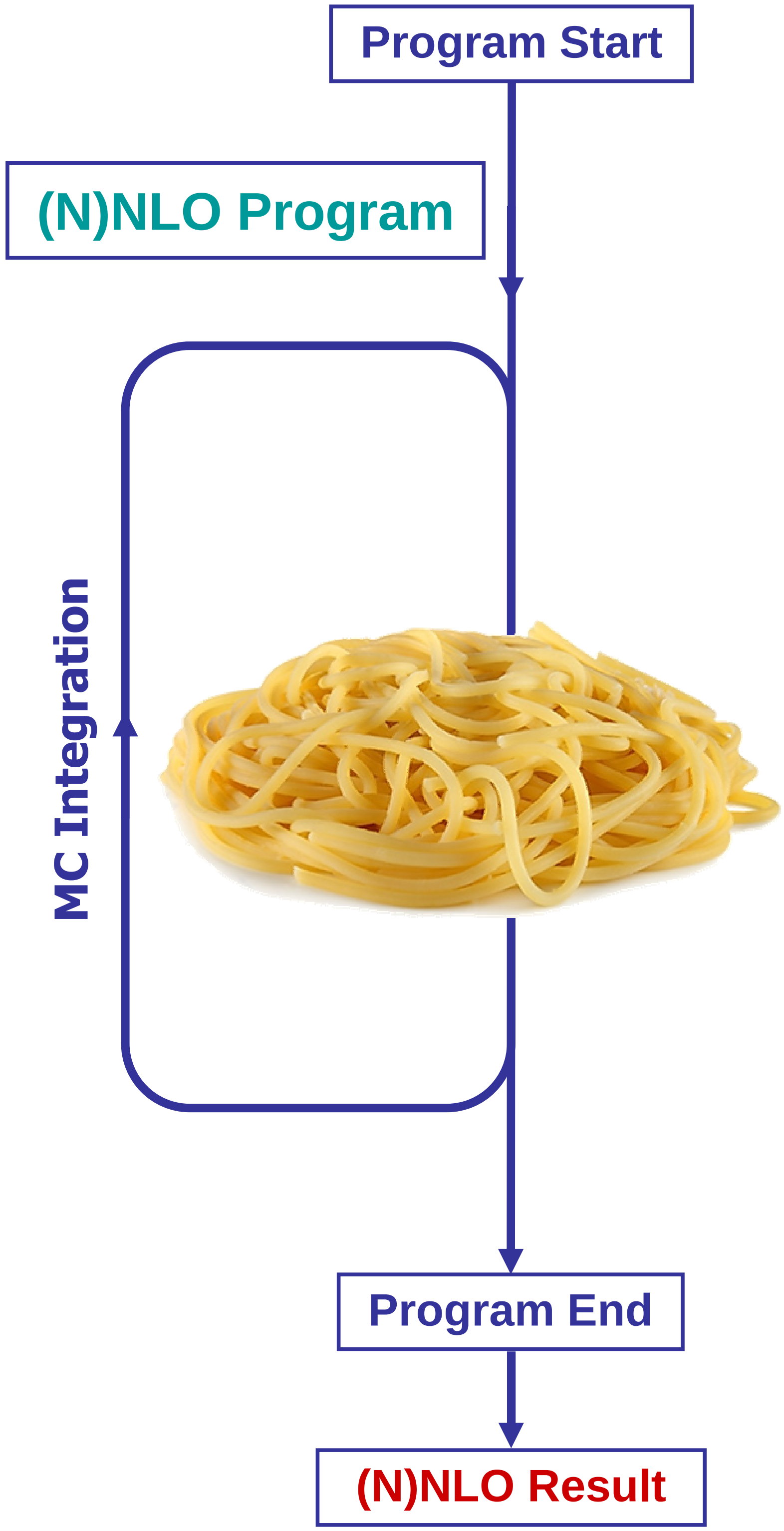



NLO program B




Think about a general interface to any kind of theoretical program

# Application procedure of new ‘fastNLO Toolkit’



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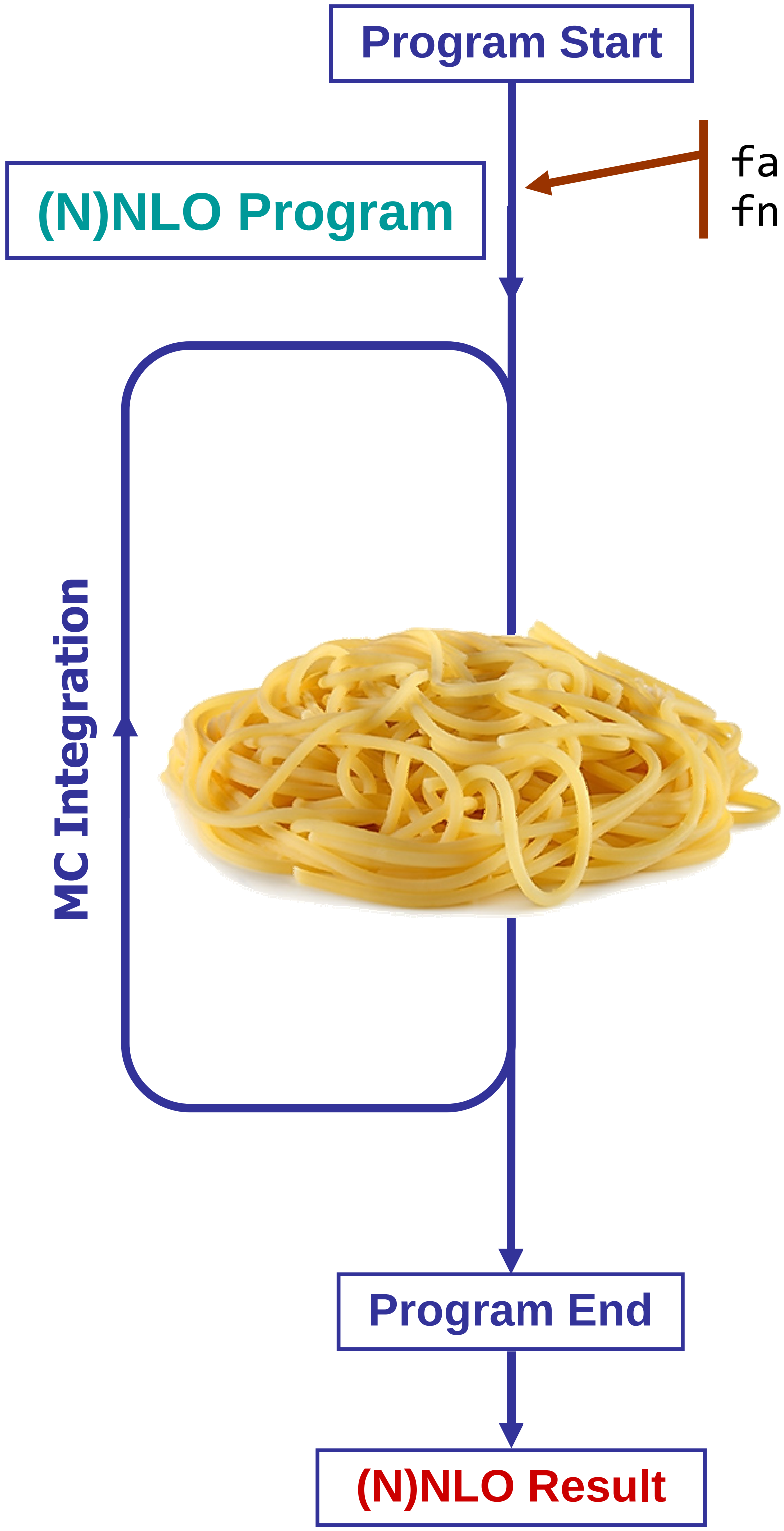


```
fastNLOCreate fnlo(„steering.str“);
fnlo.SetOrderOfCalculation(int order);
```

Initialize fastNLO class(es)

# Application procedure of new ‘fastNLO Toolkit’

Program Start

(N)NLO Program

```
fastNLOCreate fnlo(„steering.str“);
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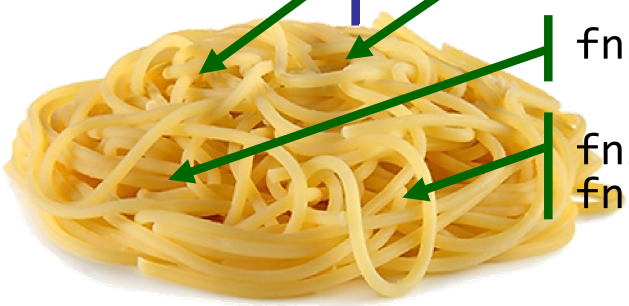
Initialize fastNLO class(es)

MC Integration



```
fnlo.fEvent.SetProcessID(int id);

fnlo.fEvent.SetX1(double x1);
fnlo.fEvent.SetX2(double x2);

fnlo.fEvent.SetWeight(double w);

fnlo.fScenario.SetObservable0(double pt);
fnlo.fScenario.SetObsScale1(double s1);
```

Pass the process specific variables during the ‘event loop’ to fastNLO

- Order does not matter
- Many other convenient implementations possible

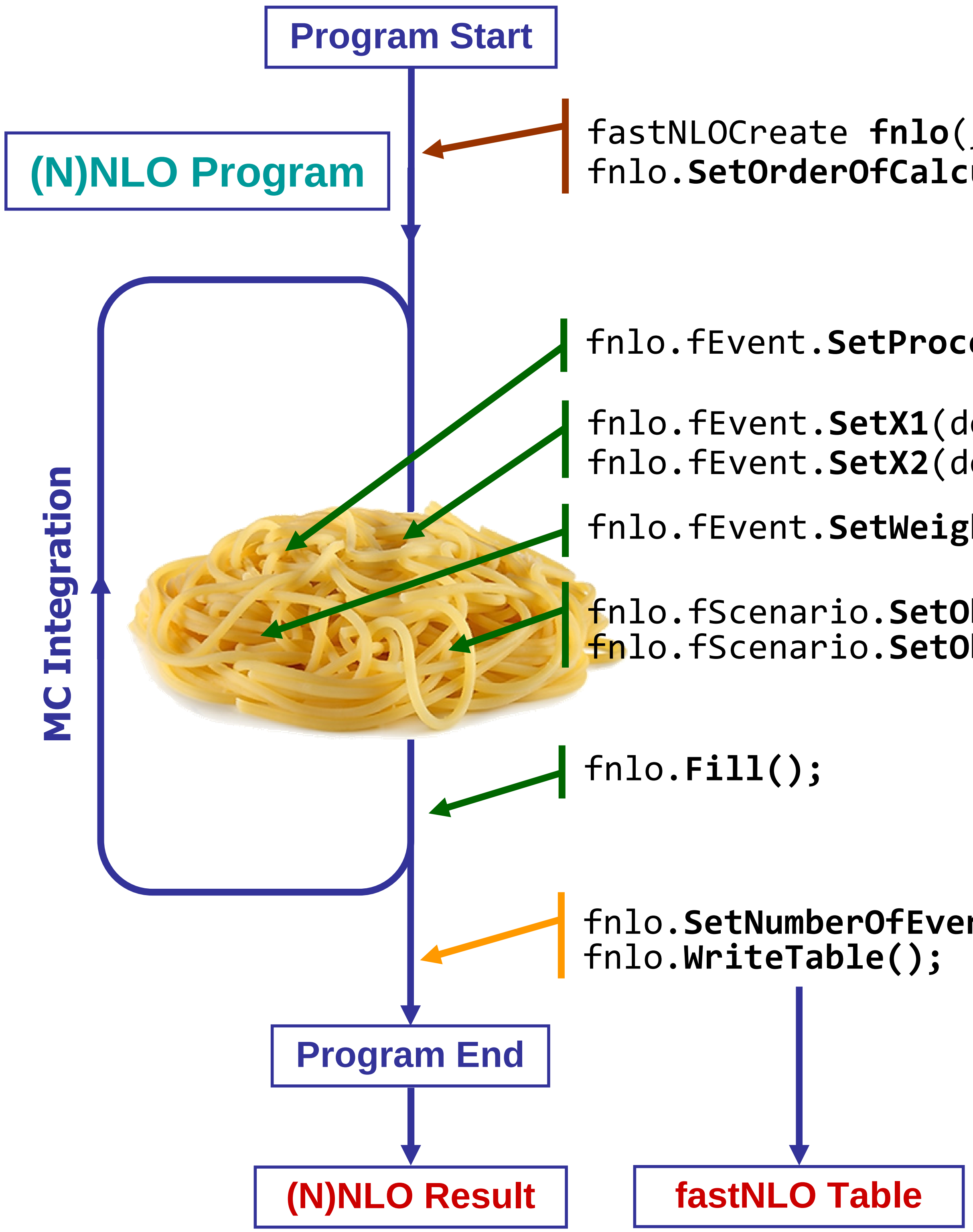
```
fnlo.Fill();
```

Pass all information to fastNLO

Program End

(N)NLO Result

# Application procedure of new ‘fastNLO Toolkit’

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```

Pass the process specific variables during the ‘event loop’ to fastNLO

- Order does not matter
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```
fnlo.Fill();
```

Pass all information to fastNLO

```
fnlo.SetNumberOfEvents(double nevents);
fnlo.WriteTable();
```

Set normalization of the MC integration and write table

Minimum implementation:
11 lines of code

**Convenient implementation of fastNLO into any (N)NLO program possible**

# New developments for fastNLO toolkit

### 1) Creation of fastNLO tables

- One stand-alone c++ library
  - No third party packages needed
  - Optimally: Only 11 lines of code necessary
  - Depending on (N)NLO program: many implementations possible
- Parameters are specified in steering card
  - Binning (also double- or triple- differentially)
- Performance optimized
  - Caching of interpolation values
  - Automatic optimization of grids to phase space
- PDF parton combinations for different subprocesses
  - Specified in steering
  - Stored in table

### 2) fastNLO table format

- Further contributions are forseen
  - EW corrections, etc…
- PDF combinations are stored in table
- Storage of uncertainties soon available
- QEDPDFs or p-Pb collisions also forseen

### 3) Evaluating fastNLO tables

- New interface in PYTHON
- Many interfaces to PDF and $\alpha_s$ routines
  - LHAPDF5, LHAPDF6, Hoppet, QCDNUM, ALPHAS, CRunDec, …
- Improved speed for flexible-scale tables
- Caching of PDFs and $\alpha_s$ values for (even) faster re-evaluation

```
// FastNLO example code in c++ for reading CMS incl.
// jets (PRL 107 (2011) 132001) with CT10 PDF set

fastNLOLHAPDF fnlo("fnl1014.tab","CT10.LHgrid",0);
fnlo.PrintCrossSections();
```

# Summary and Outlook

**fastNLO enables fast re-evalution of perturbative calculations**

- Convenient for scale or PDF studies (e.g. uncertainties)
- Mandatory tool for phenomenological analysis (e.g. $\alpha_s$ or PDF fits)

**Scale-independent concept successfully applied in NNLO**

**fastNLO is applicable to a wide range of processes and corrections**

**New tool: ‘fastNLO toolkit’**

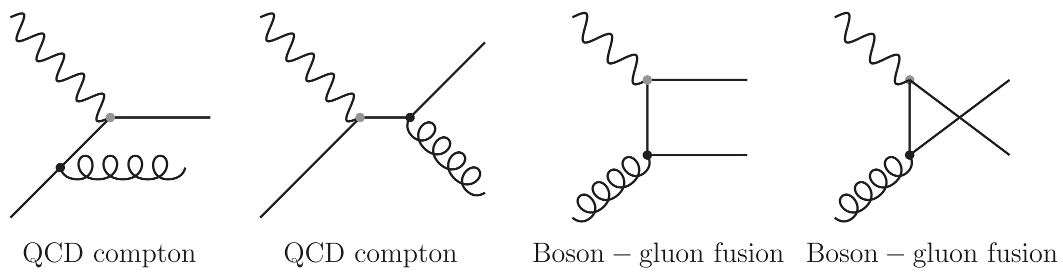
- Facilitates creation of tables and interface to other (N)NLO programs
- Very flexible code:
  Only few modifications in (N)NLO programs are needed (O(11) lines of code)

**Code and manual is released soon after conference**

- Python interface available for reading tables
- pre-release version of ‘fastNLO toolkit’ is available on request
- more information: http://fastnlo.hepforge.org

# Application of flexible scale concept

**Inclusive jet production in DIS**




**Two scales are stored in table**

- $Q^2$
- $p_T$ of the jet

**Any function of the two can be used as scale**

**Renormalization and factorization scale can be varied seperately**

**Choose for scale study**

- $\mu_r^2 = Q^2 + p_T$
- $\mu_f^2 = Q^2$
- Color code shows 5% change in cross section w.r.t. to scale factor of 1

**Other scale choices also shown without scale factors**