Determination of the strong coupling constant $\alpha_s(m_Z)$ in next-to-next-to-leading order QCD using H1 jet cross section measurements

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Why α_s?

***Strong coupling α_s** enters in the calculation of every process that involves the strong interaction*

World average value

$\alpha_s(m_Z) = 0.1181 \pm 0.0011$ [PDG2016]

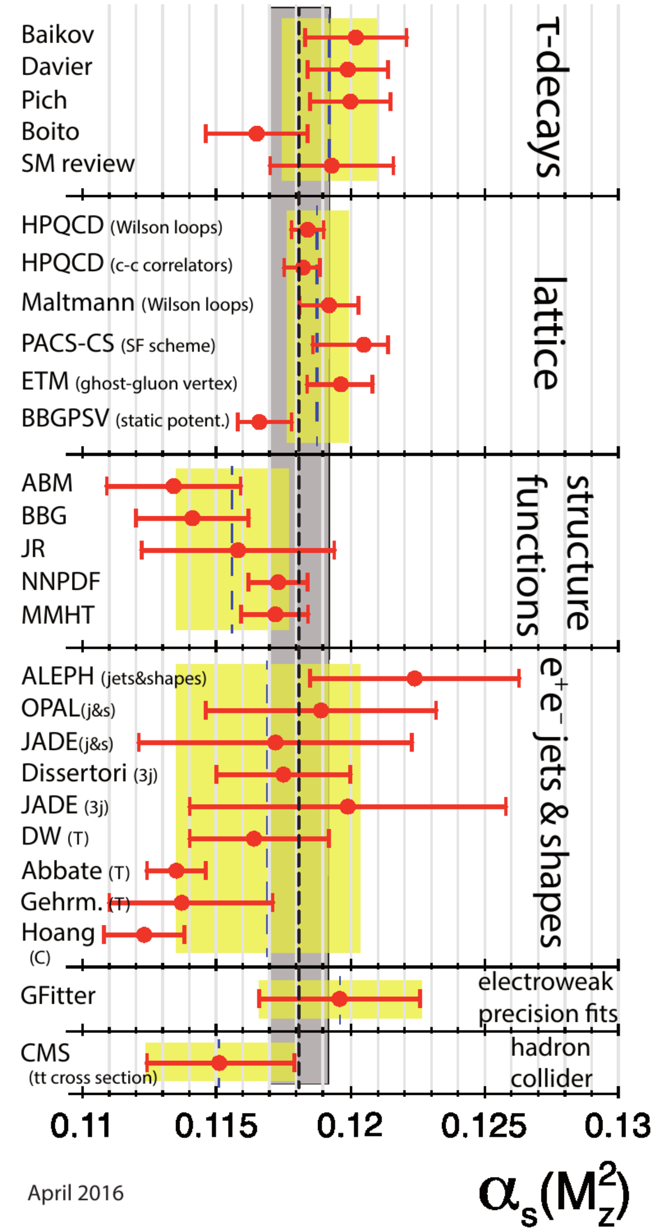
~0.9% relative uncertainty

Uncertainty on α_s

-> non-negligible uncertainties on many observables: e.g. Higgs production cross sections, branching ratios, ...

Jet measurements

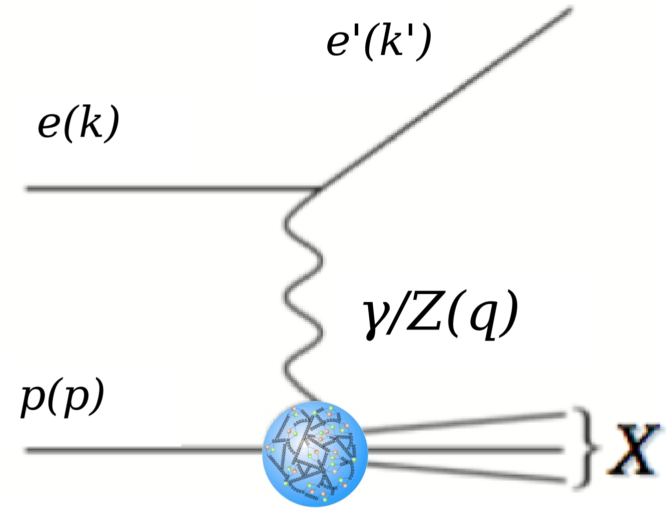
- Direct constraint on α_s
- So far no NNLO results available



Deep-inelastic *ep* scattering

Neutral current scattering (NC)

$ep \rightarrow e'X$



Kinematic variables

Photon virtuality

$$Q^2 = -q^2 = -(k - k')^2$$

Inelasticity

$$y = \frac{p \cdot q}{p \cdot k}$$

HERA *ep* collider in Hamburg

Data taking periods

- HERA I: 1994 – 2000
- HERA II: 2003 – 2007
- $\sqrt{s}$ = 300 or 319 GeV

H1 Experiment at HERA

H1 multi-purpose detector

Asymmetric design

Trackers

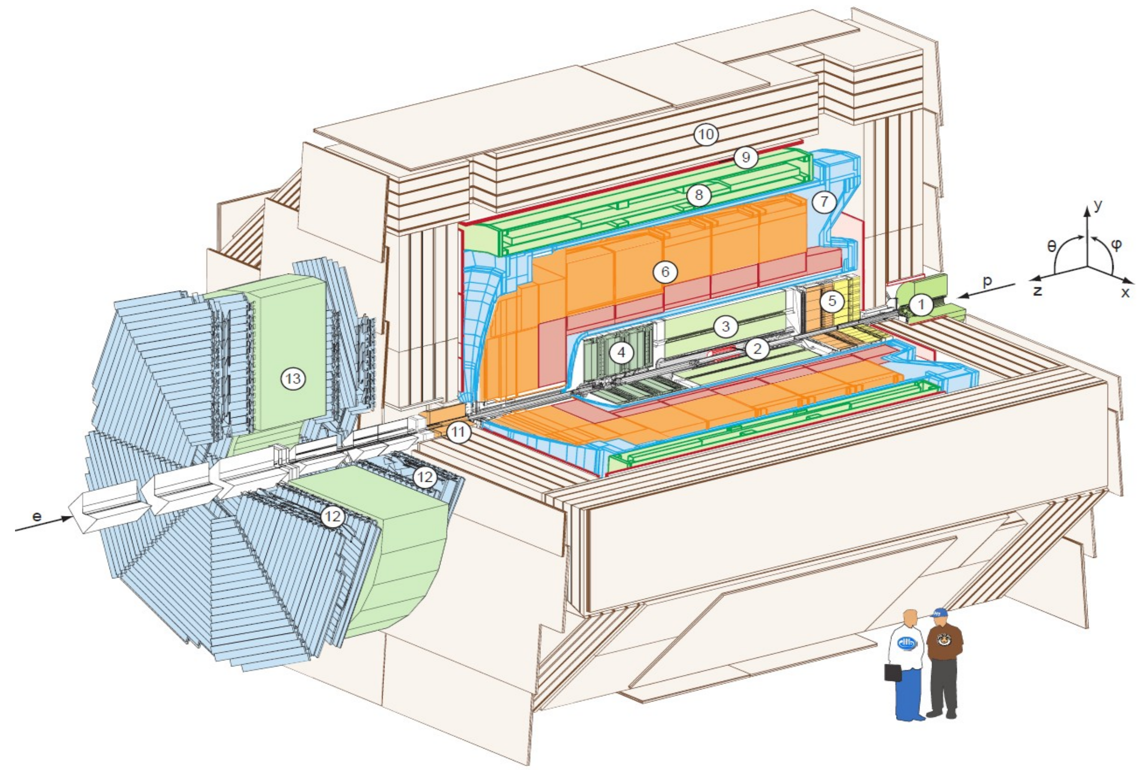
- Silicon tracker,
- Jet chambers
- Proportional chambers

Calorimeters

- Liquid Argon sampling calorimeter
- SpaCal: scintillating fiber calorimeter

Superconducting solenoid, 1.15T

Muon detectors

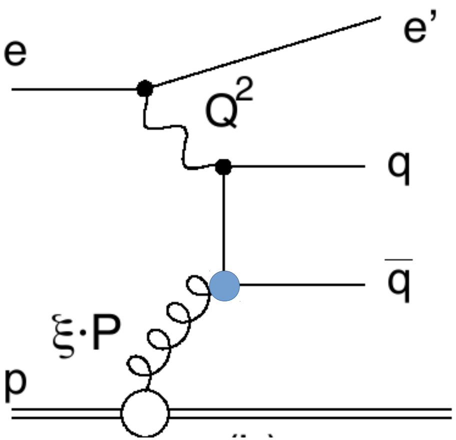


Drawing of the H1 experiment

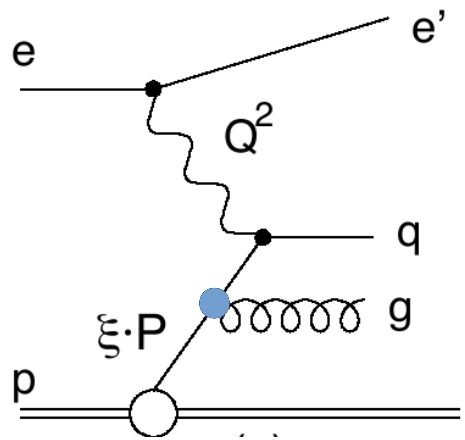
High experimental precision

- Overconstrained system in NC DIS
- Electron measurement: 0.5 – 1% scale uncertainty
- Jet energy scale: 1%

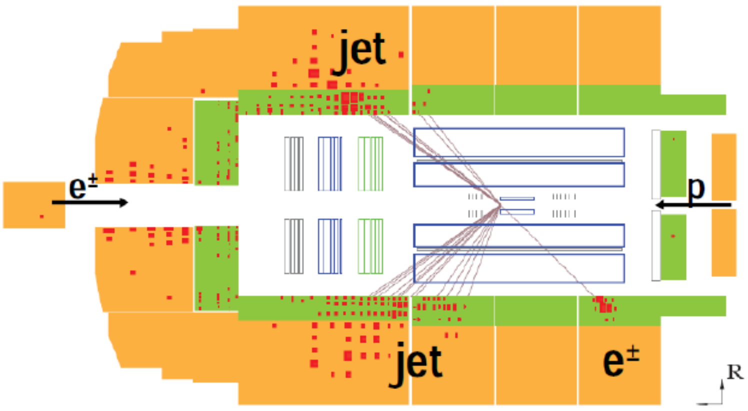
Jet production in DIS



Boson-gluon fusion



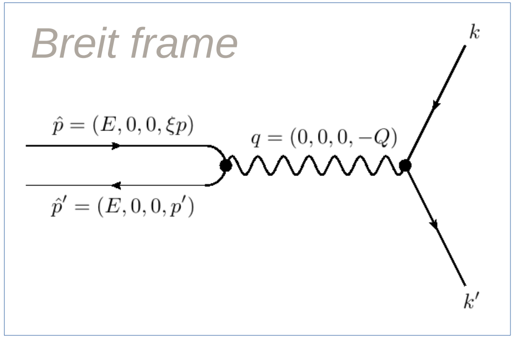
QCD Compton



Exemplary event display

Jets in DIS measured in Breit frame

- ep -> 2jets
- Virtual boson collides 'head-on' with parton from proton
- Boson-gluon fusion dominant process
  QCD compton important only for high-p_T jets (high-x)



Jet measurement sensitive to α_s and gluon density

Inclusive jet cross sections by H1

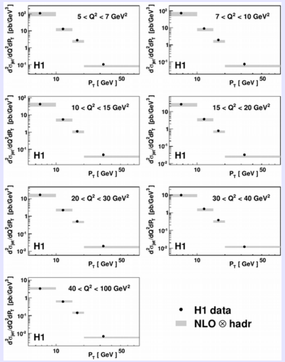
Inclusive jet cross sections

- $d\sigma/dQ^2dP_T^{jet}$
- 300 GeV, HERA-I & HERA-II
- low-Q^2 (<100 GeV^2) and high-Q^2 (>150 GeV^2) regions

Consistency

- kt-algorithm, R=1
- $-1.0 < \eta < 2.5$
- P_T ranges from 4.5 to 50 GeV

HERA-I low-Q^2



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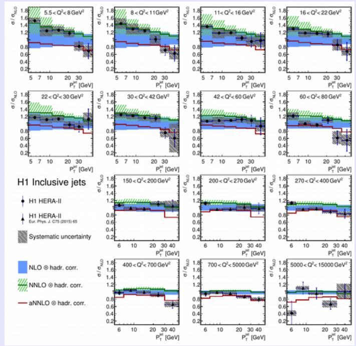
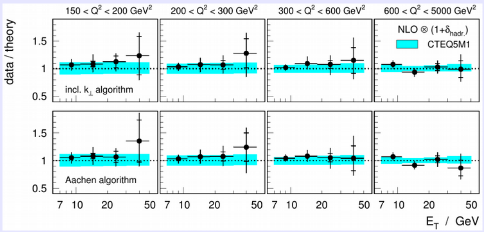
HERA-II low-Q^2



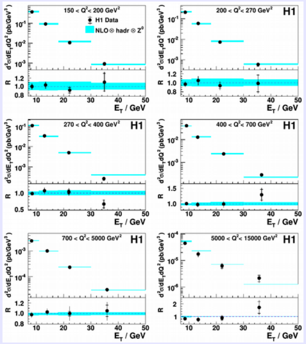
arXiv:1611.03421

300 GeV high-Q^2



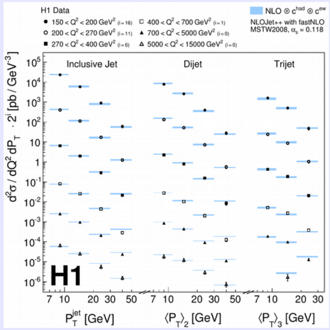
Eur.Phys.J.C19 (2001) 289

HERA-I high-Q^2



Phys.Lett.B653 (2007) 134

HERA-II high-Q^2



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arXiv:1611.03421

Dijet cross section by H1

Dijet definitions

- $\langle p_T \rangle$ greater than 5,7 or 8.5 GeV
- P_T jet greater 4, 5 or 7 GeV
- Asymmetric cuts on p_T^{jet1} and p_T^{jet2}
- M_{12} cut for two data sets

Dijet cross sections

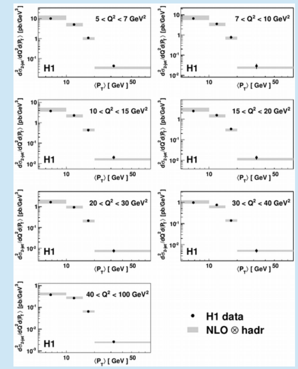
- $d\sigma/dQ^2 d\langle p_T \rangle$
- 300 GeV, HERA-I & HERA-II
- low-Q^2 and high-Q^2

Earlier studies

All inclusive jet and dijet data have been employed for α_s extractions previously

-> Data and uncertainties well-understood
-> NNLO theory is new

HERA-I low-Q^2



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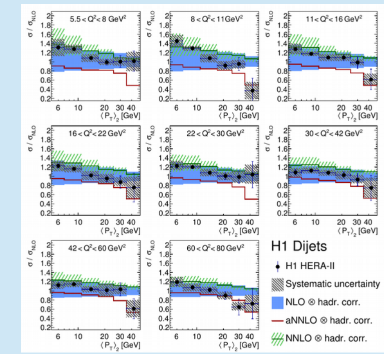
HERA-II low-Q^2



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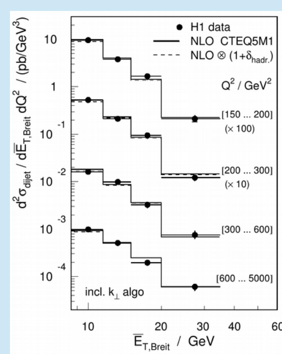
300 GeV high-Q^2



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HERA-I high-Q^2

Dijet cross sections not statistically independent from HERA-II analysis

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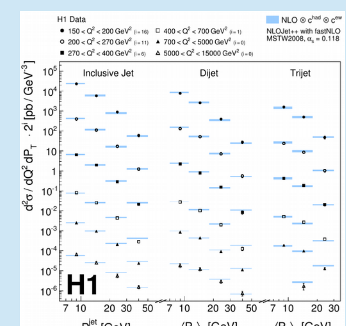
HERA-II high-Q^2



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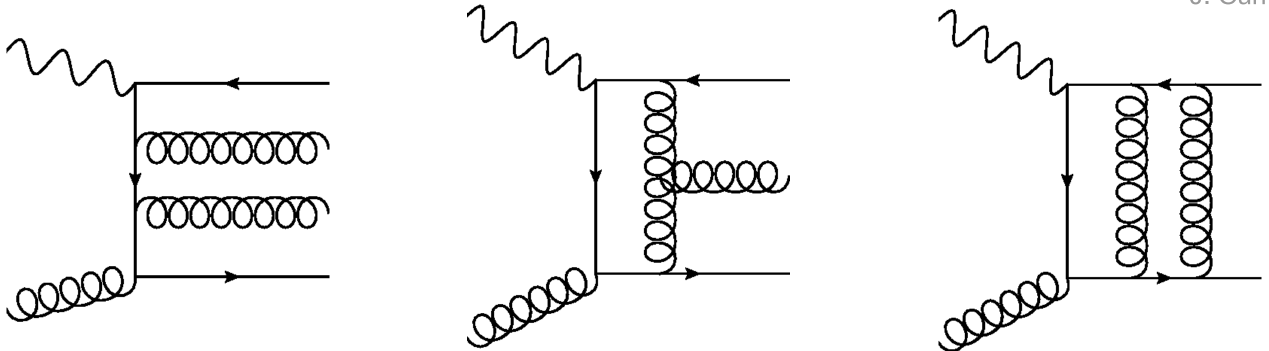
DIS jet production in NNLO

J. Currie, et al. [RPL 117 (2016) 042001]
J. Currie, et al. [JHEP 1707 (2017) 018]



Double-real   Real-virtual   Double-virtual

$$\mathrm{d}\sigma^{RR,S}_{NNLO} \approx \underbrace{X(\{p_X\})}_{\text{antenna}} \overbrace{\mathrm{d}\Phi_3(\{p_X\})}^{\text{Antenna PS}} \times \underbrace{|\mathcal{M}(\{\widetilde{p_m}\})|^2}_{\text{reduced ME}} \overbrace{\mathrm{d}\Phi_m(\{\widetilde{p_m}\})}^{\text{reduced PS}} \times \underbrace{\mathcal{J}(\{\widetilde{p_m}\})}_{\text{jet function}}$$

A bit of history

- 1973 asymptotic freedom of QCD
  [PRL 30(1973) 1343 & 1346]
- 1993 NLO studies of DIS jet cross sections
  [Phys. Rev. D49 (1994) 3291]
- 2016 NNLO corrections for DIS jets
  [Phys. Rev. Lett. 117 (2016) 042001], [arXiv:1703.05977]

Antenna subtraction

- Cancellation of IR divergences with local subtraction terms
- Construction of (local) counter terms
- Move IR divergences across different phase space multiplicities

Scale dependence of NNLO cross sections

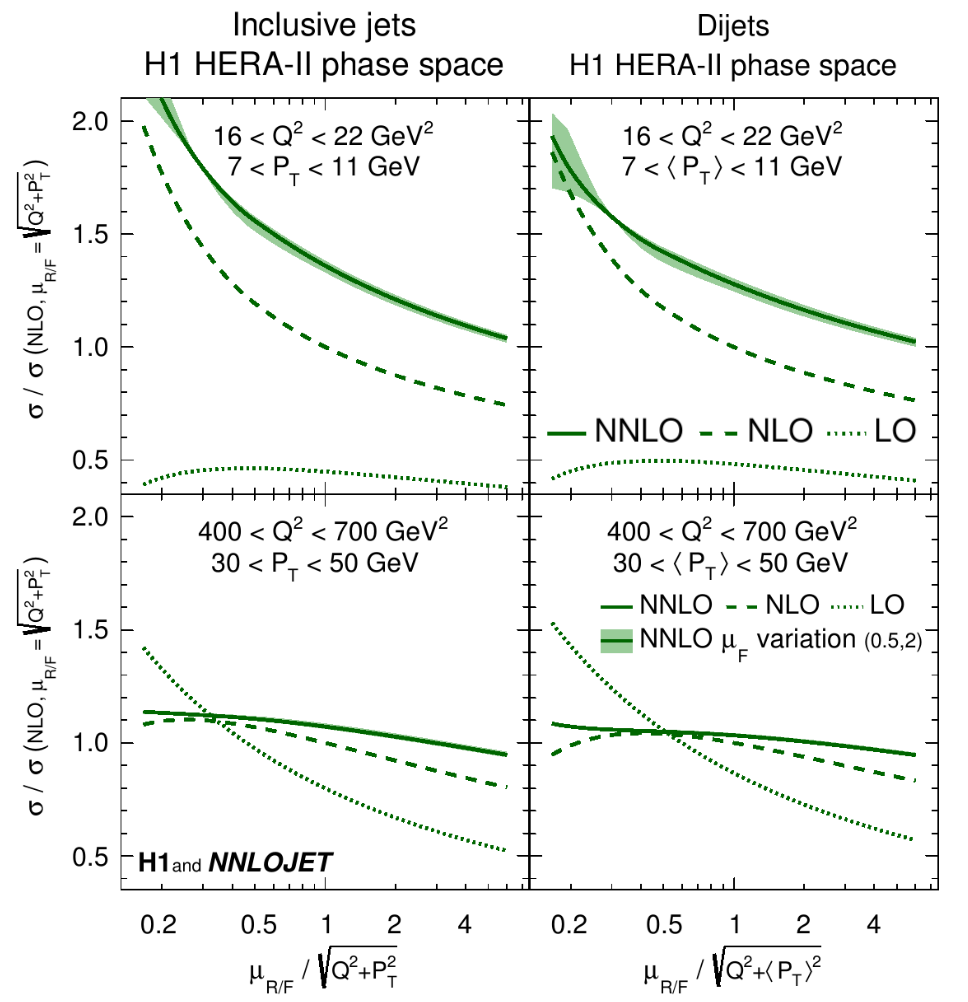
Simultaneous variation of μ_R and μ_F

At lower scales

- Significant NNLO k-factors
- NNLO with reduced scale dependence
- Inclusive jets with higher scale dependence than dijets

At higher scales

- NNLO with reduced scale dependence
- μ_F dependence very small



α_s-fit methodology

α_s determined in χ_2-minimisation

- $\alpha_s(m_Z)$ is a free parameter to NNLO theory prediction σ_i

$$\chi^2 = \sum_{i,j} \log \frac{\varsigma_i}{\sigma_i} (V_{\rm exp} + V_{\rm had} + V_{\rm PDF})^{-1}_{ij} \log \frac{\varsigma_j}{\sigma_j}$$

ς_i	H1 jet data
σ_i	NNLO theory
V	covariance matrices

- NNLO theory is sensitive to $\alpha_s(m_Z)$

$$\sigma_i = \sum_{n=1}^{\infty} \sum_{k=g,q,\bar{q}} \int dx f_k(x, \mu_F) \hat{\sigma}^{(n)}_{i,k}(x, \mu_R, \mu_F) \cdot c_{\rm had}$$

Hard ME's

$$\hat{\sigma}^{(n)}_{i,k} = \alpha_s^n(\mu_R) \tilde{\sigma}^{(n)}_{i,k}(x, \mu_R, \mu_F)$$

PDFs $\frac{\partial f}{\partial \alpha_s} = \frac{\mathcal{P} \otimes f}{\beta}$

- α_s dependence of PDF is accounted for by using $\mu_{F,0}$=20GeV and applying DGLAP

Perform fits to

- All inclusive jet data sets (137 data points)
- All dijet data sets (103 data points)
- All H1 jet data taken together (denoted as 'H1 jets')
  (exclude HERA-I dijet data as correlations to inclusive jets are not known)

Strong coupling in NNLO from jets

***α_s* from individual data sets**

- High experimental precision
- Scale uncertainty is largest (theory) error
- All fits with good χ^2
  -> consistency of data

Main result

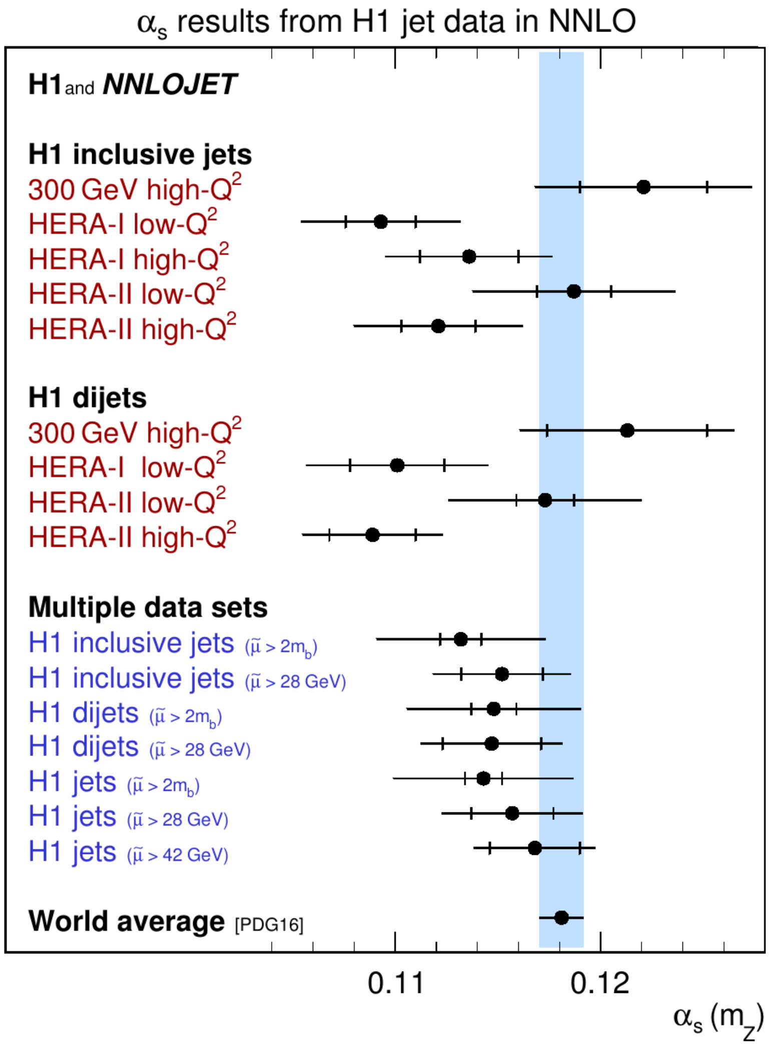
- Inclusive jets & dijets
  μ>28GeV, 91 data points

$$\alpha_s(m_Z) = 0.1157\,(20)_{\rm exp}\,(6)_{\rm had}\,(3)_{\rm PDF}\,(2)_{{\rm PDF}\alpha_s}\,(3)_{\rm PDFset}\,(27)_{\rm scale}$$

- Moderate exp. precision (due to μ>28GeV)
- Scale uncertainty dominates
- PDF uncertainties negligible

Smallest exp. uncertainty

- Fit to all data: $\Delta\alpha_s = (9)_{\rm exp}$



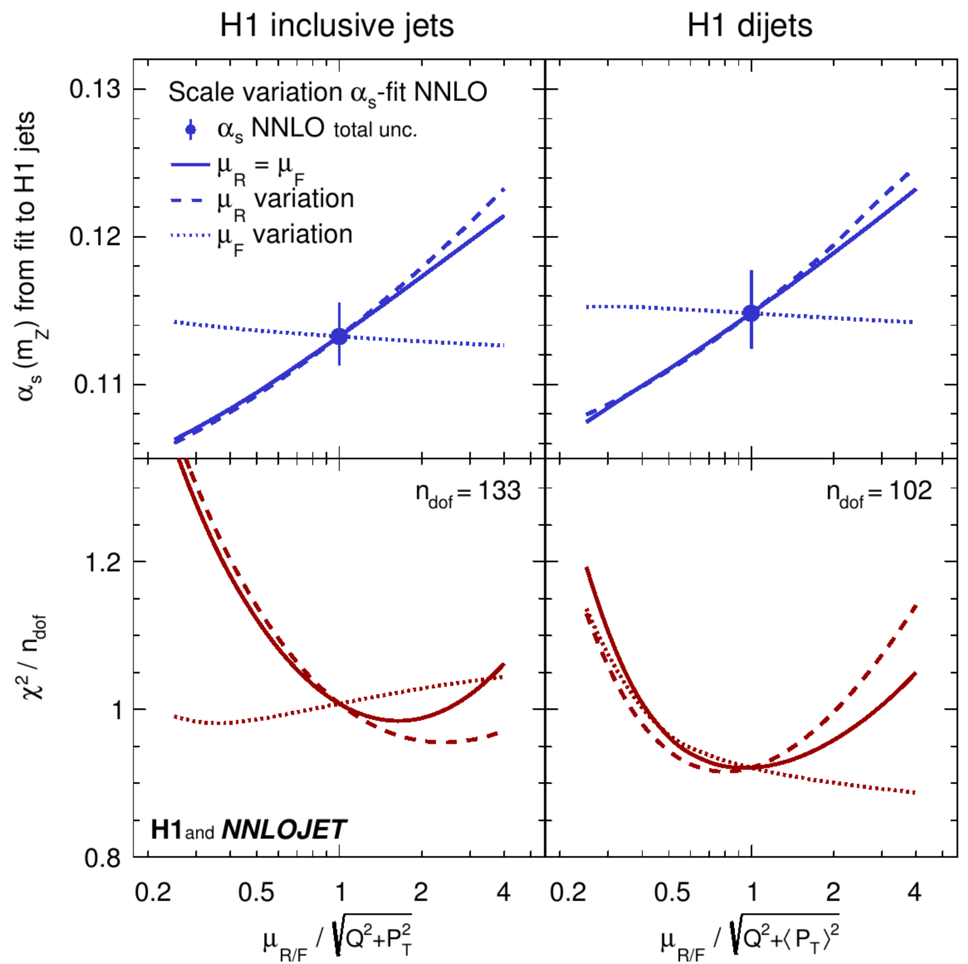
Scale dependence of α_s fit

α_s results as a function of scale factors

- Smooth results for studied scale variations
- μ_R variation with more impact than μ_F

χ^2 values

- somewhat a 'technical parameter'
  -> not intended to be a parabolas
- χ^2 values increase for large scale factors
  -> large scale factors disvafored



Scale choice for α_s fit

Study scales calculated from Q^2 and p_T

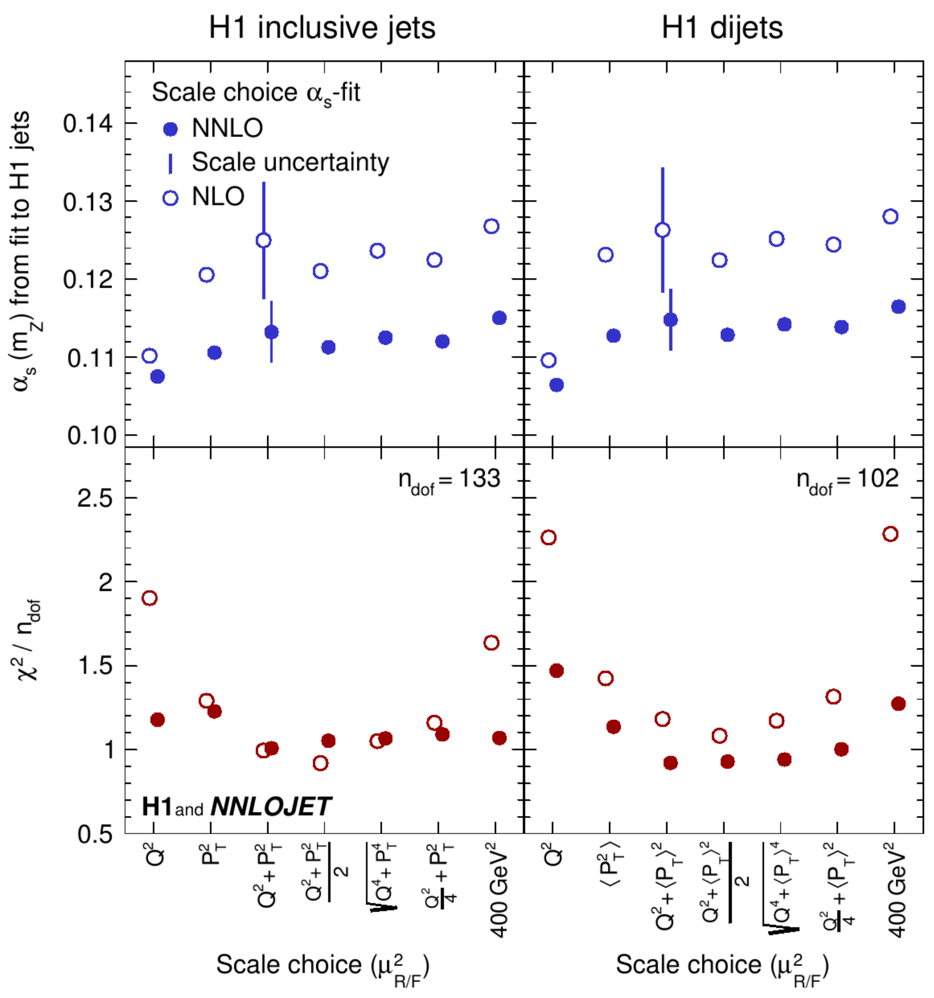
'p_T' refers to: p_T^{jet} or $<p_T>$

α_s results and χ^2 values

- Spread of results covered by scale uncertainty
- χ^2 values are similar for different choices
  -> NNLO with small 'scale dependence'

NLO matrix elements

- Large scale uncertainty
- Relevant dependence of result on scale choice
- Mainly larger χ^2 values than NNLO
- Larger fluctuation of χ^2 values than NNLO



NNLO with reduced scale dependence

Dependence on the PDF

PDF is an external input to NNLO calculation

PDF fitting groups differ

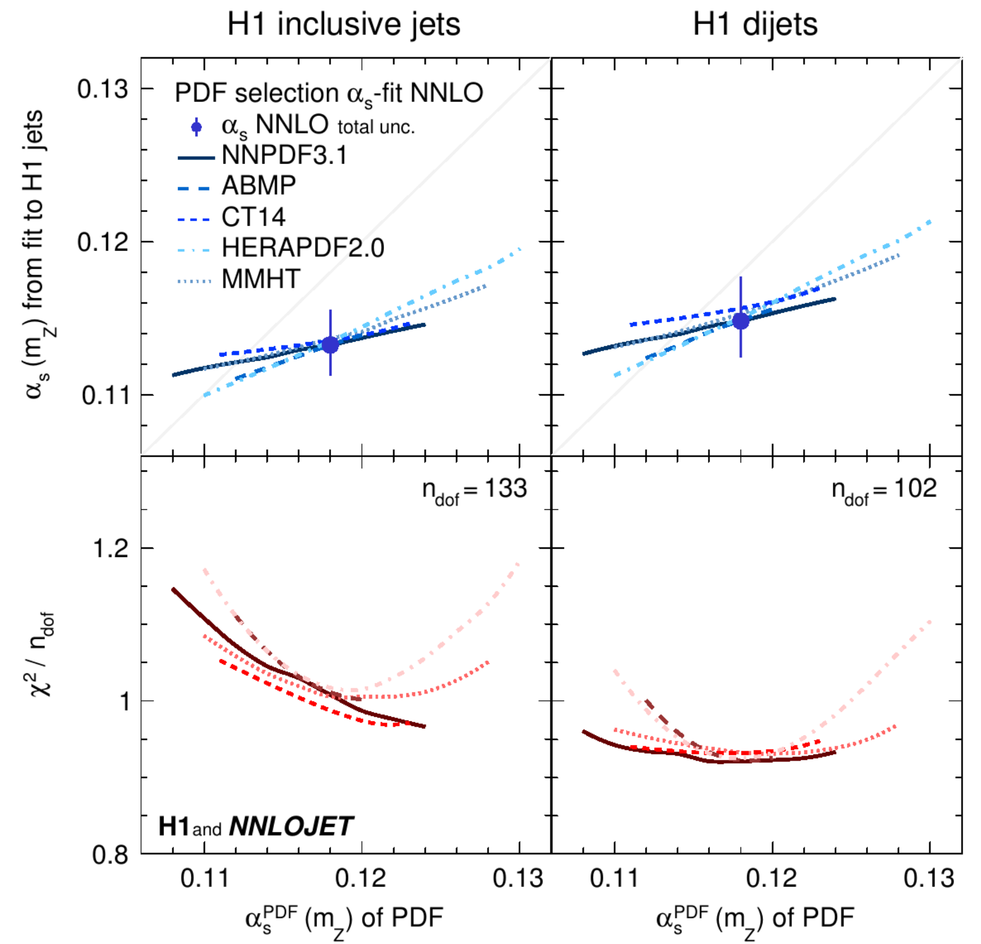
- choice of input data sets,
  PDF parameterisations, model parameters, fit methodology, etc...
- Though:
  different PDFs appear to be quite consistent

Choice of α_s for PDF determination

- $\alpha^{PDF}_s(m_Z)$ important input parameter to PDF fit
- Small correlation with fitted results

Our (main) α_s result

- almost independent on PDF assumptions



Comparison of NNLO predictions with data

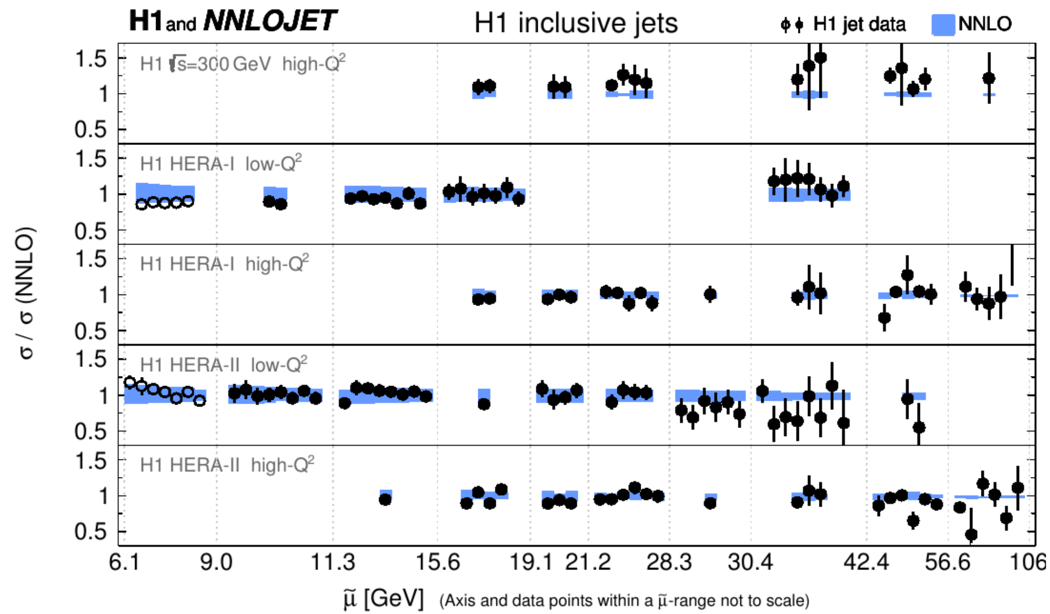
All H1 jet cross section data compared to NNLO predictions

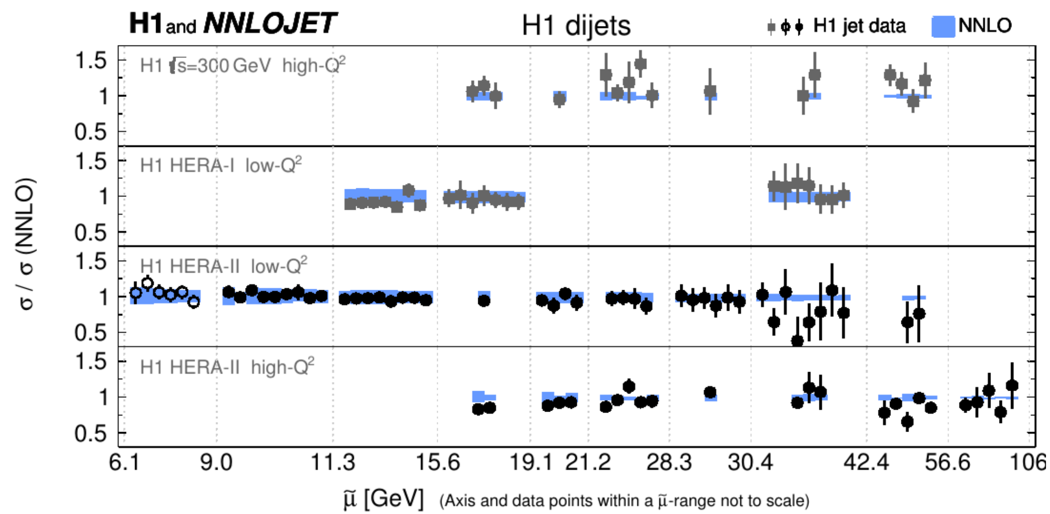
- Inclusive jets
- Dijets

Overall good agreement

- NNLO describes all data very well
- Also justified of course by good χ^2 values of the fits

Great success of pQCD





Tests of running of strong coupling

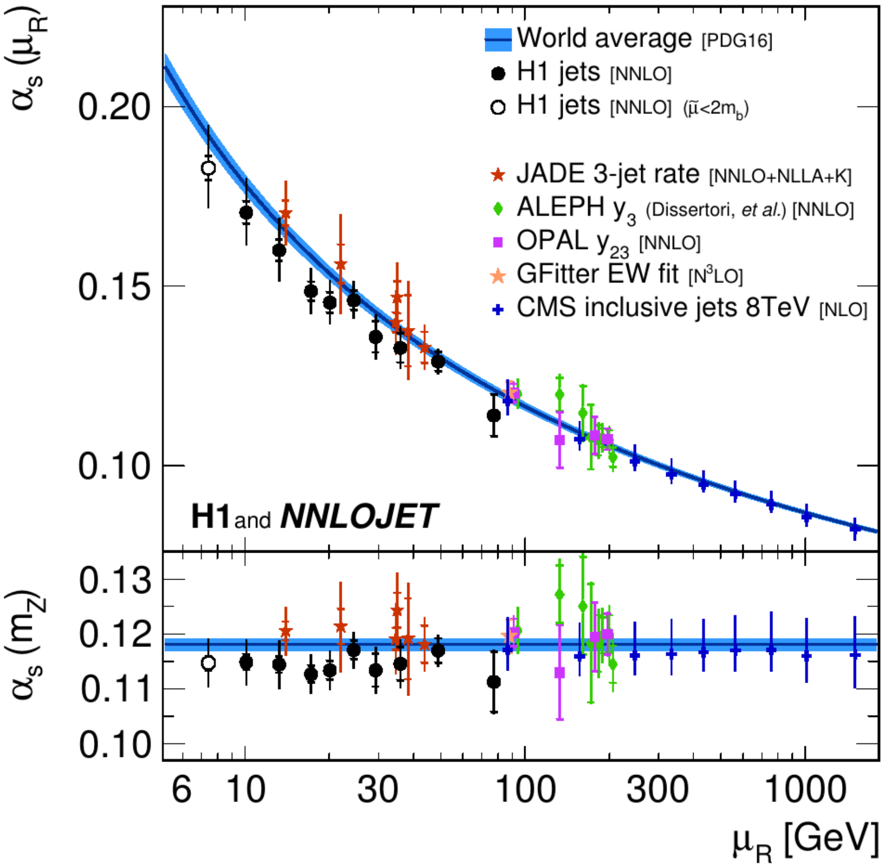
Test running of strong coupling

- Perform fits to groups of data points at similar scale
- Assumes running to be valid within the limited range covered by interval
- All fits have good χ^2

Results

- Consistency with expectation at all scales
- Scale uncertainty dominates at lower μ
- Consistency of inclusive jets and dijets (backup)

Most precise test in range $7 < \mu < 90$ GeV



Alternative α_s fitting approach

'PDF+α_s-fit'
H1PDF2017

Alternative α_s fitting approach: 'PDF+α_s-fit'

Simultaneous fit PDFs and α_s

- PDFs are predominantly determined from H1 inclusive DIS data

Perform H1 alone PDF fit: H1PDF2017

- Use (all) H1 inclusive DIS data
- Use (all) H1 normalised jet cross section data

-> 1529 data points

Normalised jet cross sections

- Jet cross sections normalised to inclusive DIS
- Correlations of jets and inclusive DIS cancel

PDFs are parameterised as

$$xf(x)|_{\mu_0} = f_A x^{f_B}(1-x)^{f_C}(1+f_D x + f_E x^2)$$

Cross section: ~ PDF ⊗σ

$$\sigma_i = \sum_{k=g,q,\overline{q}} \int dx f_k(x,\mu_F)\hat{\sigma}_{i,k}(x,\mu_R,\mu_F) \cdot c_{\text{had},i}$$

Normalised jets

Data set [ref.]	Q^2 domain	Inclusive jets	Dijets	Normalised inclusive jets	Normalised dijets	Stat. corr. between samples
300 GeV [17]	high-Q^2	✓	✓	–	–	–
HERA-I [23]	low-Q^2	✓	✓	–	–	–
HERA-I [21]	high-Q^2	✓	–	✓	–	–
HERA-II [15]	low-Q^2	✓	✓	✓	✓	✓
HERA-II [15, 24]	high-Q^2	✓	✓	✓	✓	✓

Inclusive NC & CC DIS

Data set [ref.]	Lepton type	$\sqrt{s}$ [GeV]	Q^2 range [GeV^2]	NC cross sections	CC cross sections	Lepton beam polarisation
Combined low-Q^2 [64]	e^+	301,319	(0.5) 12 – 150	✓	–	–
Combined low-E_p [64]	e^+	225,252	(1.5) 12 – 90	✓	–	–
94 – 97 [61]	e^+	301	150 – 30 000	✓	✓	–
98 – 99 [62, 63]	e^-	319	150 – 30 000	✓	✓	–
99 – 00 [63]	e^+	319	150 – 30 000	✓	✓	–
HERA-II [65]	e^+	319	120 – 30 000	✓	✓	✓
HERA-II [65]	e^-	319	120 – 50 000	✓	✓	✓

PDF+α_s-fit – H1PDF2017 [NNLO]

Result for PDFs

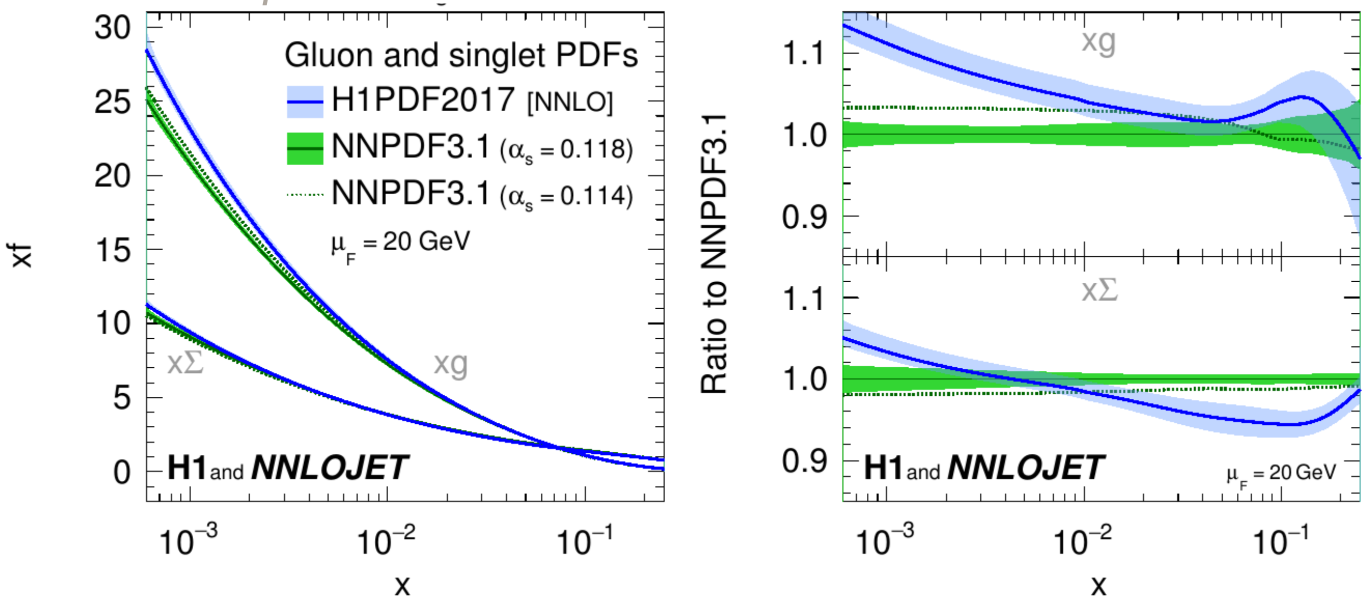
- Set of PDFs determined with high precision
- Despite α_s is a <u>free parameter</u> to the fit: precision is competitive with gloabl PDF fitters
- Gluon at lower *x*-values tends to be higher

  -> nowadays: also favored by small-x resummed PDFs

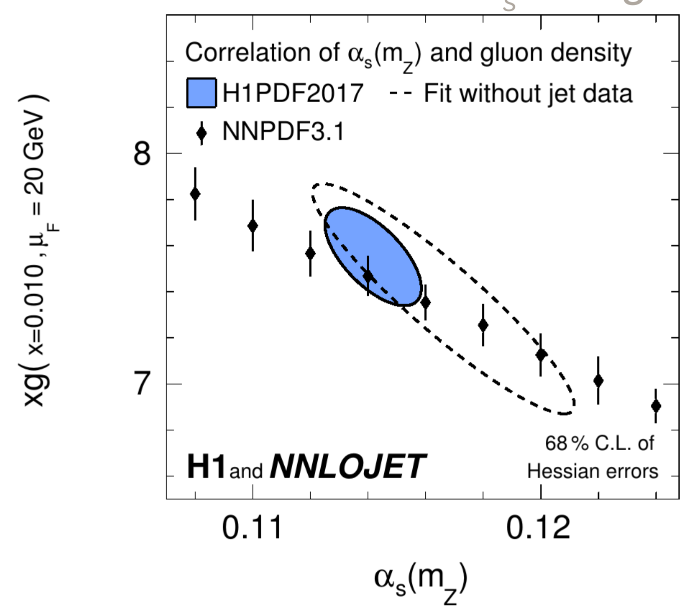
PDF+α_s-fit

- Using H1 jet data allows a precise determination of the <u>gluon PDF</u> and <u>α_s</u>
- χ^2/ndf ~ 1.01

Comparison of H1PDF2017 and NNPDF3.1



Correlation of α_s and g



Results

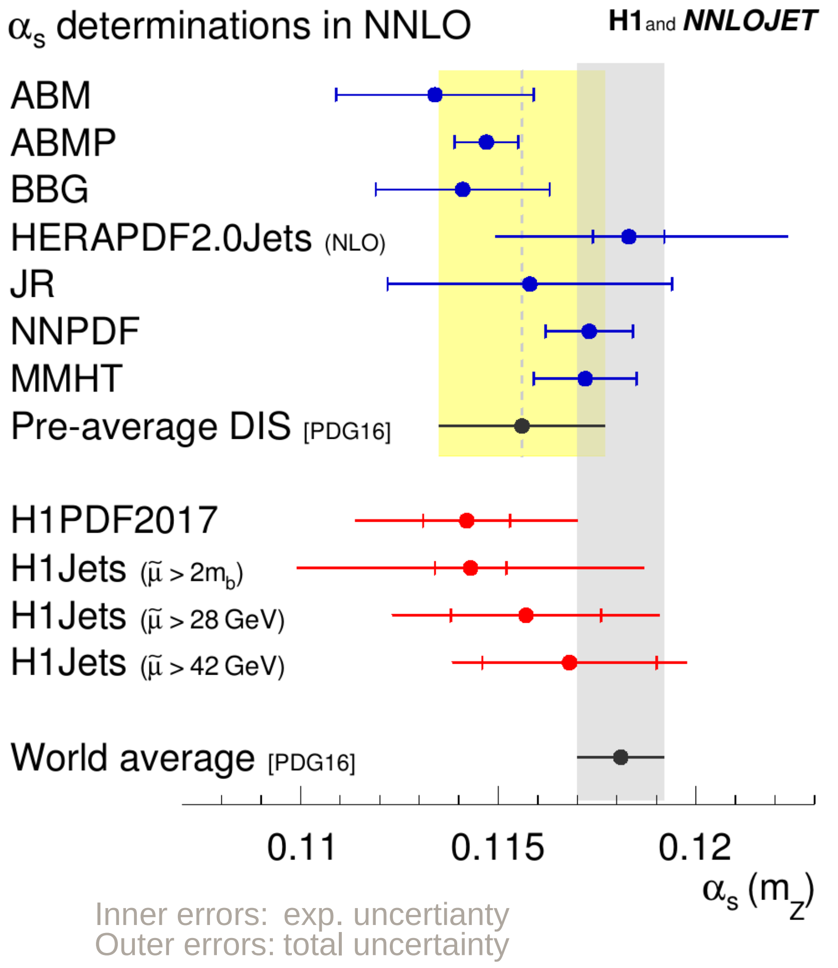
α_s determined in PDF+α_s-fit

$$\alpha_s(m_Z) = 0.1142\,(11)_{\text{exp,had,PDF}}\,(2)_{\text{mod}}\,(2)_{\text{par}}\,(26)_{\text{scale}}$$

- High experimental precision
- Moderate theory uncertainty from NNLO

Comparison

- Higher precision than most of other (comparable) determinations
  -> PDF groups commonly determine exp. uncertainties (only)
  -> We further estimate scale uncertainties
- All H1 results consistent
- Results competitive with world average
- All results from DIS data tend to be lower than world average value



Summary

All H1 jet data confronted with NNLO predictions

- NNLO provides improved description w.r.t. NLO
- Quantitative comparison of all data
- NNLO predictions studied in great detail

NNLO used for determination of $\alpha_s(m_Z)$

- α_s-fit $\quad \alpha_s(m_Z) = 0.1157\,(20)_{\rm exp}\,(6)_{\rm had}\,(3)_{\rm PDF}\,(2)_{{\rm PDF}\alpha_s}\,(3)_{\rm PDFset}\,(27)_{\rm scale}$
- α_s+PDF-fit $\quad \alpha_s(m_Z) = 0.1142\,(11)_{\rm exp,had,PDF}\,(2)_{\rm mod}\,(2)_{\rm par}\,(26)_{\rm scale}$
- High experimental and theoretical precision

NNLO predictions for jets are used for PDF fits for the first time

- Successful determination of gluon-density and $\alpha_s(m_Z)$ simultaneously
- Competitive precision of PDFs and $\alpha_s(m_Z)$
- H1PDF2017 available at LHAPDF

Fruitful collaboration of theoreticians and experimentalists (H1 & NNLOJET)

Study of total uncertainty

Scale uncertainties at various scales μ

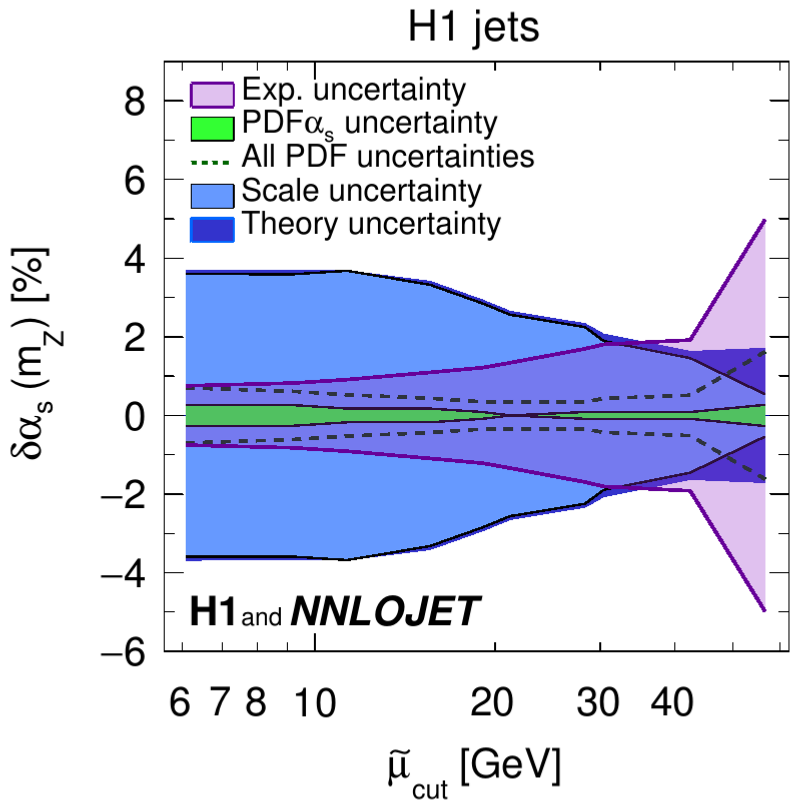
- At low-μ: large scale uncertainties...
- ... but also high sensitivity to $\alpha_s(m_Z)$

Fits imposing a cut on scale μ

- Repeat α_s fits:
  successively cut away data below μ_{cut}

Results

- Scale uncertainty decreases with μ_{cut}
- Exp. uncetainty increases with μ_{cut}



Cut on μ can balance between exp. and theoretical uncertainties at constant total precision

$\alpha_s(m_Z)$ dependence of cross sections

Jet cross sections directly sensitive to α_s

$$\sigma_i = \sum_{n=1}^{\infty} \sum_{k=g,q,\bar{q}} \int dx\, f_k(x,\mu_F)\,\hat{\sigma}_{i,k}^{(n)}(x,\mu_R,\mu_F) \cdot c_{\text{had}}$$

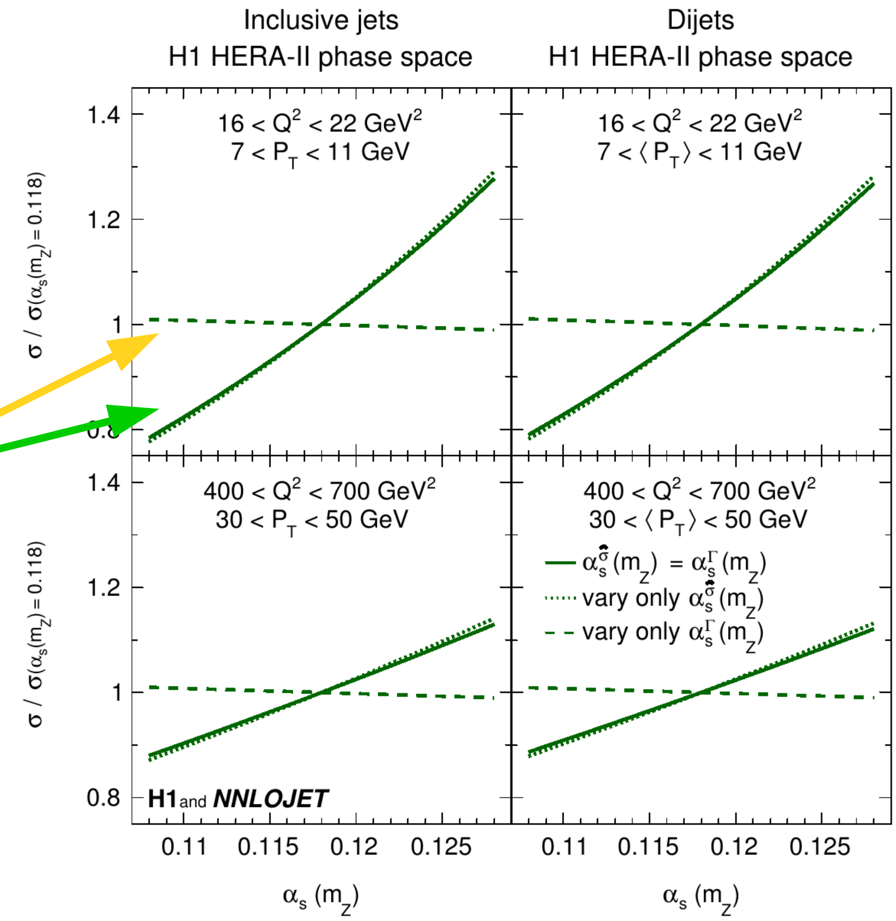
Two α_s-dependencies

PDFs $\frac{\partial f}{\partial \alpha_s} = \frac{\mathcal{P} \otimes f}{\beta}$

Hard ME's

$$\hat{\sigma}_{i,k}^{(n)} = \alpha_s^n(\mu_R)\,\tilde{\sigma}_{i,k}^{(n)}(x,\mu_R,\mu_F)$$

- *Predominant α_s-sensitivity from ME's*
- *PDF's with almost negligible sensitivity*



α_s dependencies separately fitted

Fits to

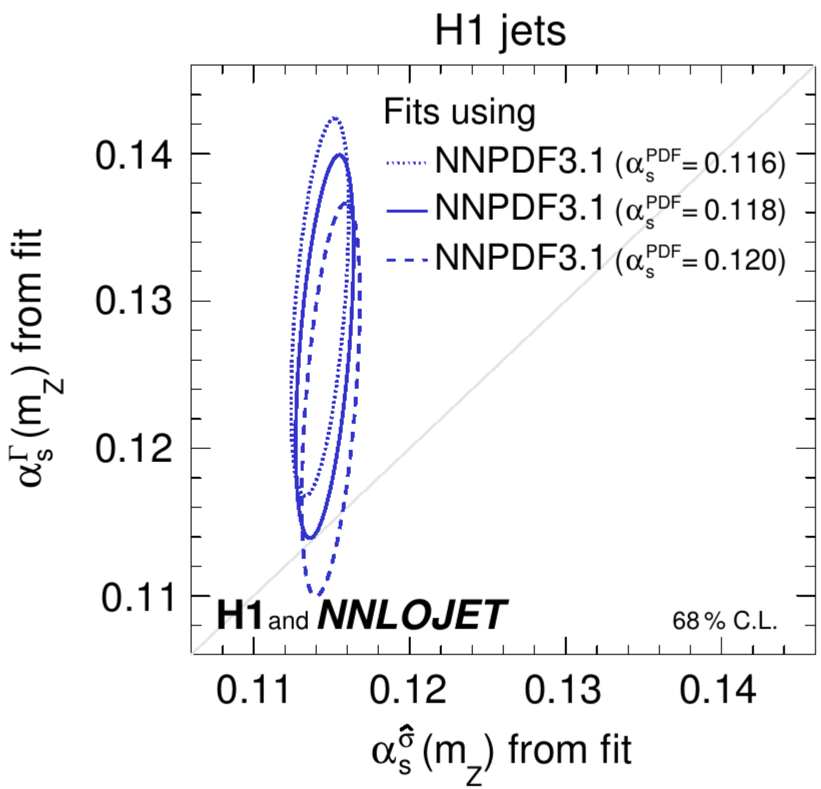
- Inclusive jet and dijet data fitted together
- Fits performed for different PDFs

Fits with two free α_s parameters

$$\sigma_i = f(\alpha_s^f(m_Z)) \otimes \hat{\sigma}_k(\alpha_s^{\hat{\sigma}}(m_Z)) \cdot c_{\rm had}$$

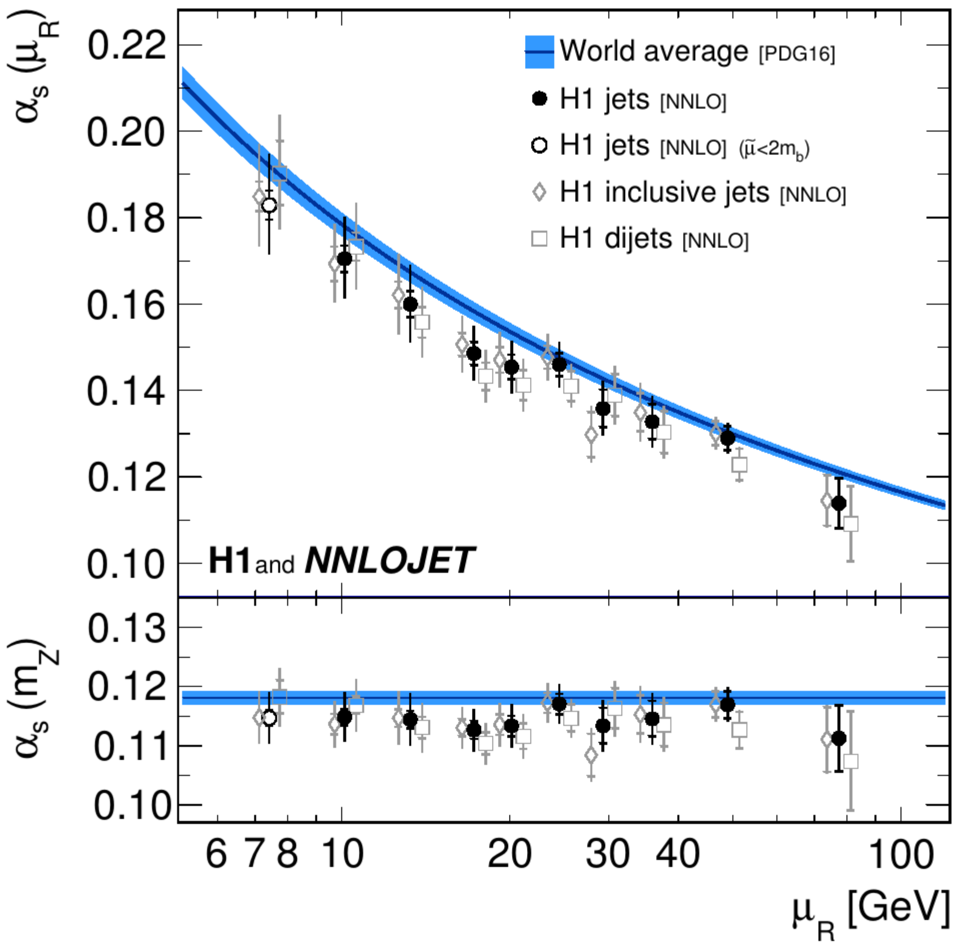
Results

- Most sensitivity arises from matrix elements
- Best-fit α_s-values in PDF's and ME's are consistent
- Anti-correlation between $\alpha_s^{PDF}(m_Z)$ and $\alpha_s^{\Gamma}(m_Z)$



$\alpha_s(m_Z)$ values from H1 jet cross sections

Data	$\tilde{\mu}_{cut}$	**$\alpha_s(m_Z)$ with uncertainties**	**th**	**tot**	χ^2/n_{dof}
Inclusive jets					
300 GeV high-Q^2	$2m_b$	$0.1221\,(31)_{exp}\,(22)_{had}\,(5)_{PDF}\,(3)_{PDF\alpha_s}\,(4)_{PDFset}\,(36)_{scale}$	$(43)_{th}$	$(53)_{tot}$	6.5/15
HERA-I low-Q^2	$2m_b$	$0.1093\,(17)_{exp}\,(8)_{had}\,(5)_{PDF}\,(5)_{PDF\alpha_s}\,(7)_{PDFset}\,(33)_{scale}$	$(35)_{th}$	$(39)_{tot}$	17.5/22
HERA-I high-Q^2	$2m_b$	$0.1136\,(24)_{exp}\,(9)_{had}\,(6)_{PDF}\,(4)_{PDF\alpha_s}\,(4)_{PDFset}\,(31)_{scale}$	$(33)_{th}$	$(41)_{tot}$	14.7/23
HERA-II low-Q^2	$2m_b$	$0.1187\,(18)_{exp}\,(8)_{had}\,(4)_{PDF}\,(4)_{PDF\alpha_s}\,(3)_{PDFset}\,(45)_{scale}$	$(46)_{th}$	$(50)_{tot}$	29.6/40
HERA-II high-Q^2	$2m_b$	$0.1121\,(18)_{exp}\,(9)_{had}\,(5)_{PDF}\,(4)_{PDF\alpha_s}\,(2)_{PDFset}\,(35)_{scale}$	$(37)_{th}$	$(41)_{tot}$	42.5/29
Dijets					
300 GeV high-Q^2	$2m_b$	$0.1213\,(39)_{exp}\,(17)_{had}\,(5)_{PDF}\,(2)_{PDF\alpha_s}\,(3)_{PDFset}\,(31)_{scale}$	$(35)_{th}$	$(52)_{tot}$	13.6/15
HERA-I low-Q^2	$2m_b$	$0.1101\,(23)_{exp}\,(8)_{had}\,(5)_{PDF}\,(4)_{PDF\alpha_s}\,(5)_{PDFset}\,(36)_{scale}$	$(38)_{th}$	$(45)_{tot}$	10.4/20
HERA-II low-Q^2	$2m_b$	$0.1173\,(14)_{exp}\,(9)_{had}\,(5)_{PDF}\,(5)_{PDF\alpha_s}\,(3)_{PDFset}\,(44)_{scale}$	$(45)_{th}$	$(47)_{tot}$	17.4/41
HERA-II high-Q^2	$2m_b$	$0.1089\,(21)_{exp}\,(7)_{had}\,(5)_{PDF}\,(3)_{PDF\alpha_s}\,(3)_{PDFset}\,(25)_{scale}$	$(27)_{th}$	$(34)_{tot}$	28.0/23
H1 inclusive jets	$2m_b$	$0.1132\,(10)_{exp}\,(5)_{had}\,(4)_{PDF}\,(4)_{PDF\alpha_s}\,(2)_{PDFset}\,(40)_{scale}$	$(40)_{th}$	$(42)_{tot}$	134.0/133
H1 inclusive jets	28 GeV	$0.1152\,(20)_{exp}\,(6)_{had}\,(2)_{PDF}\,(2)_{PDF\alpha_s}\,(3)_{PDFset}\,(26)_{scale}$	$(27)_{th}$	$(33)_{tot}$	44.1/60
H1 dijets	$2m_b$	$0.1148\,(11)_{exp}\,(6)_{had}\,(5)_{PDF}\,(4)_{PDF\alpha_s}\,(4)_{PDFset}\,(40)_{scale}$	$(41)_{th}$	$(42)_{tot}$	93.9/102
H1 dijets	28 GeV	$0.1147\,(24)_{exp}\,(5)_{had}\,(3)_{PDF}\,(2)_{PDF\alpha_s}\,(3)_{PDFset}\,(24)_{scale}$	$(25)_{th}$	$(35)_{tot}$	30.8/43
H1 jets	$2m_b$	$0.1143\,(9)_{exp}\,(6)_{had}\,(5)_{PDF}\,(5)_{PDF\alpha_s}\,(4)_{PDFset}\,(42)_{scale}$	$(43)_{th}$	$(44)_{tot}$	195.0/199
H1 jets	28 GeV	$0.1157\,(20)_{exp}\,(6)_{had}\,(3)_{PDF}\,(2)_{PDF\alpha_s}\,(3)_{PDFset}\,(27)_{scale}$	$(28)_{th}$	$(34)_{tot}$	63.2/90
H1 jets	42 GeV	$0.1168\,(22)_{exp}\,(7)_{had}\,(2)_{PDF}\,(2)_{PDF\alpha_s}\,(5)_{PDFset}\,(17)_{scale}$	$(20)_{th}$	$(30)_{tot}$	37.6/40
H1PDF2017 [NNLO]	$2m_b$	$0.1142\,(11)_{exp,NP,PDF}\,(2)_{mod}\,(2)_{par}\,(26)_{scale}$		$(28)_{tot}$	1539.7/1516



World average [PDG16]
H1 jets [NNLO]
H1 jets [NNLO] ($\tilde{\mu}$<2m$_b$)
H1 inclusive jets [NNLO]
H1 dijets [NNLO]
H1 and NNLOJET
$\alpha_s(\mu_R)$
$\alpha_s(m_Z)$
0.22
0.20
0.18
0.16
0.14
0.12
0.10
0.13
0.12
0.11
0.10
6 7 8 10 20 30 40 100
μ_R [GeV]

Data set [ref.]	$\sqrt{s}$ [GeV]	$\mathcal{L}$ [pb^{-1}]	DIS kinematic range	Inclusive jets	Dijets $n_{\mathrm{jets}} \geq 2$
300 GeV [17]	300	33	$150 < Q^2 < 5000\,\mathrm{GeV}^2$ $0.2 < y < 0.6$	$7 < P_{\mathrm{T}}^{\mathrm{jet}} < 50\,\mathrm{GeV}$	$P_{\mathrm{T}}^{\mathrm{jet}} > 7\,\mathrm{GeV}$ $8.5 < \langle P_{\mathrm{T}} \rangle < 35\,\mathrm{GeV}$
HERA-I [23]	319	43.5	$5 < Q^2 < 100\,\mathrm{GeV}^2$ $0.2 < y < 0.7$	$5 < P_{\mathrm{T}}^{\mathrm{jet}} < 80\,\mathrm{GeV}$	$5 < P_{\mathrm{T}}^{\mathrm{jet}} < 50\,\mathrm{GeV}$ $5 < \langle P_{\mathrm{T}} \rangle < 80\,\mathrm{GeV}$ $m_{12} > 18\,\mathrm{GeV}$ $(\langle P_{\mathrm{T}} \rangle > 7\,\mathrm{GeV})^*$
HERA-I [21]	319	65.4	$150 < Q^2 < 15000\,\mathrm{GeV}^2$ $0.2 < y < 0.7$	$5 < P_{\mathrm{T}}^{\mathrm{jet}} < 50\,\mathrm{GeV}$	–
HERA-II [15]	319	290	$5.5 < Q^2 < 80\,\mathrm{GeV}^2$ $0.2 < y < 0.6$	$4.5 < P_{\mathrm{T}}^{\mathrm{jet}} < 50\,\mathrm{GeV}$	$P_{\mathrm{T}}^{\mathrm{jet}} > 4\,\mathrm{GeV}$ $5 < \langle P_{\mathrm{T}} \rangle < 50\,\mathrm{GeV}$
HERA-II [15, 24]	319	351	$150 < Q^2 < 15000\,\mathrm{GeV}^2$ $0.2 < y < 0.7$	$5 < P_{\mathrm{T}}^{\mathrm{jet}} < 50\,\mathrm{GeV}$	$5 < P_{\mathrm{T}}^{\mathrm{jet}} < 50\,\mathrm{GeV}$ $7 < \langle P_{\mathrm{T}} \rangle < 50\,\mathrm{GeV}$ $m_{12} > 16\,\mathrm{GeV}$

Inclusive jet cross sections

Inclusive jet cross sections
- low Q^2: $4.5 < P_T < 50$ GeV
- high Q^2: $5 < P_T < 50$ GeV

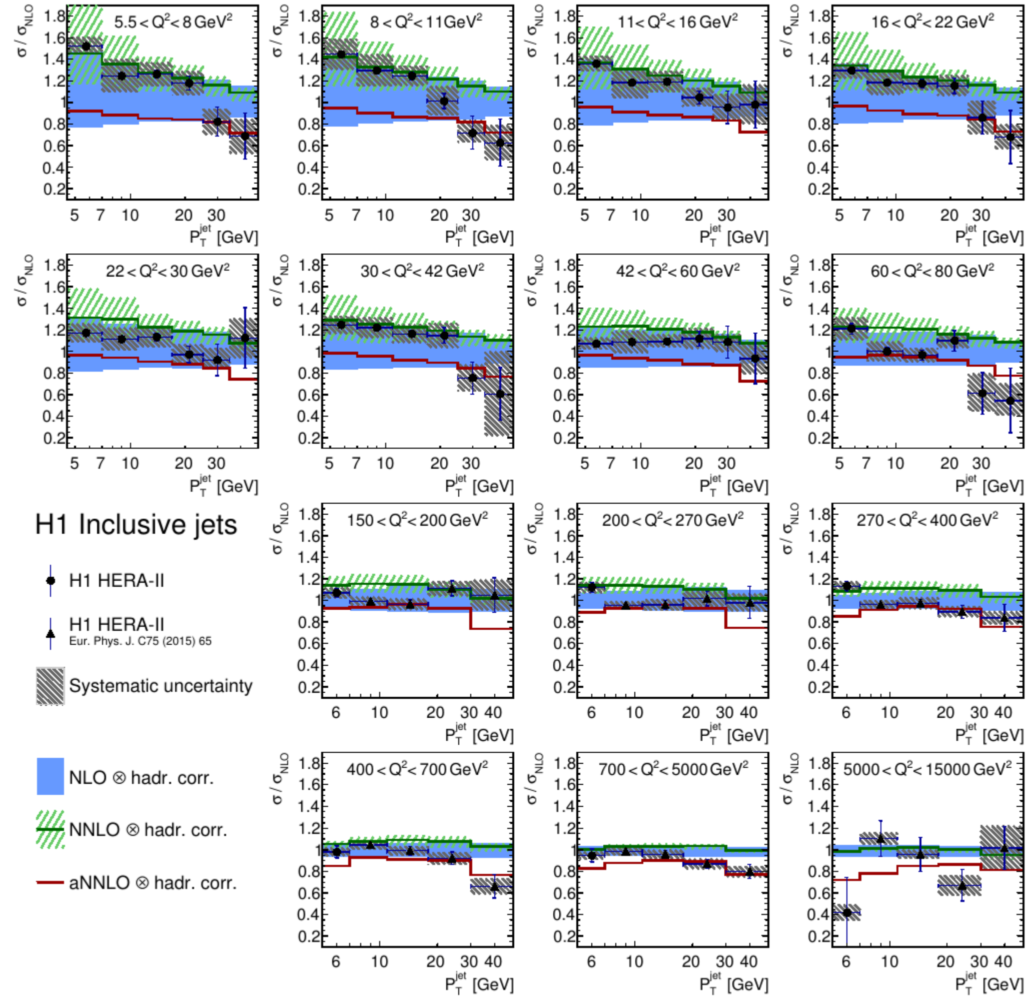
Predictions
- NLO, aNNLO & NNLO

NLO
- Data well described within uncertainties

aNNLO
- Somewhat improved shape description

NNLO
- Improved shape and normalisation
- Reduced scale uncertainties for larger values of μ_r

Also measured
- Normalised inclusive jet cross sections



Ratio of dijet cross sections to NLO

Scale uncertainty

- So-called '7-point scale variation':
  Vary μ_r and μ_f independently by factors of 2 and 0.5, but exclude variations in 'opposite' directions

Ratio to NLO prediction

- NLO give reasonable descriptions within large scale uncertainties
- aNNLO improves shape
  - aNNLO expected to improve description at high $<p_T>$
- NNLO improves shape dependence
  - NNLO predictions have smaller scale uncertainties than NLO at high-$<p_T>$