Final state interactions in DIS and other QCD phenomena

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Message: The relevance of perturbation theory

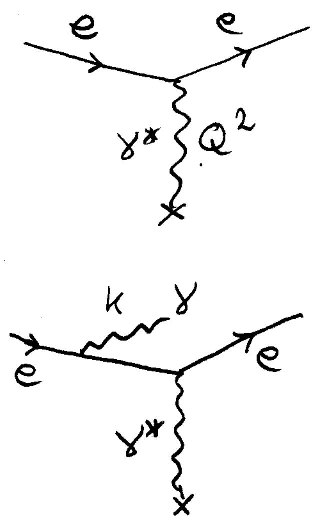
The distinction between “perturbative” and
“non-perturbative” isn’t always obvious

Ex.: Electron form factor

$$\mathcal{O}\left(\alpha^0\right): \quad F_e(Q^2) = 1 \quad \text{(electron is a pointlike particle)}$$

$$\mathcal{O}\left(\alpha\right): \quad F_e(Q^2) = 1 - \frac{\alpha}{2\pi}\log\left(\frac{Q^2}{m_e^2}\right)\log\left(\frac{Q^2}{\varepsilon^2}\right)$$

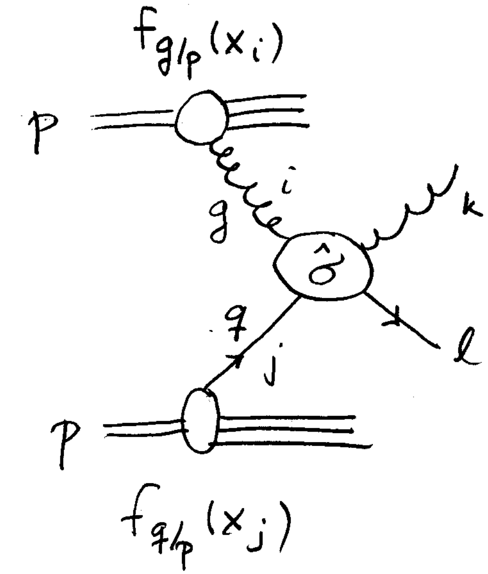
$$\mathcal{O}\left(\alpha^\infty\right): \quad F_e(Q^2) = \exp\left[-\frac{\alpha}{2\pi}\log\left(\frac{Q^2}{m_e^2}\right)\log\left(\frac{Q^2}{\varepsilon^2}\right)\right]$$



Sudakov form factor – falls faster than any power:
Sum differs qualitatively from lowest order

The physics of the Sudakov form factor is understood
– it is part of “perturbative physics”

The success of PQCD in hard processes relies on the QCD factorization theorem, which uses perturbation theory even in the soft domain



$$\sigma(pp \to 2\,jets) = \sum_{i,j=q,g} f_{i/p}(x_i, Q^2) f_{j/p}(x_j, Q^2) \widehat{\sigma}(ij \to k\ell)$$

The derivation, which is privy to the expert few, gives

$$f_{q/N}(x_B, Q^2) = \frac{1}{8\pi} \int dx^- e^{-ix_B p^+ x^-/2} \langle N(p)|\bar{q}(x^-)\gamma^+ W[x^-, 0] q(0)|N(p)\rangle_{x^+=0}$$

where the Wilson line is given by the path ordered exponential:

$$W[x^-, 0] \equiv \mathrm{P}\exp\left[\frac{ig}{2}\int_0^{x^-} dw^- A^+(w^-)\right]$$

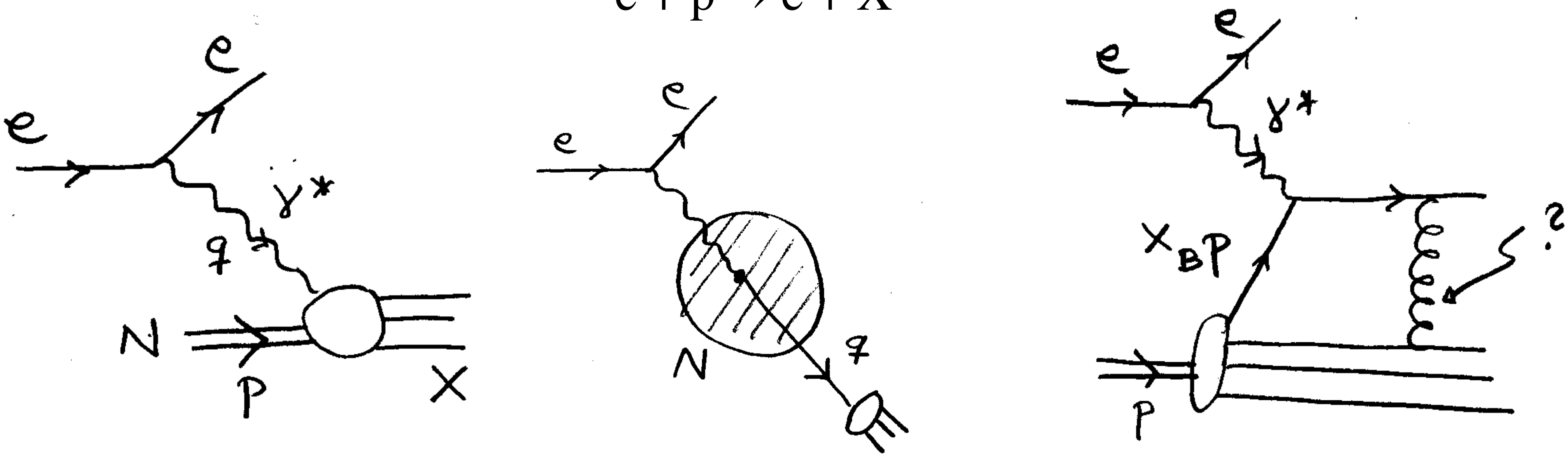
Apparently, $W[x^-,0] = 1$ in $A^+ = 0$ gauge, so that $f_{q/N}$ is given by an overlap of $x^+ = 0$ target wave functions, i.e., by the probability to find a quark in |N>.

Surprisingly, the last statement turned out to be incorrect.
Seeing why also helps understand the dynamics of QCD processes.

Brodsky
PH
Marchal
Peigné
Sannino

Deep Inelastic Scattering (DIS)

e + p → e + X



$$Q^2 = -q^2 \to \infty, \quad \nu = \frac{p\cdot q}{m_p} \to \infty, \quad x_B = \frac{Q^2}{2m_p\nu} \text{ fixed}$$

$$x^{\pm} \equiv x^0 \pm x^3 = t \pm z$$

Remarks:

- Vanishing light front time, $x^+ = t + z = 0$, does not limit t or z for particles (like the struck quark) which move with v = c along the negative z axis.

Photons reach us from the early universe in $x^+ = 0$.

- Secondary interactions occurring within the coherence ('Ioffe') length L_I of the virtual photon typically affect the DIS cross section:

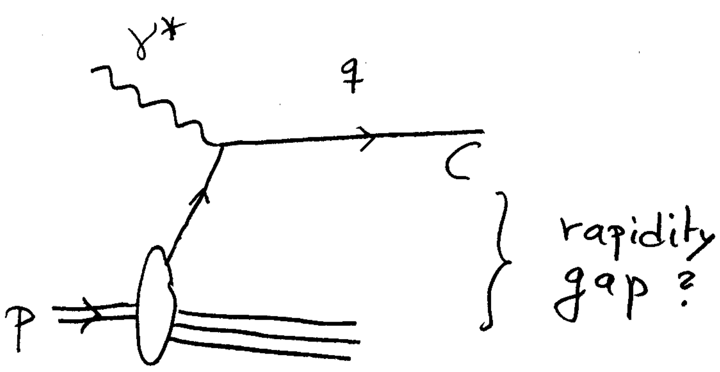
$$L_I = \frac{1}{Q} \cdot \frac{\nu}{Q} = \frac{\nu}{Q^2} = \frac{1}{2mx_B}$$

is long at small x_B

Such interactions cannot be turned off by a choice of gauge

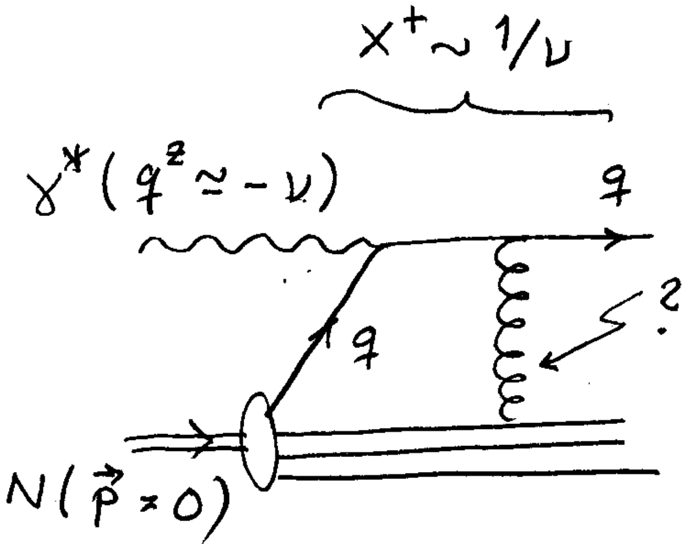
- Diffractive DIS:

In 10% of the events the struck quark seems to escape from the target without a string of hadrons: A "rapidity gap" is formed.



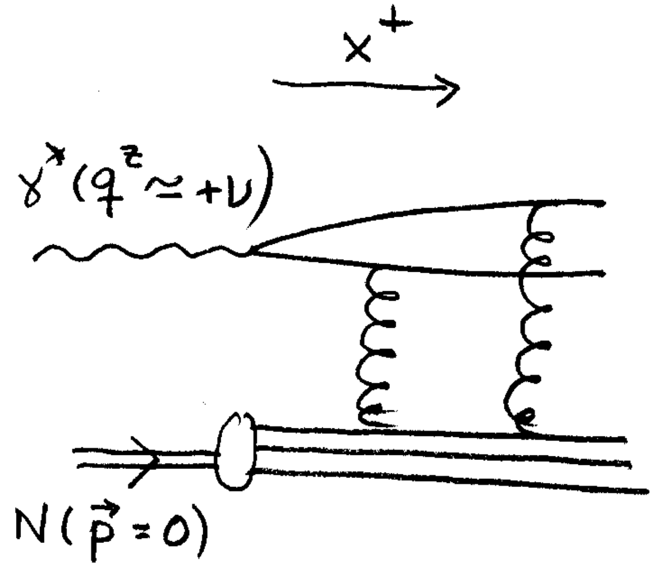
How can color be separated over long distances?

The LF time $x^+ = t + z$ development of DIS looks very different in proton rest frame, depending on whether the γ^* is moving along the -z or +z axis:



$q^Z < 0$: No rescattering

in $A^+ = 0$ gauge?



$q^Z > 0$: quark-antiquark dipole multiple scatters

The amplitudes are related by a 180° rotation around the y axis.

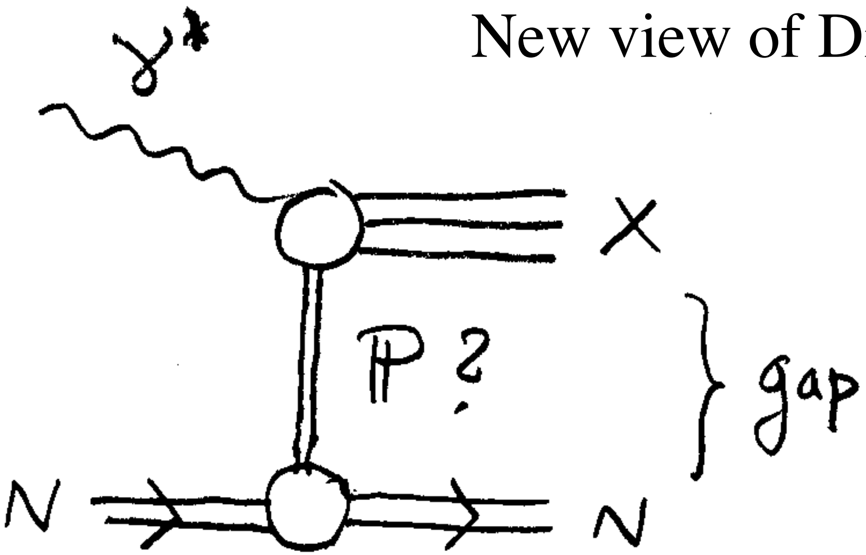
Lorentz transformation cannot be done since proton wave function is unknown.

Enter PQCD: Compare perturbative model calculations in the two frames, using a simple (quark) target.

Find: Rescattering effects persist in all frames and gauges. Gives rise to shadowing, diffraction and spin effects in DIS.

New view of Diffractive DIS (DDIS)

Brodsky
Enberg
Hoyer
Ingelman

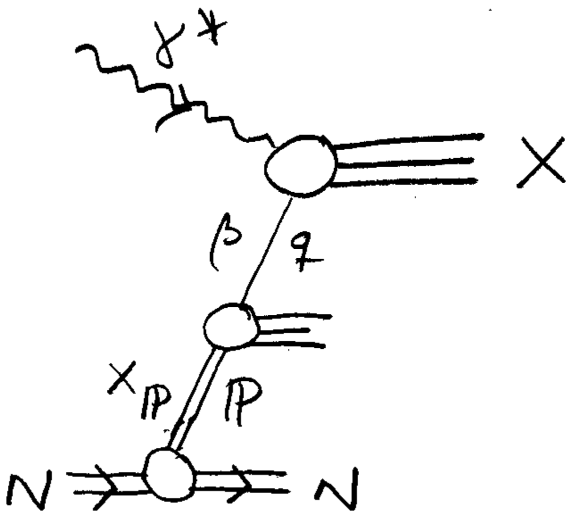


In about 10% of DIS events the proton target stays intact and is separated by a large rapidity gap from the diffractive system X.

What is this 'Pomeron' exchange?

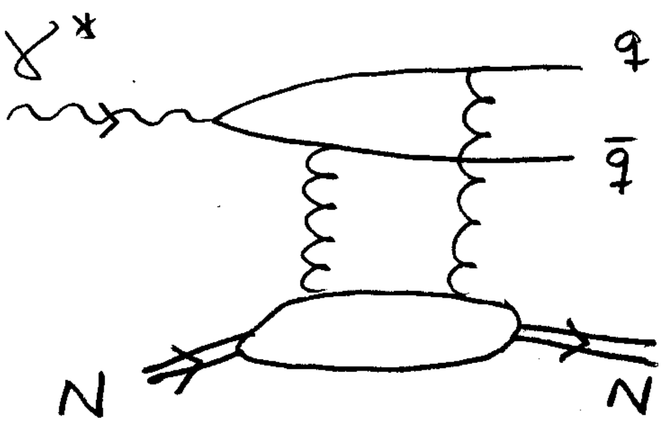
Ingelman-Schlein picture

Virtual photon probes the pomeron as a constituent of the proton target:



Two-gluon exchange models

Dipole scattering, formulated in the $q^z > 0$ frame:



The QCD factorization theorem has been extended to the diffractive structure function: $F_2^{(D)} = \sum_{i=q,G} f_{i/p}^D \otimes \widehat{\sigma}_i$ where the subprocess cross section σ_i is

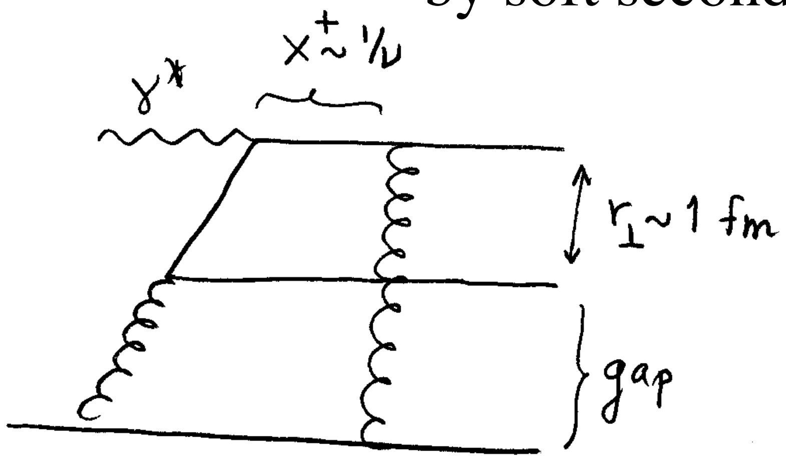
Collins

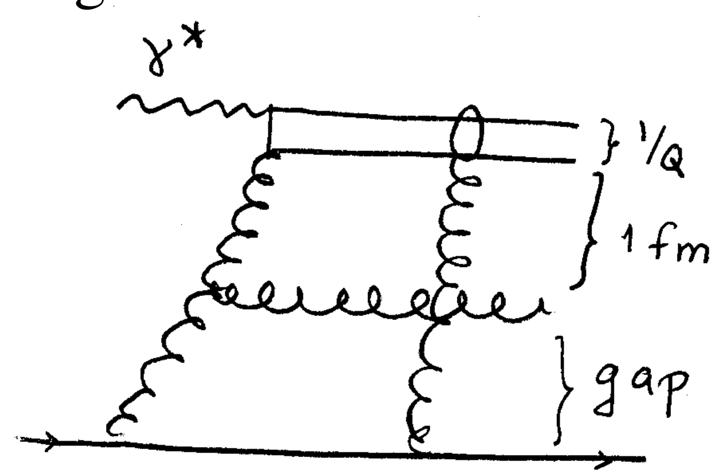
the same as in ordinary DIS, and the diffractive parton distribution f^D parton has the same Q^2 dependence as the inclusive f.

→ DDIS involves hard scattering on a single quark or gluon

In the presence of rescattering effects, diffraction is readily understood as due to color neutralization from soft rescattering.

Note: Diffractive qq or gg dipole must have large size, ~ 1 fm, to be resolved by soft secondary gluon exchange.





Consequences of DDIS arising from soft rescattering:

- The rescattering will not resolve hard gluon emission at γ^* vertex

=> Q^2 evolution identical to DIS (as required by the factorization theorem)

- The x_B (or W) dependence is given by standard gluon distribution, hence

$$\frac{\sigma_{DDIS}(x_B, Q^2)}{\sigma_{DIS}(x_B, Q^2)}$$

is independent of x_B and Q^2, in agreement with data

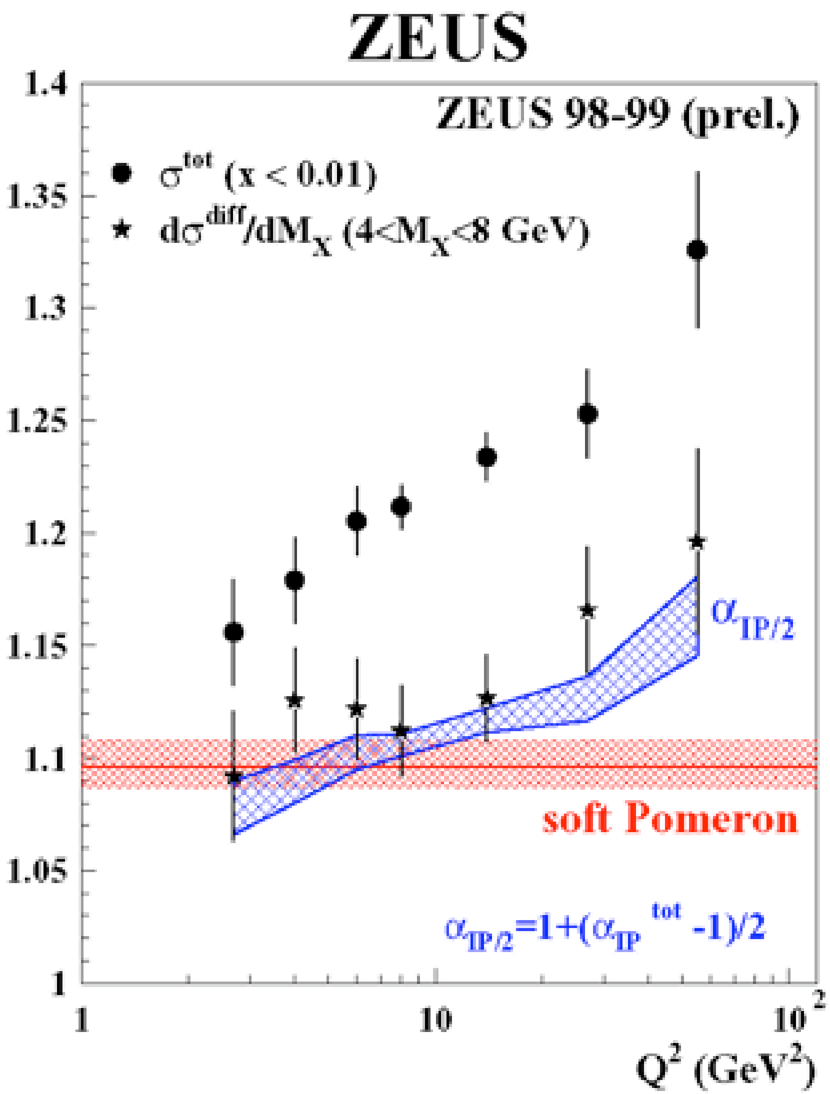
- Note: In a Pomeron Regge model, the ratio would depend on x_B, which is ruled out by the data.

$$\frac{\sigma_{DDIS}^{Regge}(x_B, Q^2)}{\sigma_{DIS}^{Regge}(x_B, Q^2)} = x_B^{1-\alpha_{I\!P}}$$

- Similarly, we would have if both gluons were hard, which is also ruled out.

$$f_{q/p}^{D}(x_B, Q^2) \propto \left[f_{q/p}(x_B, Q^2)\right]^2$$

Compare $\alpha_{I\!P}$ for diffractive and total $\gamma^* p$ scattering

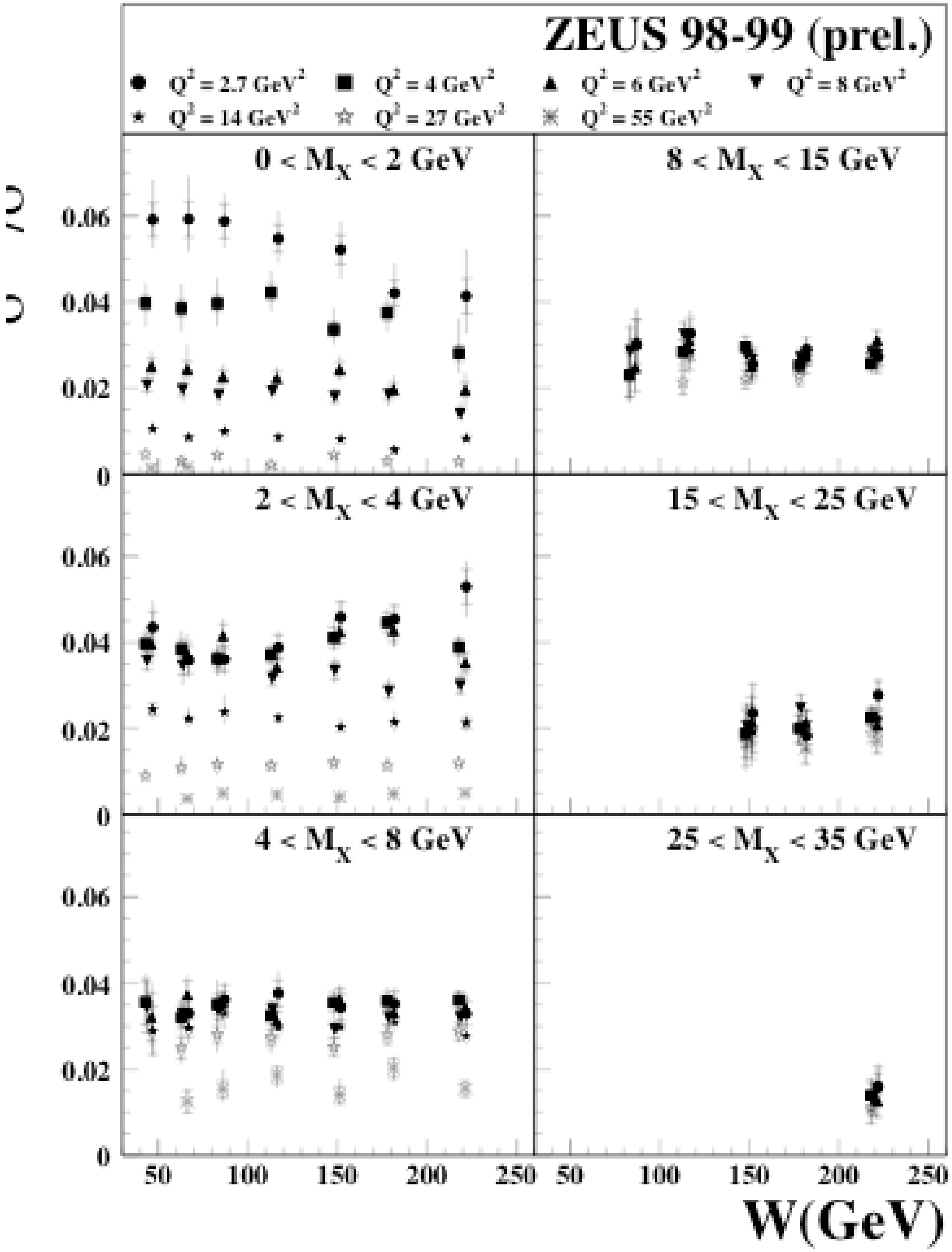


- $$\sigma^{tot}_{\gamma^* p} = \frac{4\pi^2\alpha}{Q^2} \cdot F_2(x, Q^2)$$
  $$\sim \frac{1}{W^2} Im T_{\gamma^* p \to \gamma^* p}(W^2, t=0) \sim (W^2)^{\alpha^{tot}_{I\!P}(0)-1}$$
  (Optical theorem)

- $$\frac{d^2\sigma^{diff}}{dM_X dt} \sim \quad |T_{\gamma^* p \to \gamma^* p}|^2 \quad \sim (W^2)^{2(\alpha^{diff}_{I\!P}(0)-1)}$$
  at $t = 0$

- Data ($4 < M_X < 8$ GeV) show
  $$\Longrightarrow \alpha^{diff}_{I\!P} \approx 1 + (\alpha^{tot}_{I\!P} - 1)/2$$

$$r_{tot}^{diff} = \frac{\int_{M_a}^{M_b} dM_X d\sigma^{diff}_{\gamma^* p \to XN, M_N < 2.3 GeV}/dM_X}{\sigma^{tot}_{\gamma^* p}}$$



- For $M_X < 2$ GeV, r_{tot}^{diff} is falling with W.
- For $M_X > 2$ GeV, r_{tot}^{diff} is constant with W.
  $\Longrightarrow$ The diffractive cross section has about the same W-dependence as σ^{tot}.
- The low M_X bins exhibit a strong decrease of r_{tot}^{diff} with increasing Q^2.
- For $M_X > 8$ GeV, no Q^2 dependence is observed.
- $\sigma^{diff}_{(M_X < 35 \text{ GeV})}/\sigma^{tot}$ at $W = 220$ GeV:
  $= 19.8^{+1.5}_{-1.4}\%$ $(Q^2 = 2.7$ GeV$^2)$
  $= 10.1^{+0.6}_{-0.7}\%$ $(Q^2 = 27$ GeV$^2)$
  $\Longrightarrow$ Slowly decreasing with Q^2

Remarks on hard diffraction via soft rescattering

- Rescattering amplitudes are dominantly imaginary, as expected for diffraction
  cf. Ingelman-Schlein constituent pomeron model: Real amplitude

- Rescattering happens within the Ioffe length (in the proton), long before Lorentz-dilated hadronization time t ~ ν (1 fm)^2
  => Secondary gluon can shield the color of the primary gluon
  This is not a Final State effect in the usual sense of the word!

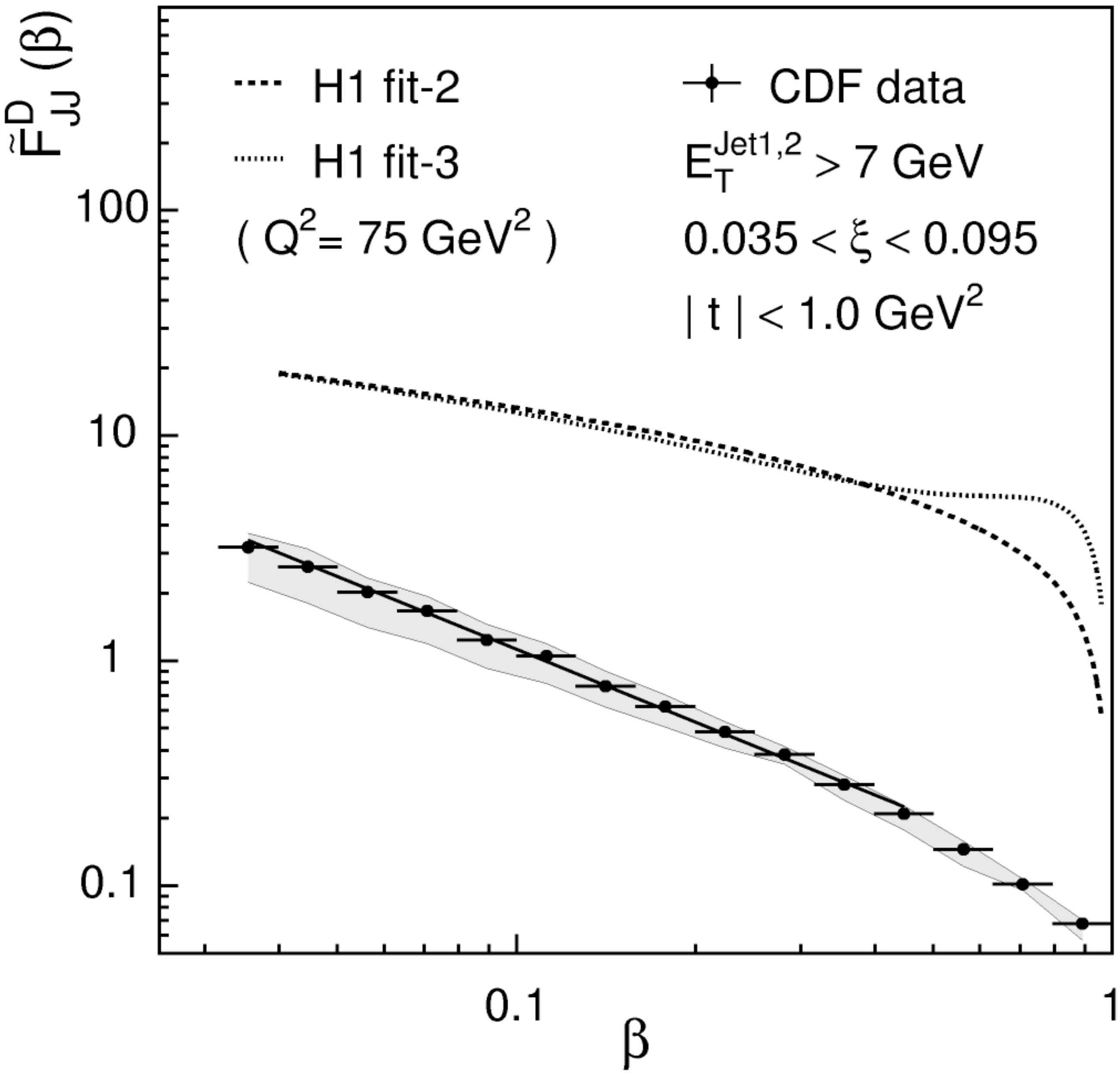
- This scenario is quite similar to the “Soft Color Interaction” (SCI) model developed previously, implemented in Monte Carlo and successfully tested on data on hard diffraction in both DIS and pp.

Edin
Ingelman
Rathsman

- Note: Hard diffraction in pp occurs 1/10 as often as in DIS. This rules out the (universal) Ingelman-Schlein constituent pomeron model.

- Note: Factorization theorem does not apply to diffraction in pp collisions, due to color interactions between projectile and target spectators. This is similar to the present rescattering picture.

CDF Phys. Rev. Lett. **84**, 5043 2000



and next a quite different application of PQCD:

PH
Peigné

Perturbative Parton Dressing

Aim:
- Use formally exact PQCD expansion
- Expand around a nontrivial (condensate) configuration

Find:
- Quark and gluon propagators with a novel analytic structure
- Partons removed from in- and out-states (no pole at $p^2 = 0$)
- Gauge invariance maintained: Photon pole at $p^2 = 0$ remains
- Chiral symmetry breaking solution for quark propagator

Method: Consider the shift $A_\mu(x) \to A_\mu(x) + \Phi_\mu$ of the gluon field in L_{QCD}

- Green functions unchanged, since
- Perturbative expansion is modified.

$$\int \mathcal{D}[A_\mu] = \int \mathcal{D}[A_\mu + \Phi_\mu]$$

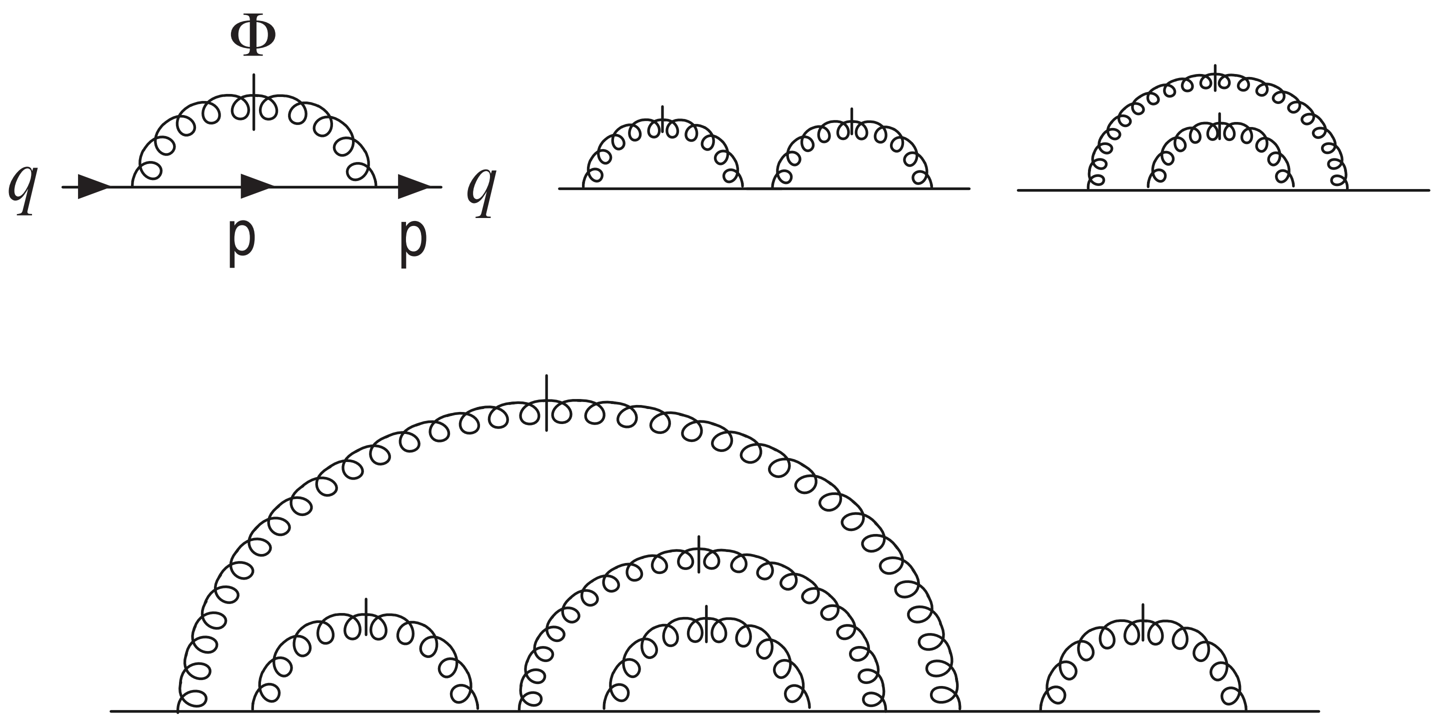
- Quark term in L_{QCD} generates $\bar{q} i \not{D} q \to \bar{q} i \not{D} q - g\, \bar{q} \not{\Phi}\, q$
- New term is gauge invariant under $q \to Uq$ provided $\Phi_\mu \to U \Phi_\mu U^\dagger$

- We take Φ_μ to be independent of x, and average over its values with a gaussian weight with a mass parameter Λ.

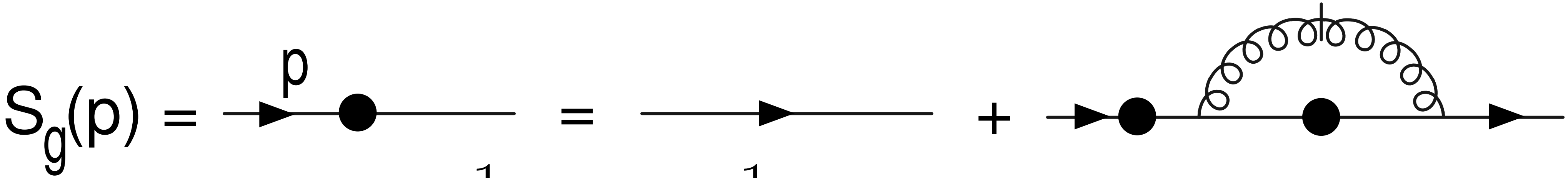
$$\int_{-\infty}^{\infty}\prod_{\mu} d\Phi_\mu \exp\left[\frac{1}{\Lambda^2}\mathrm{Tr}(\Phi_\mu\Phi^\mu)\right]$$

So far everything was formally exact. Now we dress the (massless) quark propagator by summing all its interactions with the shift field Φ_μ, at leading order in the N –› ∞ limit of a large number of colors.

Typical (planar) diagrams which contribute to the dressing:



The diagrams can be directly summed, or one may note that it satisfies a Dyson-Schwinger type equation for quark propagator



$$S_g(p) = \frac{1}{\not p} - \tfrac{1}{2}\,\mu^2\,\frac{1}{\not p}\,\gamma^\mu S_g(p)\gamma_\mu S_g(p)$$

where $\mu^2 = g^2 N \Lambda^2$. This equation is algebraic and can be solved exactly. With the general ansatz

$$S_g(p) = a(p^2)\,\not p + b(p^2)$$

we find two solutions.

Dressed quark propagator (1):

$$S_{g1}(p) = \frac{2\not{p}}{p^2 + \sqrt{p^2(p^2 - 4\mu^2)}}$$

which reduces to the standard perturbative one for $p^2 \to \infty$.

At $p^2 = 0$ it has a branch point singularity, rather than a pole.

Hence the quark does not propagate to asymptotic times:

$$|S_{g1}(t, \vec{p})| \underset{|t| \to \infty}{\sim} \mathcal{O}\left(1/\sqrt{|t|}\right)$$

Thus the interactions with the zero-momentum gluons in the perturbative vacuum effectively prevent the quark from propagating very far.

NOTE: The novel analytic structure is only seen when the vacuum interactions are summed to all orders!

Dressed quark propagator (2):

$$S_{g2}(p) = -\frac{1}{\mu^2}\left(\not{p} \pm \sqrt{p^2 + \frac{1}{2}\mu^2}\right)$$

Breaks chiral invariance! (Diverges for $\mu \to 0$)

Does not reduce to the standard perturbative propagator for $p^2 \to \infty$.

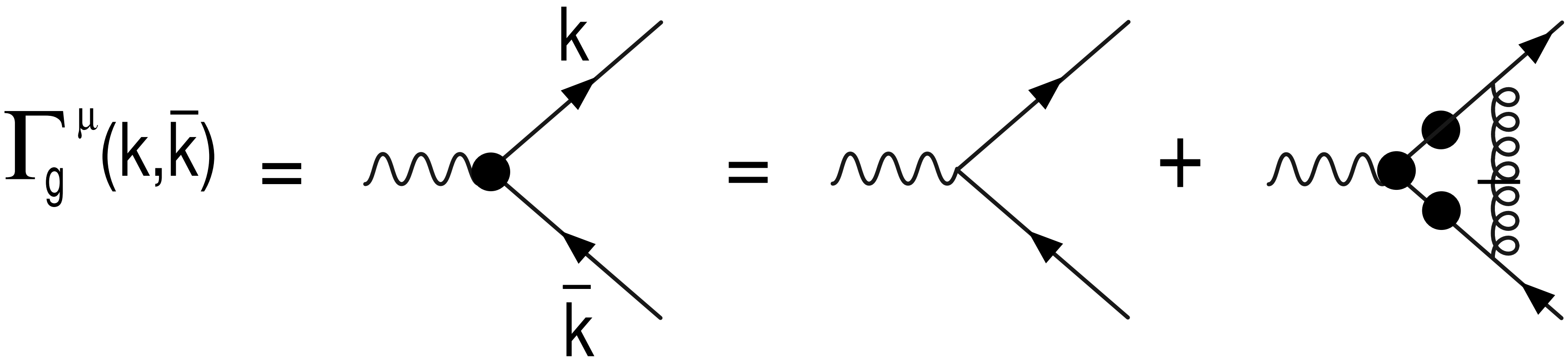
However, $S_{g2}(p) = S_{g1}(p)$ for $p^2 = -\mu^2/2$. Hence in a loop integral we may skip between solutions:

$$S_{g2}(p) \quad \text{for} \quad -\sqrt{\boldsymbol{p}^2 - \mu^2/2} \le p^0 \le \sqrt{\boldsymbol{p}^2 - \mu^2/2}$$

$$S_{g1}(p) \qquad \text{elsewhere}$$

which will give Green functions that break chiral symmetry.

Dressed quark-photon vertex



$$\Gamma_g^\mu(k,\bar{k}) = \gamma^\mu - \tfrac{1}{2}\mu^2\,\gamma^\nu S_g(k)\Gamma_g^\mu(k,\bar{k})S_g(\bar{k})\gamma_\nu$$

Linear equation with unique solution, given the quark propagator S_g .

For $S_{g1}(k)$ we find

$$\begin{aligned}\Gamma_g^\mu(k,\bar{k}) &= \frac{1}{1+2fk\cdot\bar{k}+f^2k^2\bar{k}^2}\Big\{(1+fk\cdot\bar{k})\gamma^\mu - fi\gamma_5\,\epsilon^{\mu\nu\rho\sigma}\gamma_\nu k_\rho\bar{k}_\sigma \\ &+ \frac{2f^2}{1-f^2k^2\bar{k}^2}(k^\mu \not{k}\bar{k}^2+\bar{k}^\mu\not{\bar{k}}k^2) + \frac{f(1+f^2k^2\bar{k}^2)}{1-f^2k^2\bar{k}^2}(k^\mu\not{\bar{k}}+\bar{k}^\mu\not{k})\Big\}\end{aligned}$$

where $a_p \equiv \left(1-\sqrt{1-4\mu^2/p^2}\right)/2\mu^2$ and $f \equiv \mu^2 a_k a_{\bar{k}}$

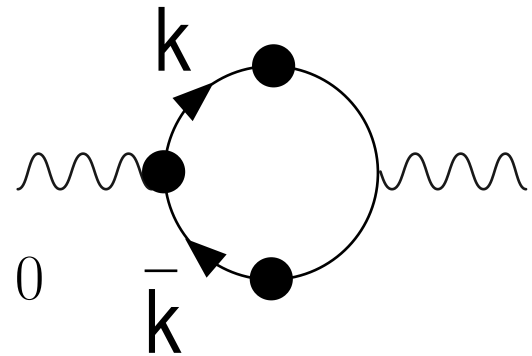
By direct calculation we may verify the Ward-Takahashi identity:

$$p_\mu \Gamma^\mu(k, \bar{k}) = S(k)^{-1} - S(\bar{k})^{-1}$$

The photon self-energy: Does the photon acquire a mass?

$$\Pi_g^{\mu\nu}(p) = ie^2 N \int \frac{d^D k}{(2\pi)^D} \mathrm{Tr}\left[\gamma^\nu S_g(k) \Gamma_g^\mu(k,\bar{k}) S_g(\bar{k})\right]$$

$$= \Pi_g(p^2)\,(p^2 g^{\mu\nu} - p^\mu p^\nu) \;\; \text{since (WT)} \;\; p_\mu \Pi_g^{\mu\nu}(p) = 0$$



Using the p → 0 form of the W-T identity,

$$\Gamma_g^\mu(k,k) = \frac{\partial S_g^{-1}(k)}{\partial k_\mu} = -S_g^{-1}(k)\left[\frac{\partial S_g(k)}{\partial k_\mu}\right] S_g^{-1}(k)$$

we have

$$\Pi_{g\mu}^{\mu}(p=0) = -ie^2 N \int \frac{d^D k}{(2\pi)^D} \frac{\partial}{\partial k_\mu} \mathrm{Tr}\left[\gamma_\mu S_g(k)\right] = 0$$

which shows that $p^2\,\Pi_g(p^2) \to 0$ for $p^2 \to 0$, hence that $m_\gamma = 0$.

This may also be verified directly from the explicit expressions.

The Φ_μ – gluon coupling

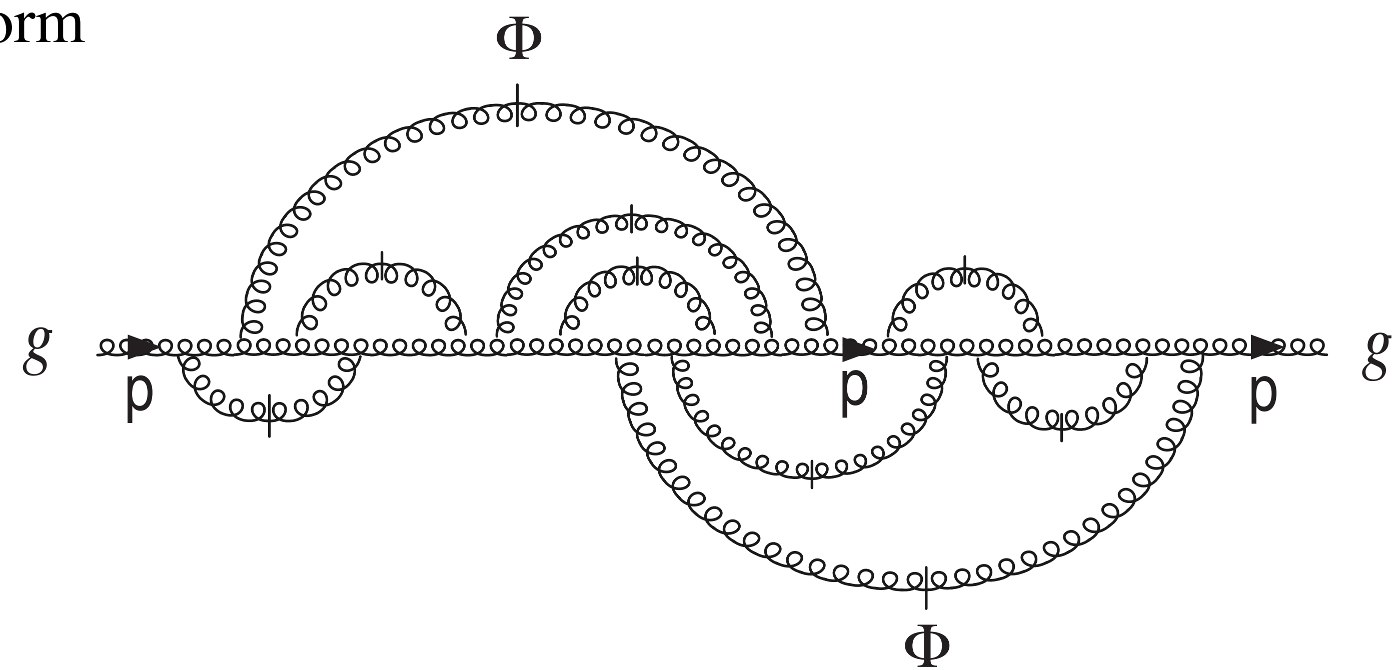
The shift $A_\mu(x) \longrightarrow A_\mu(x) + \Phi_\mu$ generates $\mathcal{L}_g \to \mathcal{L}_g + \mathcal{L}_\Phi$, where $\mathcal{L}_\Phi$ has several, separately gauge invariant terms.

We have studied the simplest one: $\mathcal{L}_\Phi^{(1)} = -\mathrm{Tr}\left(F_{\mu\nu}F_\Phi^{\mu\nu}\right)$, where

$$F_\Phi^{\mu\nu} = \partial_\mu\Phi_\nu - \partial_\nu\Phi_\mu + ig\left(\left[\Phi_\mu, A_\nu\right] - \left[\Phi_\nu, A_\mu\right]\right)$$

transforms into $U F_\Phi^{\mu\nu} U^\dagger$ under a gauge transformation U.

Taking Φ_μ constant in the covariant gauge $\partial_\mu A^\mu = 0$ we sum over all planar gluon dressings of the form



The dressed gluon propagator is

$$iD^{ab}_{g,\mu\nu}(p) = \frac{i}{p^2}\left(-g_{\mu\nu} + \frac{p_\mu p_\nu}{p^2}\right) d(-2\mu^2/p^2)$$

where

$$d(-2\mu^2/p^2) = \frac{p^2}{4\mu^2}\left[{}_2F_1(-\tfrac{1}{2},\tfrac{1}{2};2;-32\mu^2/p^2) - 1\right]$$

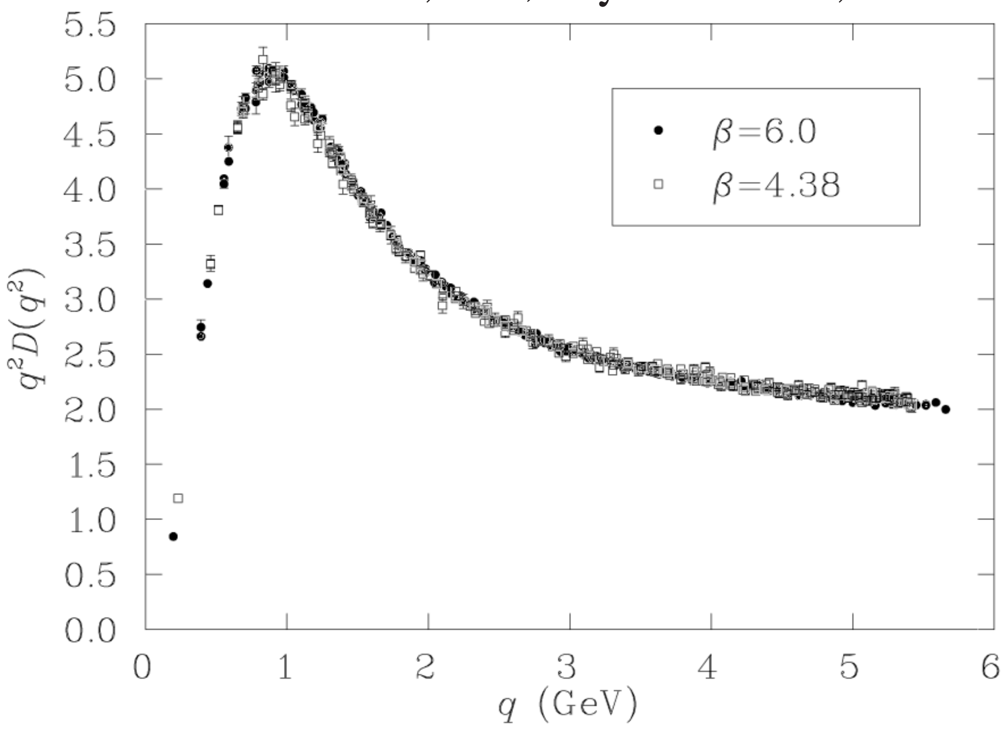
and ${}_2F_1$ is a hypergeometric function. The dressed propagator thus has a cut for -32 $\mu^2 \leq p^2 \leq 0$, and has the asymptotic limits

$$d(-2\mu^2/p^2) \;\to\; 1 \;\; \text{for } p^2 \to \infty$$

$$d(-2\mu^2/p^2) \;\to\; \frac{4}{3\pi}\frac{\sqrt{p^2}}{\mu} \;\; \text{for } p^2 \to 0$$

Interestingly, the general shape of the dressed gluon propagator resembles lattice results:

F.D.R. Bonnet, *et al*, Phys. Rev. **D64**, 034501



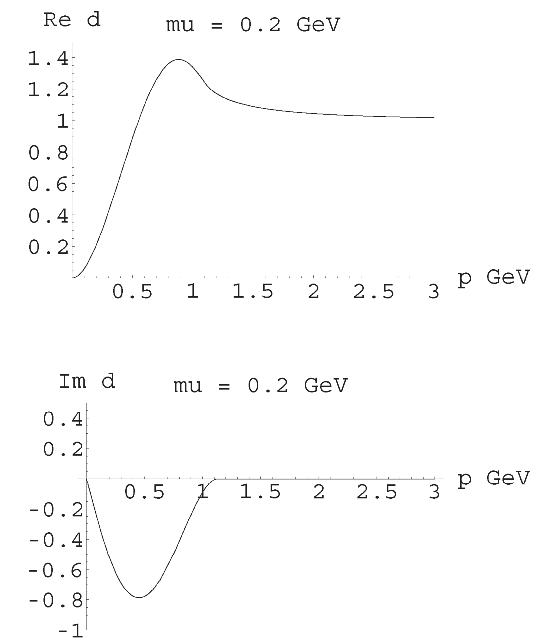


Figure 4: Our result for the gluon propagator.

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Summary

Dynamics of hard QCD processes is still incompletely understood

- Relation of parton distributions to target wave function

- Importance of interference between rescattering amplitudes
  – Transverse spin effect in DIS
  – Nuclear shadowing
  – Diffraction

The Pomeron is not a constituent of the target
– it arises from soft rescattering

PQCD helps to cast light even on confinement physics

- Color confinement may result from long distance propagation in condensate, rather than from $\alpha_s > 1$

- We may perturb around a non-empty gluon field configuration

- Method presented above maintains good features of usual PQCD, adding novel features at low p^2

- Its usefulness for understanding qualitative (analyticity, unitarity, ...) and quantitative aspects of hadron physics merits further study.