May 23-28, 2004

QNP 2004

Quark-Hadron Duality

in Structure Functions

Wally Melnitchouk

Jefferson Lab

Outline

- Introduction

- (Pre-)History
  $\longrightarrow$ duality in hadronic reactions

- Bloom-Gilman duality in $eN \rightarrow eX$
  $\longrightarrow$ resonances and scaling
  $\longrightarrow$ higher twists

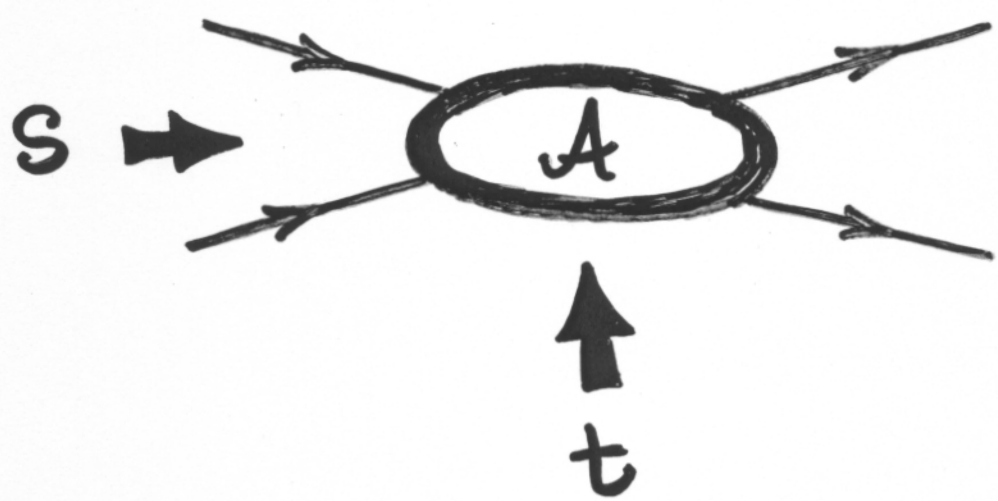
- Local duality
  $\longrightarrow$ inclusive–exclusive relations
  $\longrightarrow$ sums of squares vs. squares of sums

- Outlook

Historical Overview

Origins of duality $\longrightarrow$ 1960's Regge theory analyses of hadron-hadron scattering

Scattering amplitude $\mathcal{A}(s,t)$



Small s:

$\longrightarrow$ s-channel partial wave series

$\mathcal{A}(s,t) =$ sum of s-channel resonances

Large s:

$\longrightarrow$ density of resonances increases

$\longrightarrow$ resonances overlap

$\longrightarrow$ t-channel partial wave series more useful

$\mathcal{A}(s,t) =$ sum of Regge poles & cuts

$$\mathcal{A}(s,t) \sim s^{\alpha(t)}$$

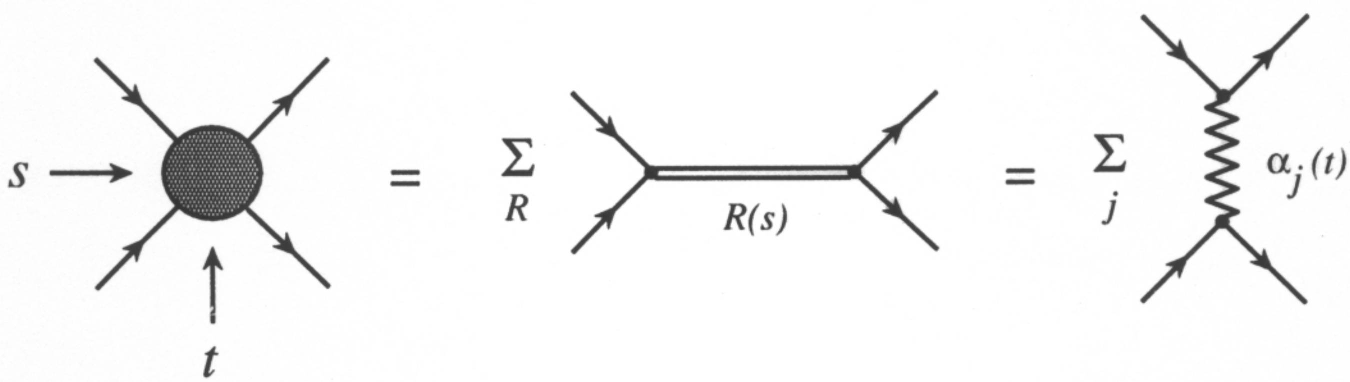
$$\alpha(t) = \alpha(0) + \alpha^{'} t$$

$\longrightarrow$ *linear Regge trajectories*

- How to merge descriptions, especially at intermediate s?

Equivalence between *s-channel resonances* $R(s)$ and *t-channel poles* $\alpha_j(t)$

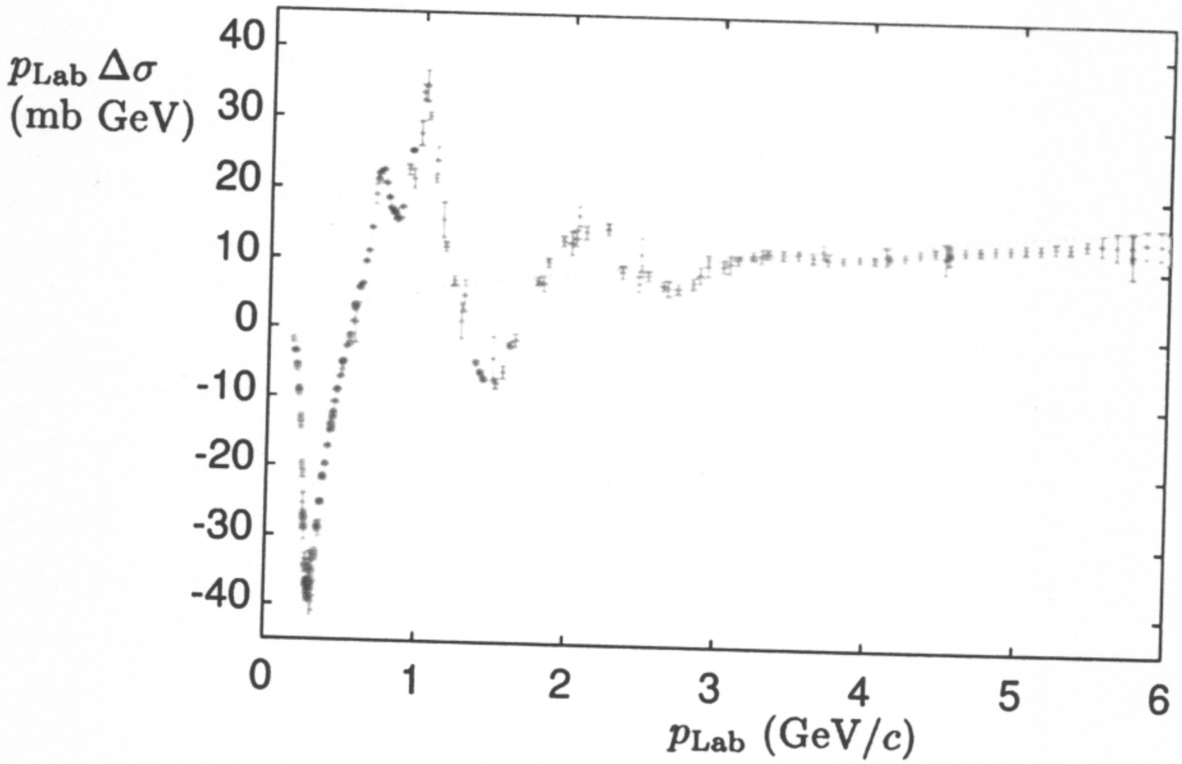
$$\sum_R \mathcal{A}_R(s,t) \approx \sum_j \mathcal{A}_j(s,t)$$



Igi (1962); Dolen, Hornm Schmidt (1968)

"Finite-energy sum rules"

Dormadie et al. (2002)



$$\Delta\sigma = \sigma(\pi^+ p) - \sigma(\pi^- p)$$

" *Two-Component Duality* "

Harari 1968, Freund 1968

If there are vacuum quantum number exchanges

$$\mathcal{A} = \sum_R \mathcal{A}_R + \mathcal{A}_{background}$$

$$\updownarrow \qquad \updownarrow$$

$$= \sum_j \mathcal{A}_j + \mathcal{A}_{Pomeron}$$

At large s

$$\sigma \sim s^{\alpha(0)-1}$$

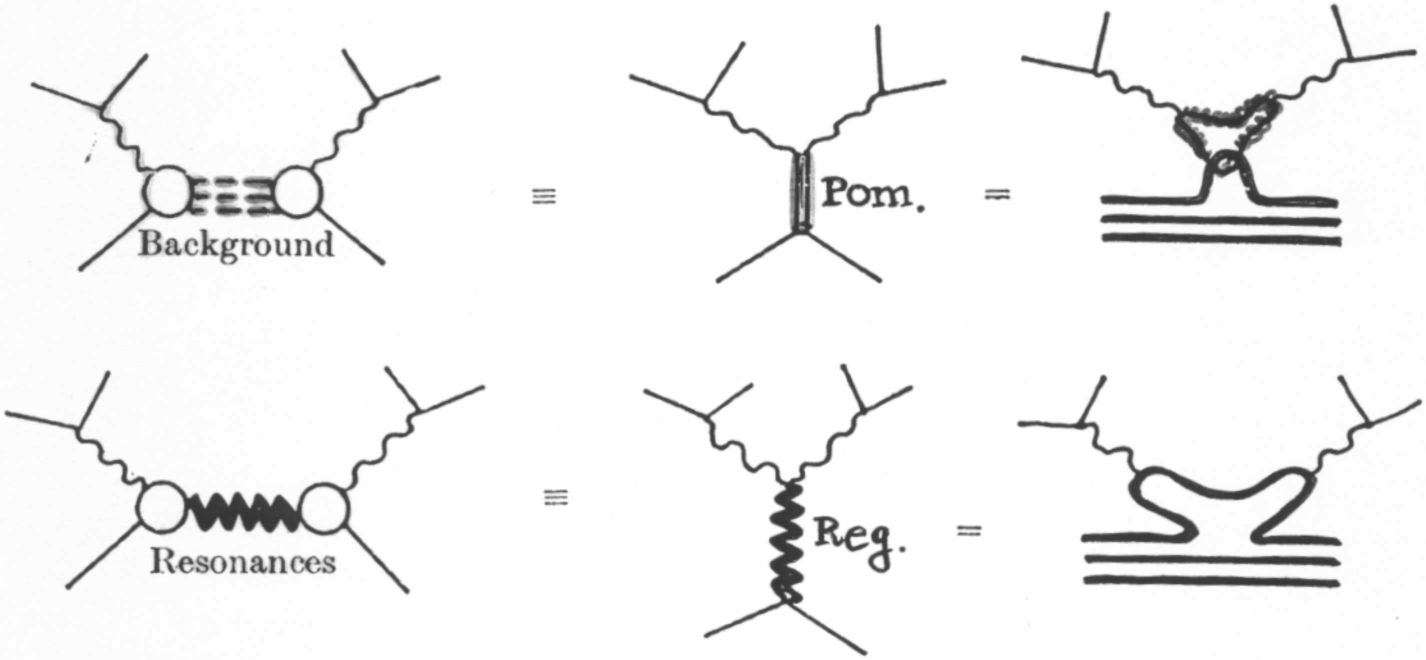
Resonances $\longrightarrow$ Reggeon exchange

$\longrightarrow \alpha_R(0) \approx 1/2$

Background $\longrightarrow$ Pomeron exchange

$\longrightarrow \alpha_P(0) \approx 1$

DUALITY DIAGRAMS FOR ep → eX



COLLINS 1977

Bloom-Gilman Duality

- Individual resonances are strongly Q^2 dependent

  $\longrightarrow N \to N^*$ transition form factors $\sim (1/Q^2)^n$

- However, *average* over resonances is aproximately Q^2 *independent*

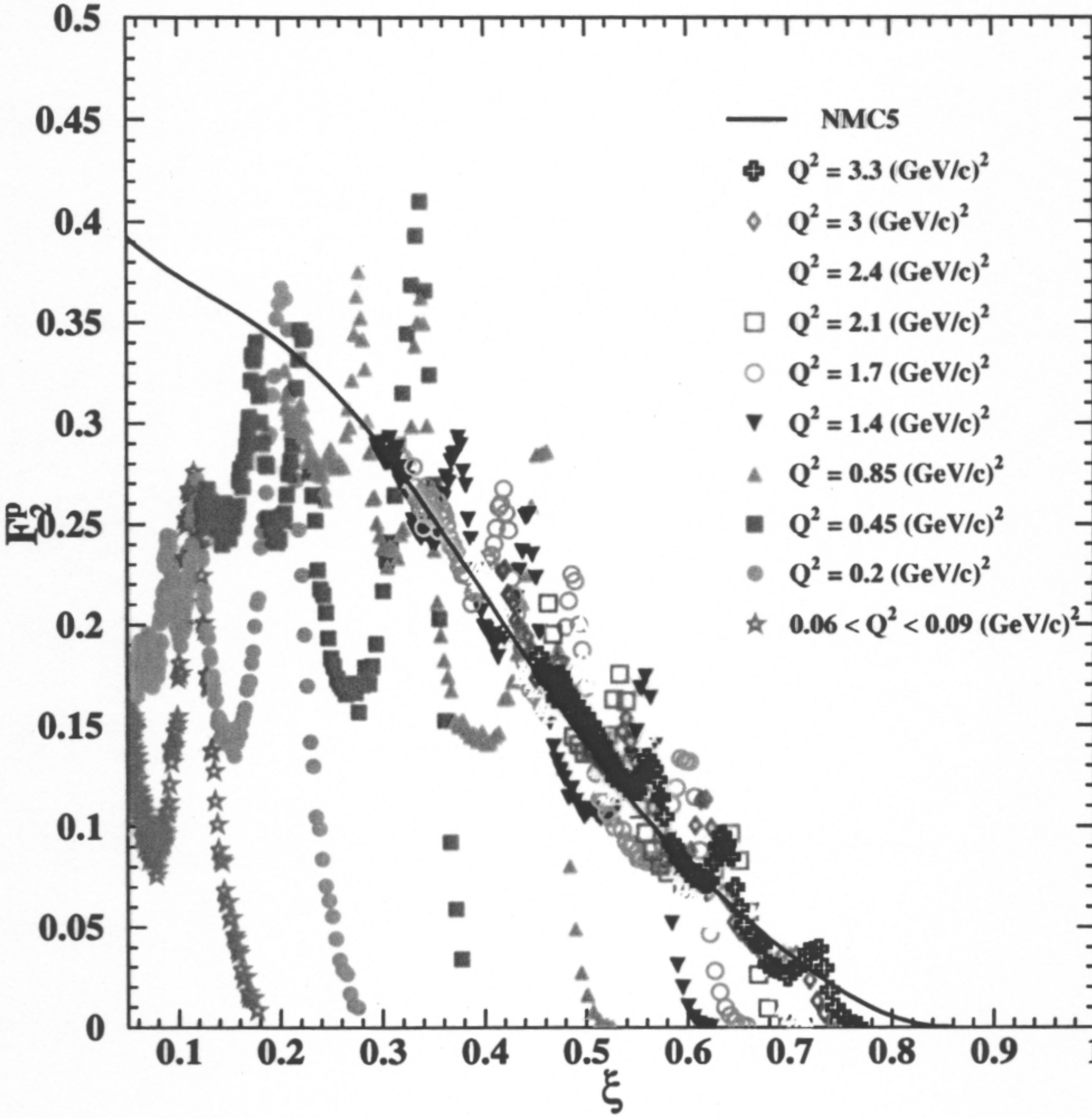
  $\longrightarrow$ resembles *scaling* (leading twist) structure function

Finite-energy sum rule for eN scattering

$$\frac{2M}{Q^2}\int_0^{\nu_m} d\nu\ \nu W_2(\nu, Q^2) \ = \ \int_1^{1+W_m^2/Q^2} d\omega'\ \nu W_2(\omega')$$

$$\omega' = 1/x + M^2/Q^2$$

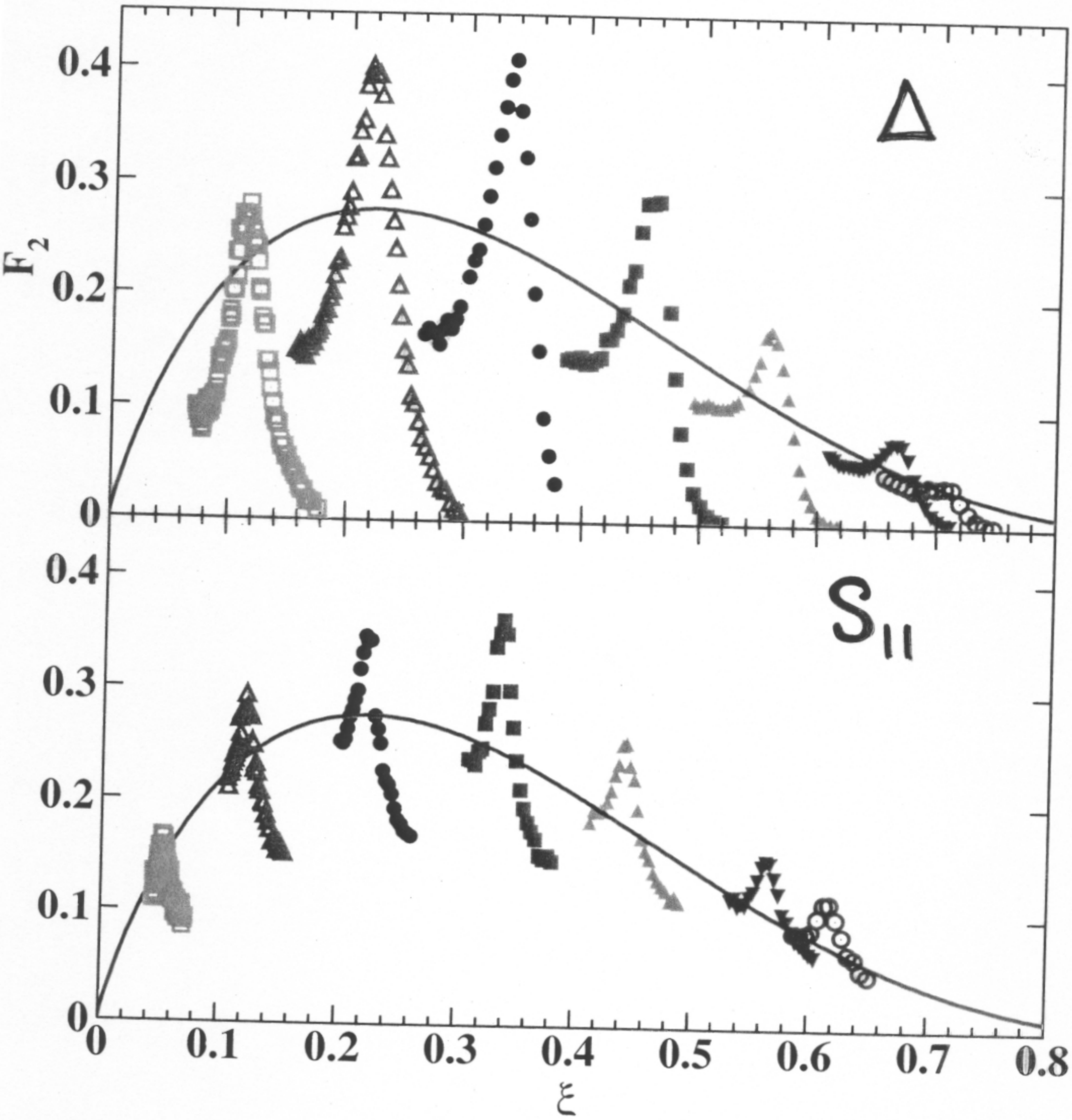
Bloom, Gilman (1970)



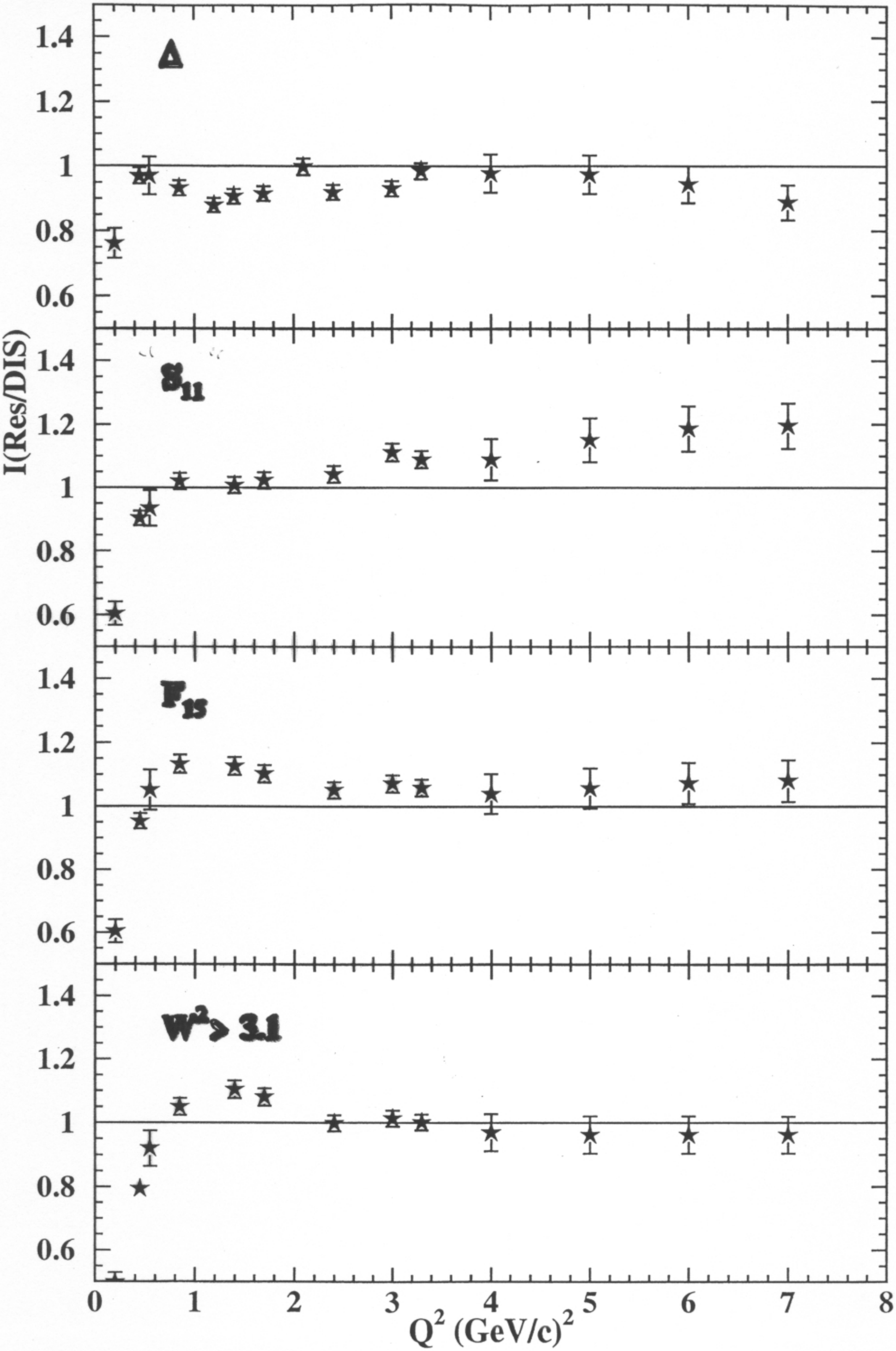
$$\xi = 2x/(1 + \sqrt{1 + 4M^2x^2/Q^2})$$

Niculescu et al. (JLab Hall C)
Phys. Rev. Lett. 85 (2000) 1185

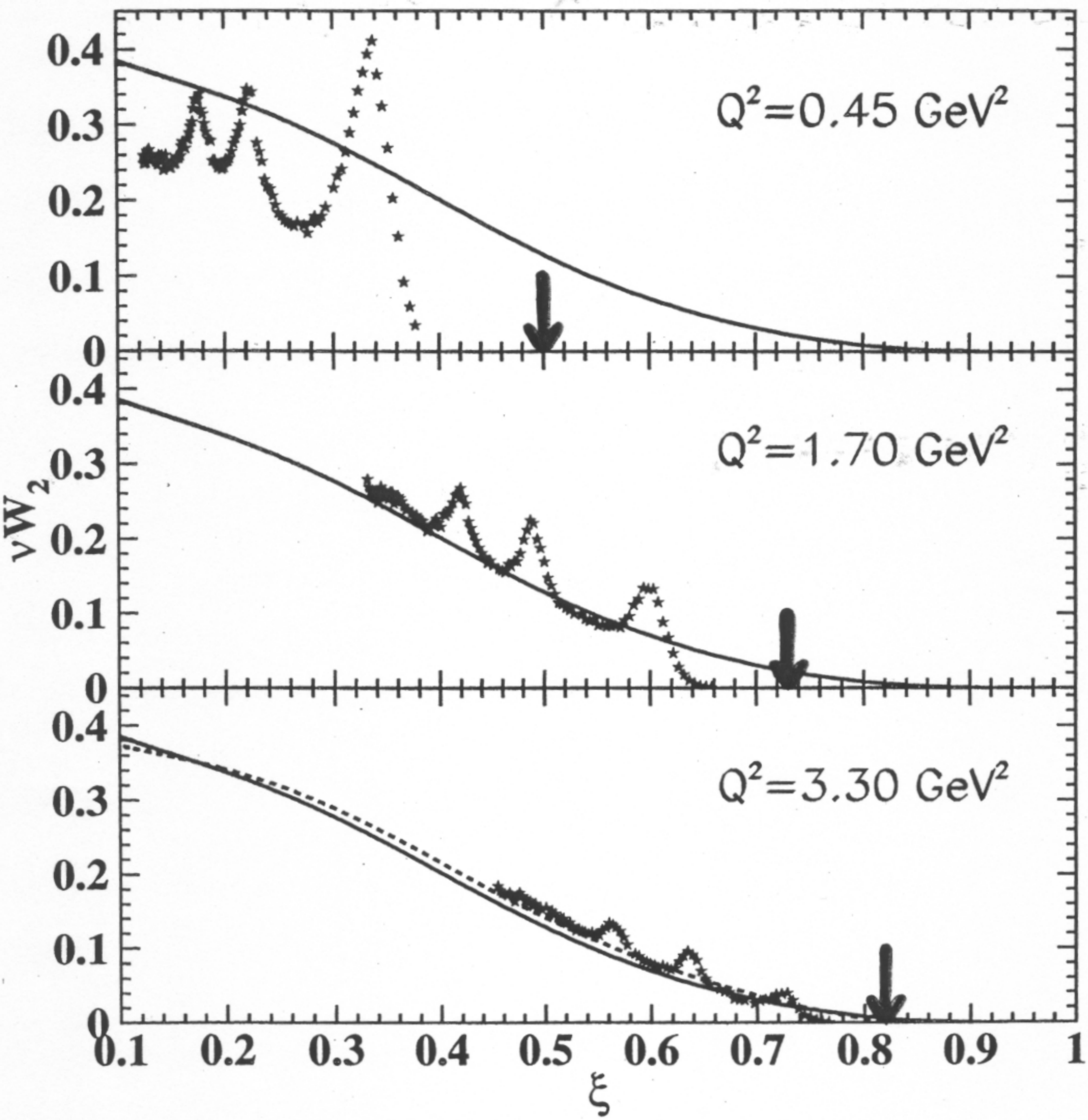
LOCAL DUALITY



$$\xi = 2x \Big/ \left(1 + \sqrt{1 + 4x^2M^2/Q^2}\right)$$



Δ
S11
F15
W2> 3.1
I(Res/DIS)
Q2 (GeV/c)2
0
1
2
3
4
5
6
7
8
0.6
0.8
1
1.2
1.4



$$\xi = 2x \Big/ \left(1 + \sqrt{1 + 4x^2M^2/Q^2}\right)$$

TARGET MASS CORRECTION

Resonances and DIS

Contribution of (narrow) resonance R to structure function:

$$\nu W_2^{(R)} \approx 2M\nu \left(G_R(Q^2)\right)^2 \delta(W^2 - M_R^2)$$

If $G_R(Q^2) \sim (1/Q^2)^N$, then for $Q^2 \gg M_R^2$

$$\nu W_2^{(R)} \sim (1 - x_R)^{2N-1}$$

with

$$x_R = \frac{Q^2}{M_R^2 - M^2 + Q^2}$$

$\Rightarrow$ As $Q^2 \to \infty$, resonances pile up at $x_R \to 1$

ELASTIC CONTRIBUTION TO STRUCTURE FNS.

$$F_1^{el} = M\tau \, G_M^2 \, \delta(\nu - Q^2/2M)$$

$$F_2^{el} = \frac{2M\tau}{1+\tau} \left(G_E^2 + \tau G_M^2 \right) \delta(\nu - Q^2/2M)$$

$$\left[\tau = Q^2/4M^2 \right]$$

DUALITY FOR ELASTIC PEAK

$$\int_1^{1+\delta\omega'} d\omega' \, F_2^{scaling}(\omega') \qquad \left[\omega' = \frac{2M\nu + M^2}{Q^2} \right]$$

$$= \frac{2M}{Q^2} \int d\nu \, F_2^{el}(\nu, Q^2)$$

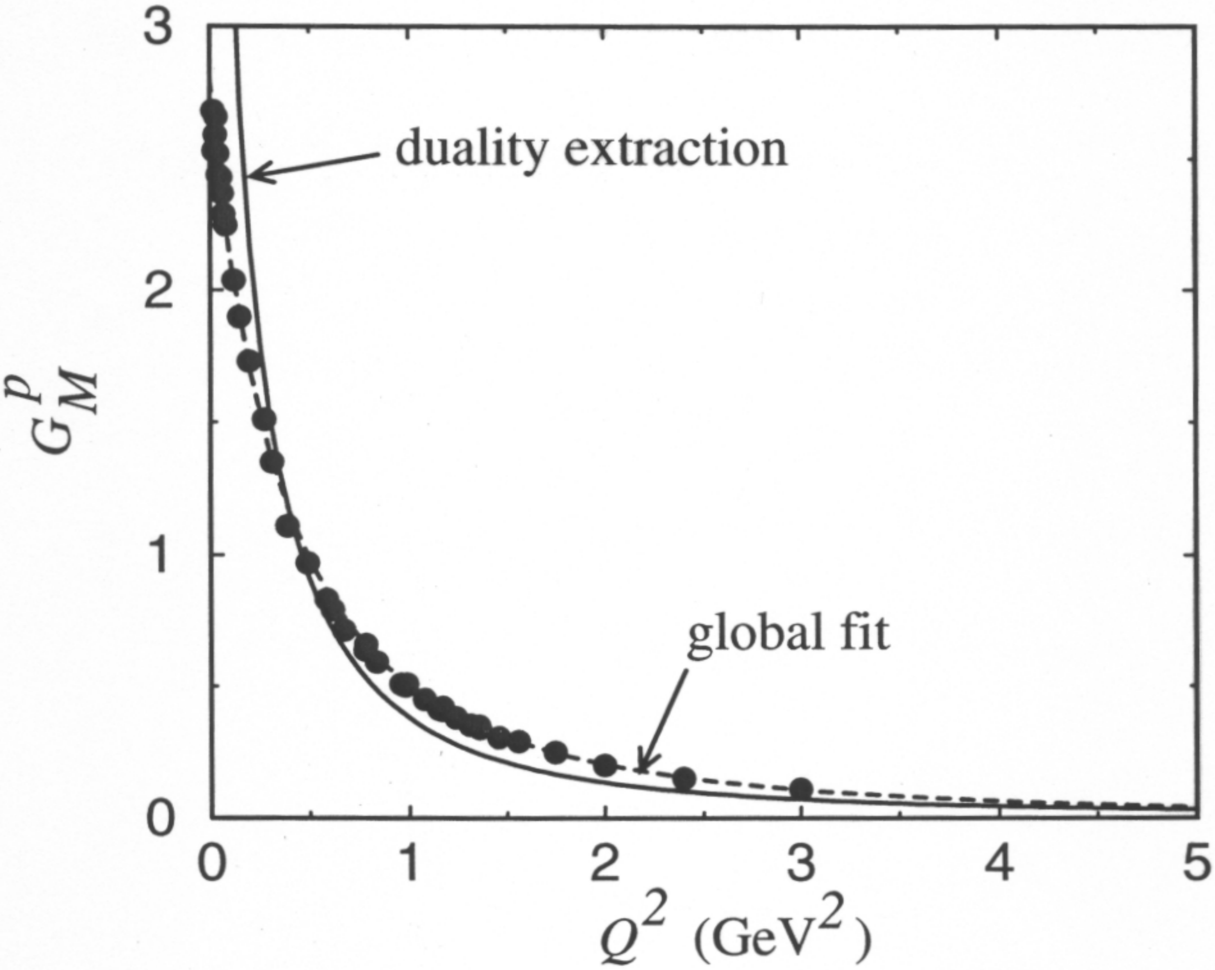
$$= \frac{G_E^2 + \tau G_M^2}{1+\tau}$$

AREA UNDER ELASTIC PEAK SAME AS INTEGRAL OF STRUCTURE FUNCTION BELOW THRESHOLD

BLOOM, GILMAN 1970

Extract magnetic form factor from JLab data on integral of F_2

(Niculescu et al, Phys. Rev. Lett. 85 (2000) 1186)



Proton magnetic form factor from local duality

Bloom-Gilman Duality & the OPE

- Operator product expansion

  ⟶ moments of structure function expanded in powers of $1/Q^2$

$$M_n(Q^2) = \int_0^1 dx\ x^{n-2}\ F_2(x,Q^2)$$

$$= A_n^{(2)} + \frac{A_n^{(4)}}{Q^2} + \frac{A_n^{(6)}}{Q^4} + \cdots$$

  ⟶ coefficients $A_n^{(\tau)}$ = matrix elements of operators with specific "*twist*" (τ)
  (twist τ= dimension – spin)

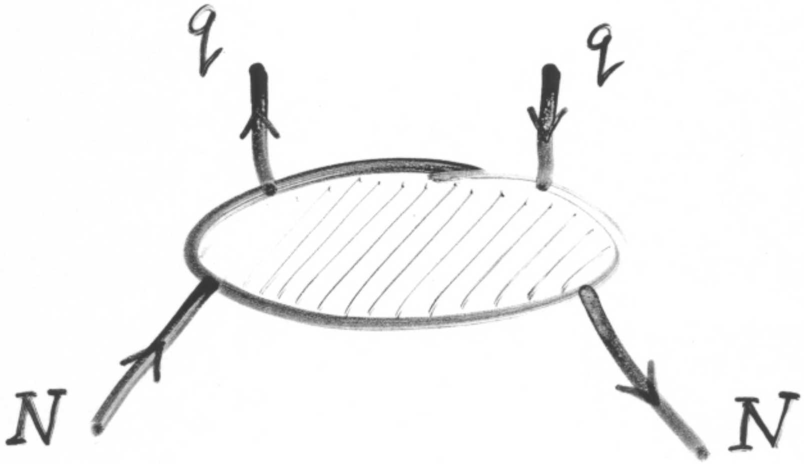
- If moment $M_n(Q^2) \approx$ independent of Q^2

  ⟶ higher twist terms $A_n^{(\tau>2)}$ small

duality ⇔ suppression of higher twists

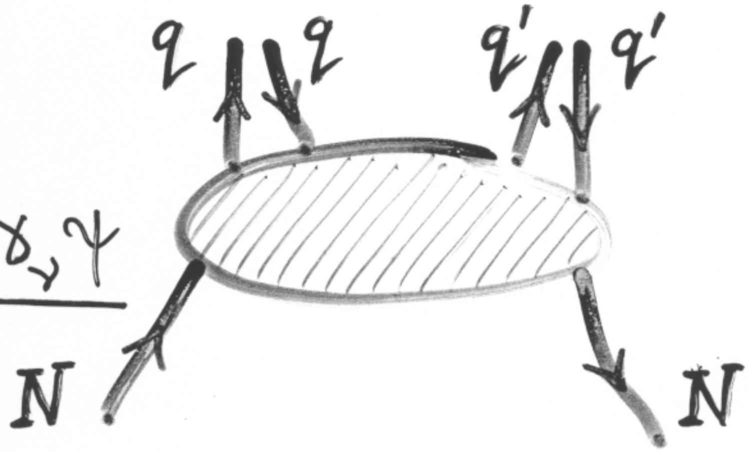
De Rujula, Georgi, Politzer (1975)

$\underline{\bar{\Psi} \gamma_\mu \Psi}$



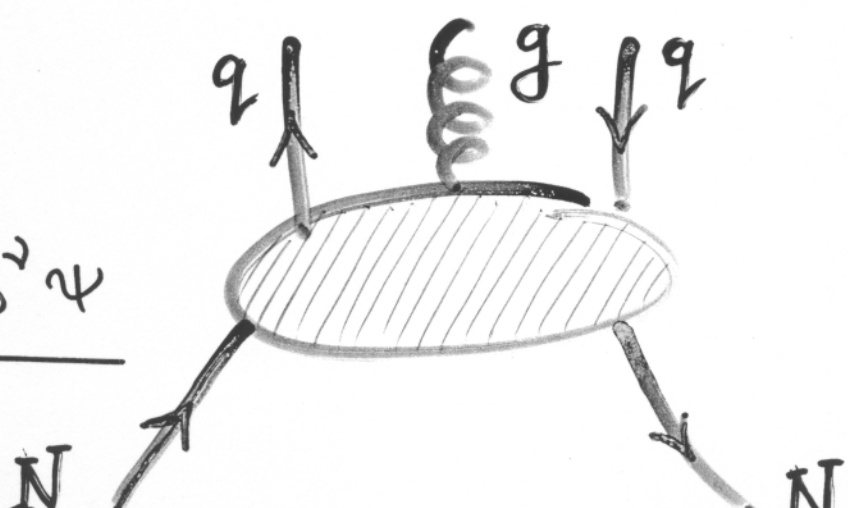
$\tau = 2$

$\underline{\bar{\Psi} \gamma_\mu \Psi \cdot \bar{\Psi} \gamma_\nu \Psi}$



$\tau > 2$

$\underline{\bar{\Psi} G_{\mu\nu} \gamma^\nu \Psi}$



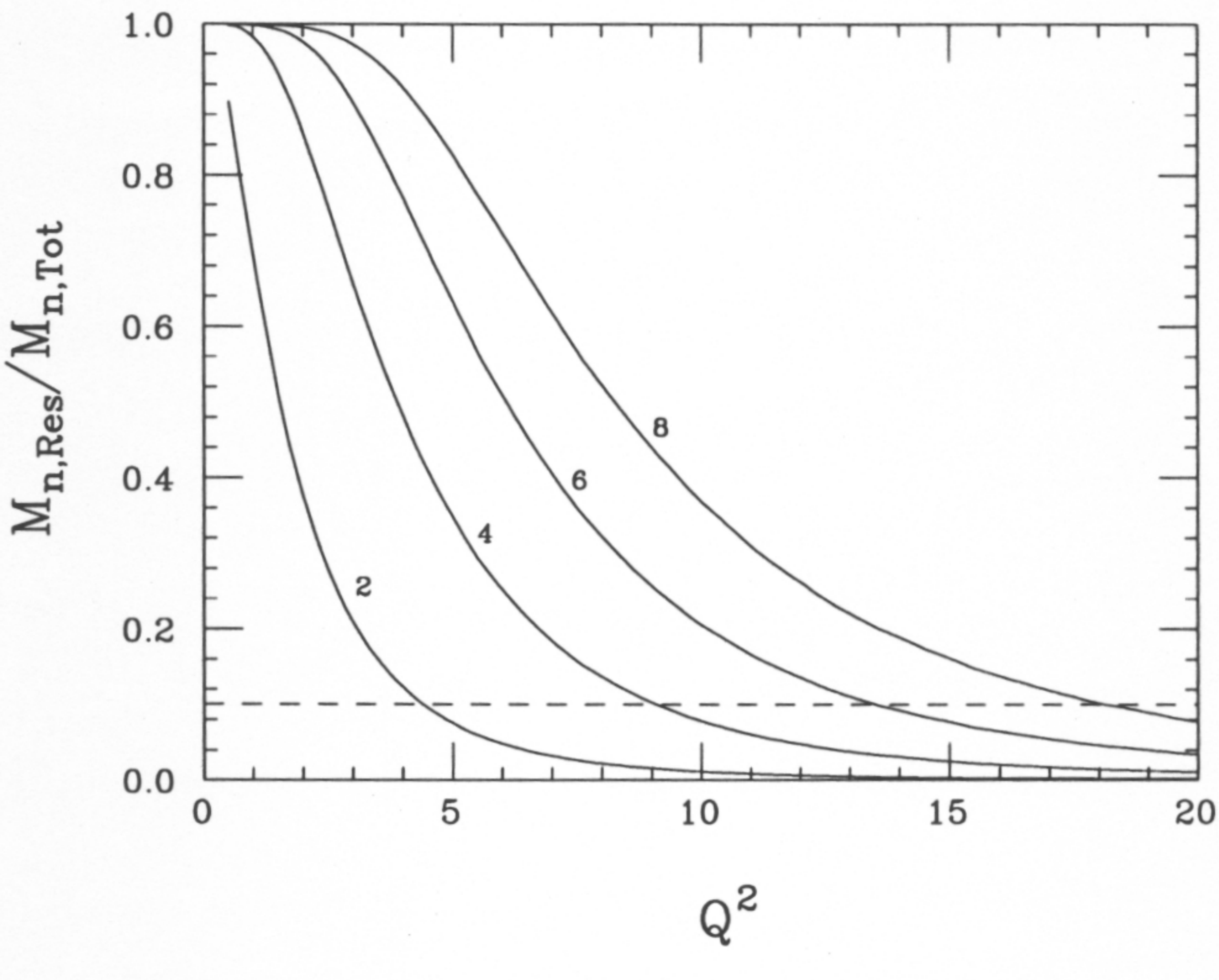
Resonances, Scaling & Higher Twist

- Bloom-Gilman duality ⇒ distinction between "resonance" and "DIS" regions is *artificial*

- *E.g.* at $Q^2 = 1$ GeV^2 about 70% of F_2 comes from $W < 2$ GeV

- *But* because of duality, resonances and DIS continuum conspire to produce $\sim$ 10% higher twist ($1/Q^2$) contribution!

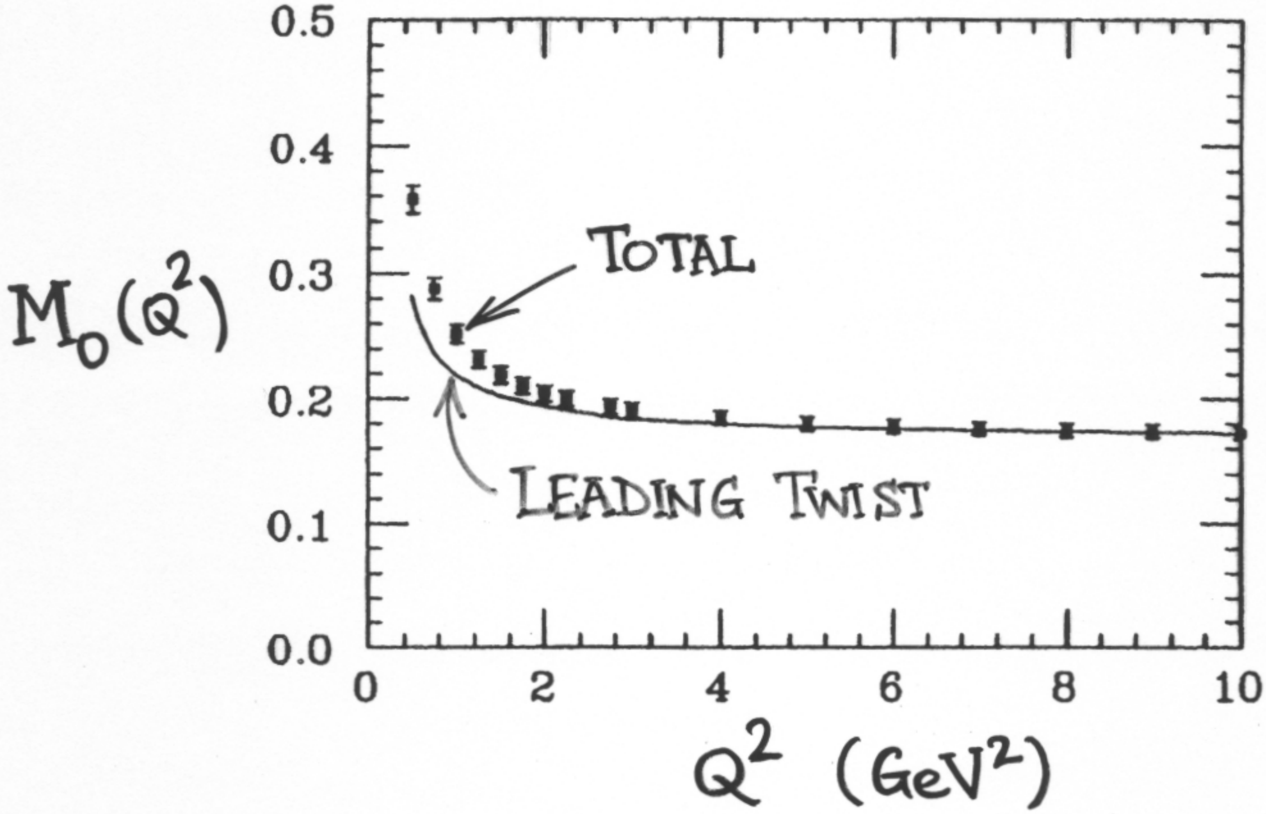
Resonances are an integral part of the scaling structure functions !

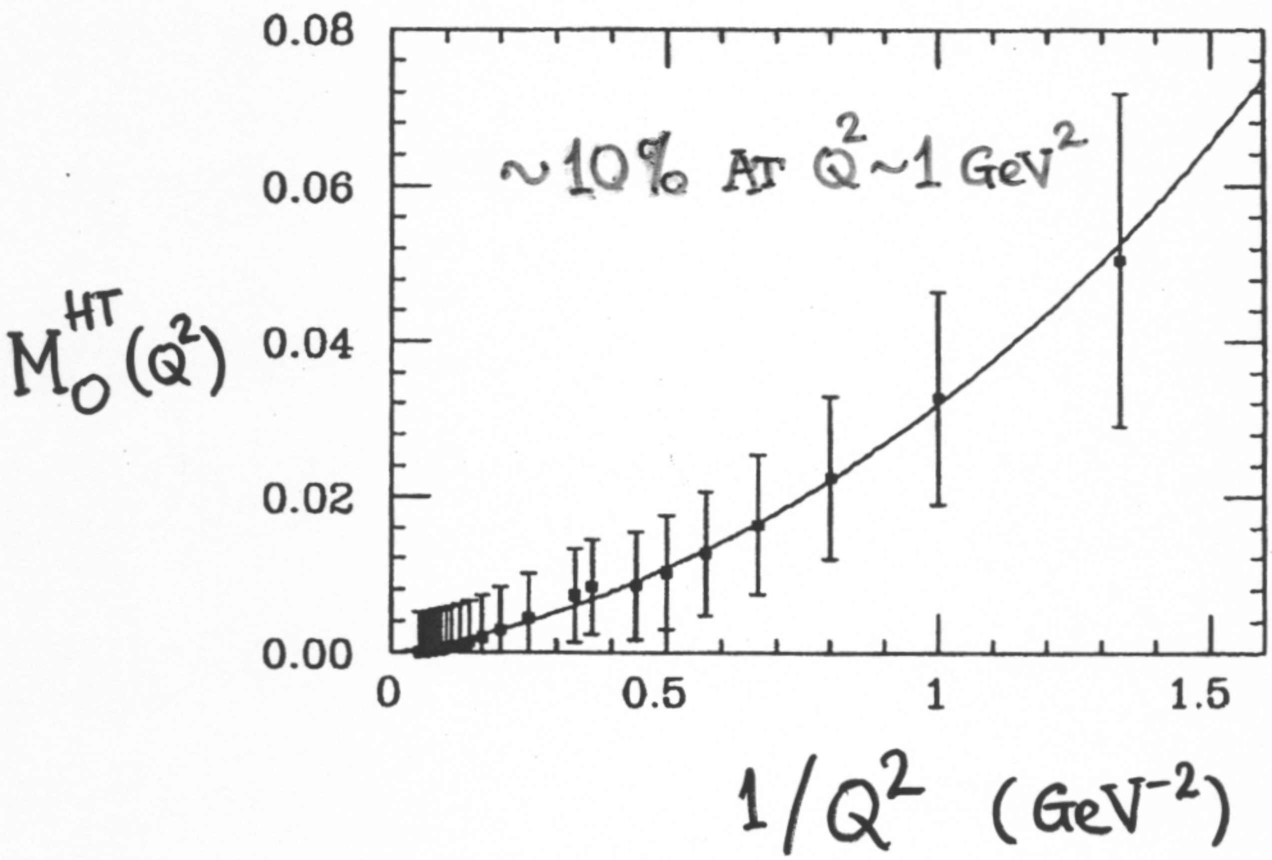


Relative contribution of resonance region ($W < 2$ GeV) to n-th moment of F_2^p

Ji, Unrau
Phys. Rev. D 52 (1995) 72

Ji, Unrau 1995





Spin Dependence of Duality

- Quark-hadron duality in *spin dependent* structure functions even more intriguing than in spin averaged case

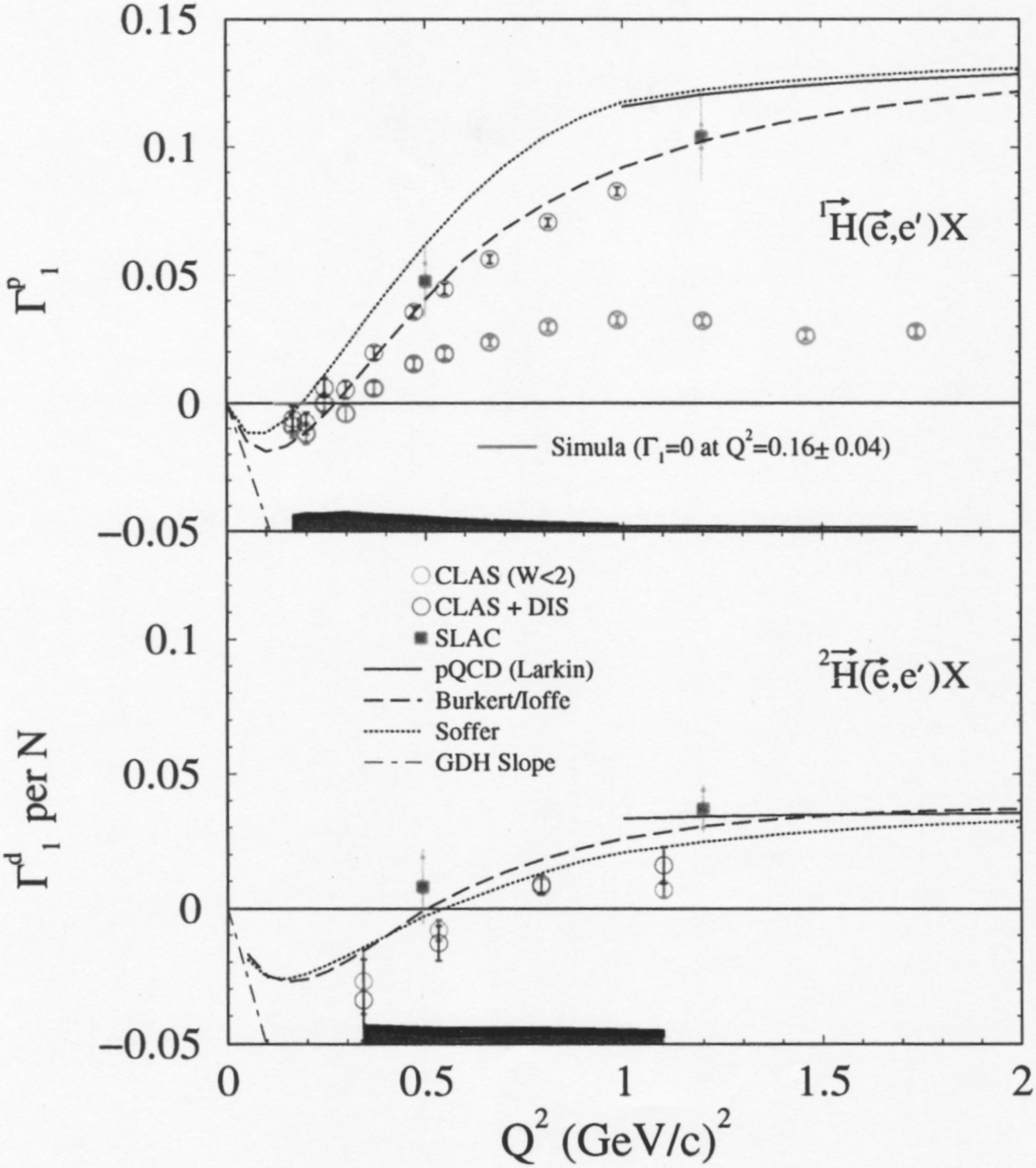
- Spin structure functions
  $\longrightarrow$ differences of cross section
  $\longrightarrow$ *need not be positive*
  (e.g. Δ contribution to g_1 negative at low Q^2)

- Dramatic transition from (negative) GDH integral at $Q^2 = 0$ to (positive) Bjorken integral at $Q^2 \geq 2$ GeV^2 for proton

- Does duality work at all for spin-dependent observables?

The First Moment of g_1

$$\Gamma_1(Q^2) \equiv \int_0^1 g_1(x, Q^2)dx$$



OPE FOR MOMENT OF g_1

$$\Gamma_1(Q^2) = \int_0^1 dx \; g_1(x, Q^2)$$

INCLUDING ELASTIC

$$= \mu_2 + \frac{\mu_4}{Q^2} + \frac{\mu_6}{Q^4} + \cdots$$

TWIST-2

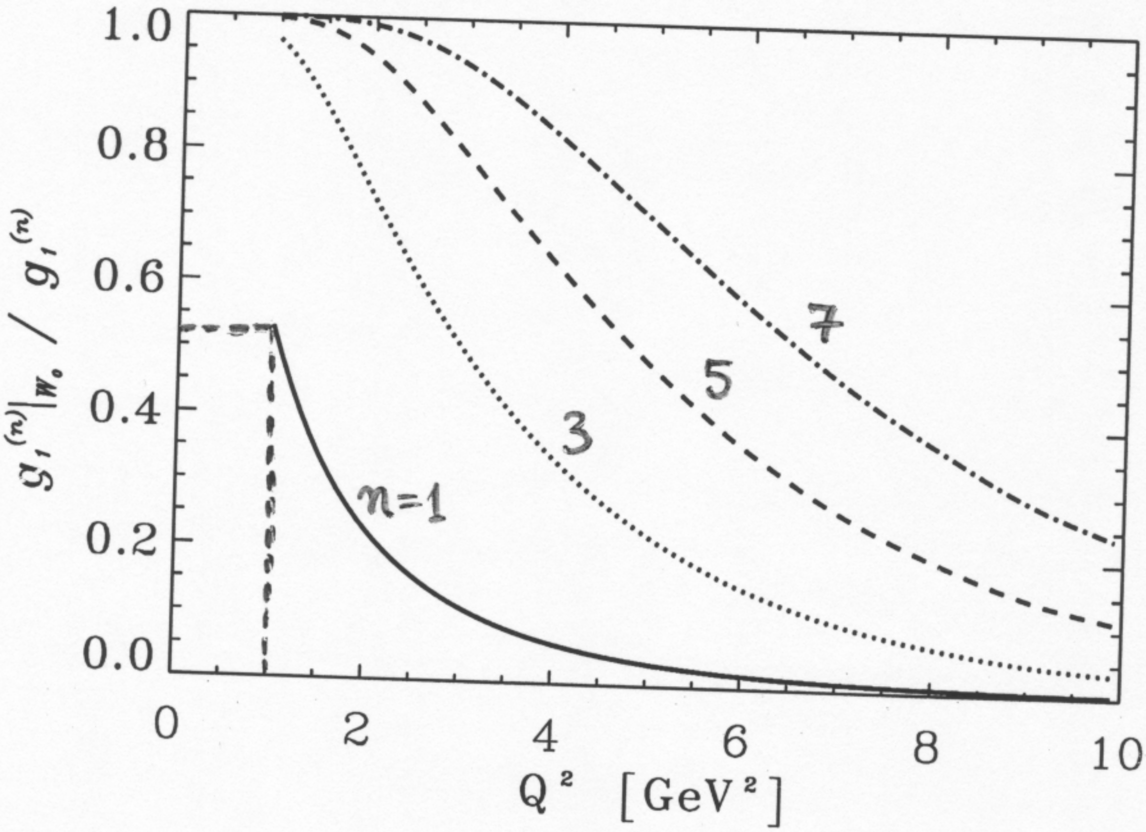
$$\mu_2^{p(n)} = \left(\pm\frac{1}{12} g_A + \frac{1}{36} a_8\right) C_{ns}(Q^2)$$

triplet (g_A) — octet (a_8)

$$+ \frac{1}{9} \Delta\Sigma \; C_s(Q^2)$$

RGI singlet axial charge ($\Delta\Sigma$)

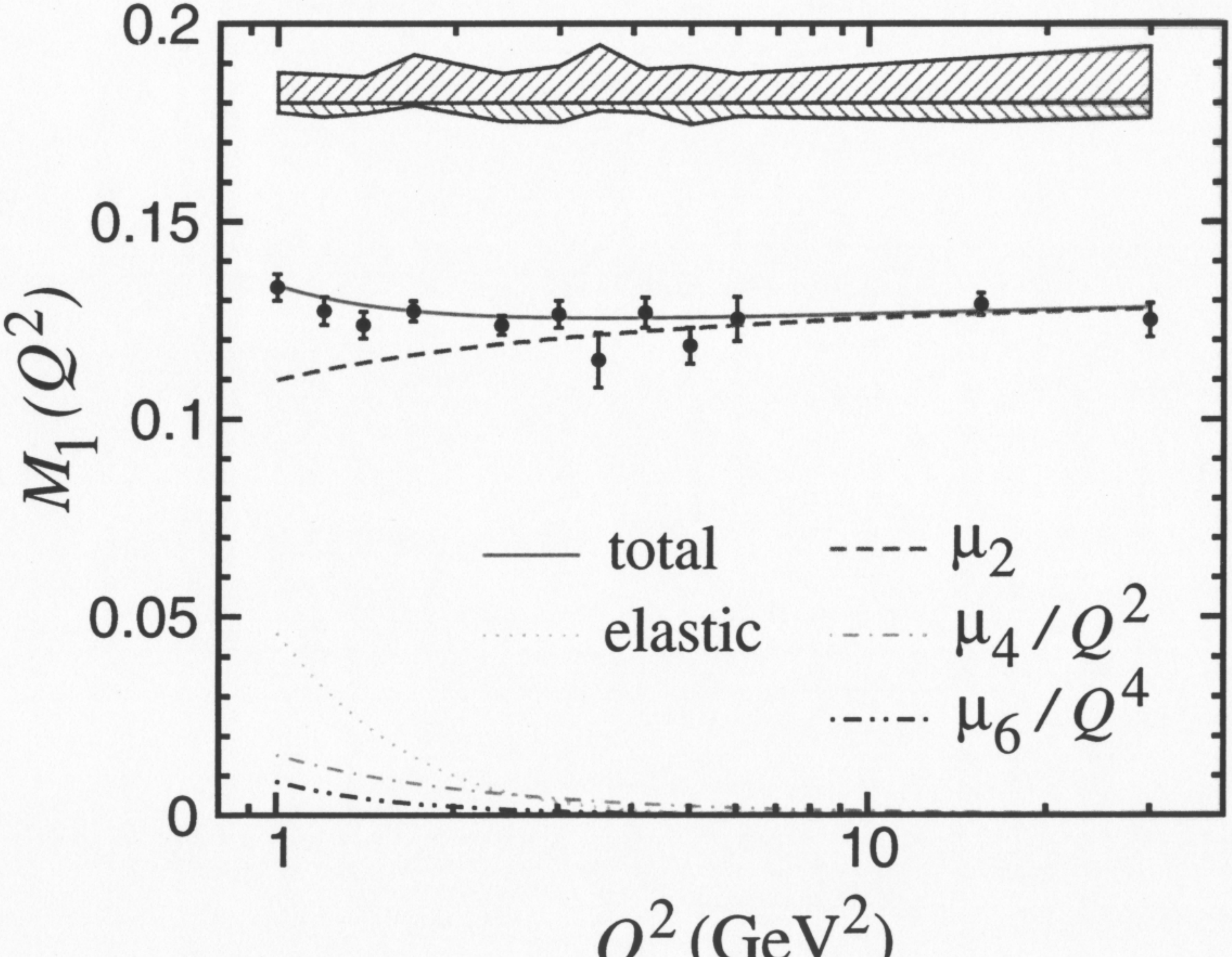
$$\Gamma_1^{p(n)}(Q^2) = \int_0^1 dx\; x^{n-1}\, g_1^p(x, Q^2)$$



Contribution to moments of g_1^p from the resonance region ($W < 2$ GeV), normalized to total moments

Edelmann, Piller, Kaiser, Weise
Nucl. Phys. A665 (2000) 125

Osipenko, WM, ... hep-ph/0404195



Higher twists

Leading $1/Q^2$ correction to $\Gamma_1(Q^2)$

$$\mu_4(Q^2) = \frac{1}{9}M^2\left(a_2(Q^2) + 4d_2(Q^2) + 4f_2(Q^2)\right)$$

$\longrightarrow$ Target mass correction a_2 (twist-2)

$$a_2\ S^{\{\mu}P^{\nu}P^{\lambda\}} = \frac{1}{2}\sum_q e_q^2\ \langle P,S|\bar{\psi}_q\ \gamma^{\{\mu}iD^{\nu}iD^{\lambda\}}\psi_q|P,S\rangle$$

$\longrightarrow$ Twist-3 correction d_2

$$d_2\ S^{[\mu}P^{\{\nu]}P^{\lambda\}} = \frac{1}{2}\sum_q e_q^2\ \langle P,S|g\ \bar{\psi}_q\ \tilde{G}^{\mu\{\nu}\gamma^{\lambda\}}\ \psi_q|P,S\rangle$$

$\longrightarrow$ Twist-4 contribution

$$f_2\ M^2 S^{\mu} = \frac{1}{2}\sum_q e_q^2\ \langle P,S|g\ \bar{\psi}_q\ \tilde{G}^{\mu\nu}\gamma_{\nu}\ \psi_q|P,S\rangle$$

In terms of structure functions ...

- Target mass correction a_2 calculated from leading twist part of g_1

$$a_2 \;=\; 2\int_0^1 dx\; x^2\; g_1(x,Q^2)$$

- Twist-3 correction d_2 extracted from g_1 and g_2

$$d_2 \;=\; \int_0^1 dx\; x^2\; \left(2\; g_1(x,Q^2) \;+\; 3\; g_2(x,Q^2)\right)$$

- Note x^2 weighting

$\longrightarrow$ emphasizes high x region

$\longrightarrow$ more important role played by nucleon resonances

Color polarizabilities

Twist-3 and -4 matrix elements from components of dual field strength tensor:

$$8M^2\, d_2\, \vec{S} \;=\; \langle P,S|\; \vec{j}_a \times \vec{E}_a \;+\; 2j_a^0 \vec{B}_a\; |P,S\rangle$$

$$2M^2\, f_2\, \vec{S} \;=\; \langle P,S|\; \vec{j}_a \times \vec{E}_a \;-\; j_a^0 \vec{B}_a\; |P,S\rangle$$

quark current $j_a^\alpha = -g\bar{\psi}\gamma^\alpha t_a \psi$

[Mankiewicz, Schafer *et al.*; Balitsky, Braun *et al.*]

- Color electric and magnetic polarizabilities

$$\chi_E\; 2M^2\vec{S} \;\equiv\; \langle P,S|\; \vec{j}_a \times \vec{E}_a\; |P,S\rangle$$

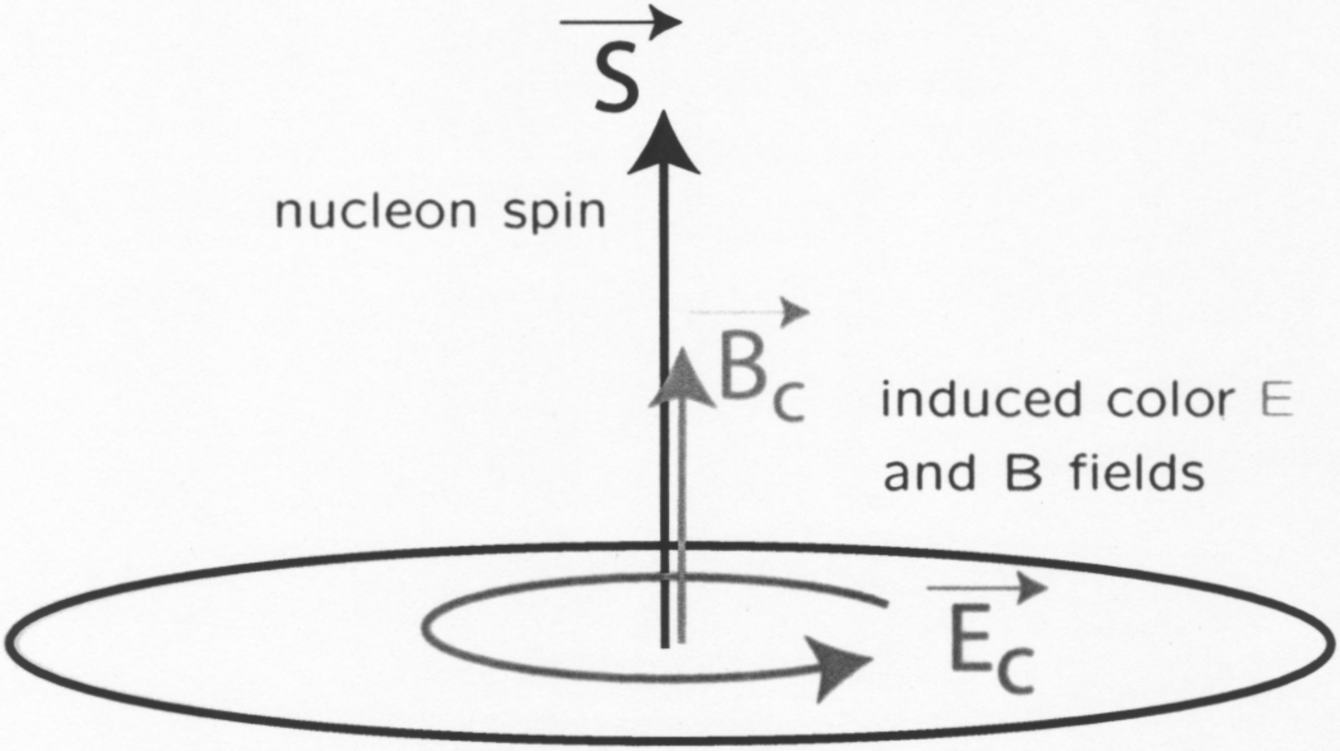
$$\chi_B\; 2M^2\vec{S} \;\equiv\; \langle P,S|\; j_a^0\; \vec{B}_a\; |P,S\rangle$$

- In terms of d_2 and f_2

$$\chi_E \;=\; \frac{2}{3}(4d_2 \;+\; 2f_2)$$

$$\chi_B \;=\; \frac{2}{3}(4d_2 \;-\; f_2)$$

Color Polarizabilities



Response of (nonperturbative) gluon field
to nucleon polarization

ANALYSIS OF Γ_1^p [UP TO $O(1/Q^4)$]

$$f_2^p = 0.039 \pm 0.022\,(\text{stat}) \pm 0.009\,(\text{sys}) \pm 0.030\,(\text{low } x) \pm 0.010\,(\alpha_s)$$

$$\frac{\mu_6}{M^4} = 0.011 \pm 0.013\,(\text{stat}) \pm 0.010\,(\text{total sys})$$

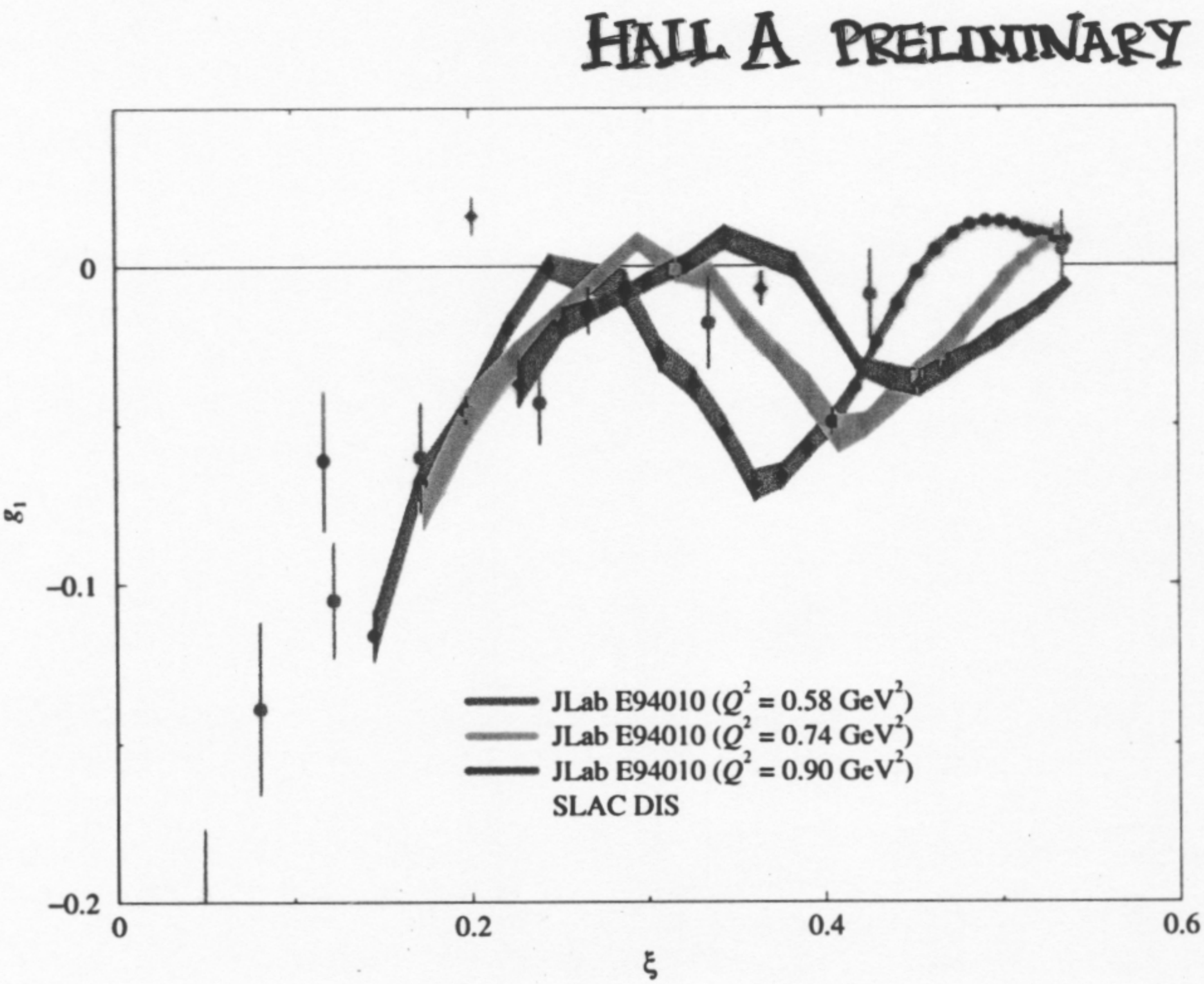
$$\chi_E^p = 0.026 \pm 0.015\,(\text{stat}) \pm 0.021\,(\text{sys})$$

$$\chi_B^p = -0.013 \pm 0.007\,(\text{stat}) \pm 0.011\,(\text{sys})$$

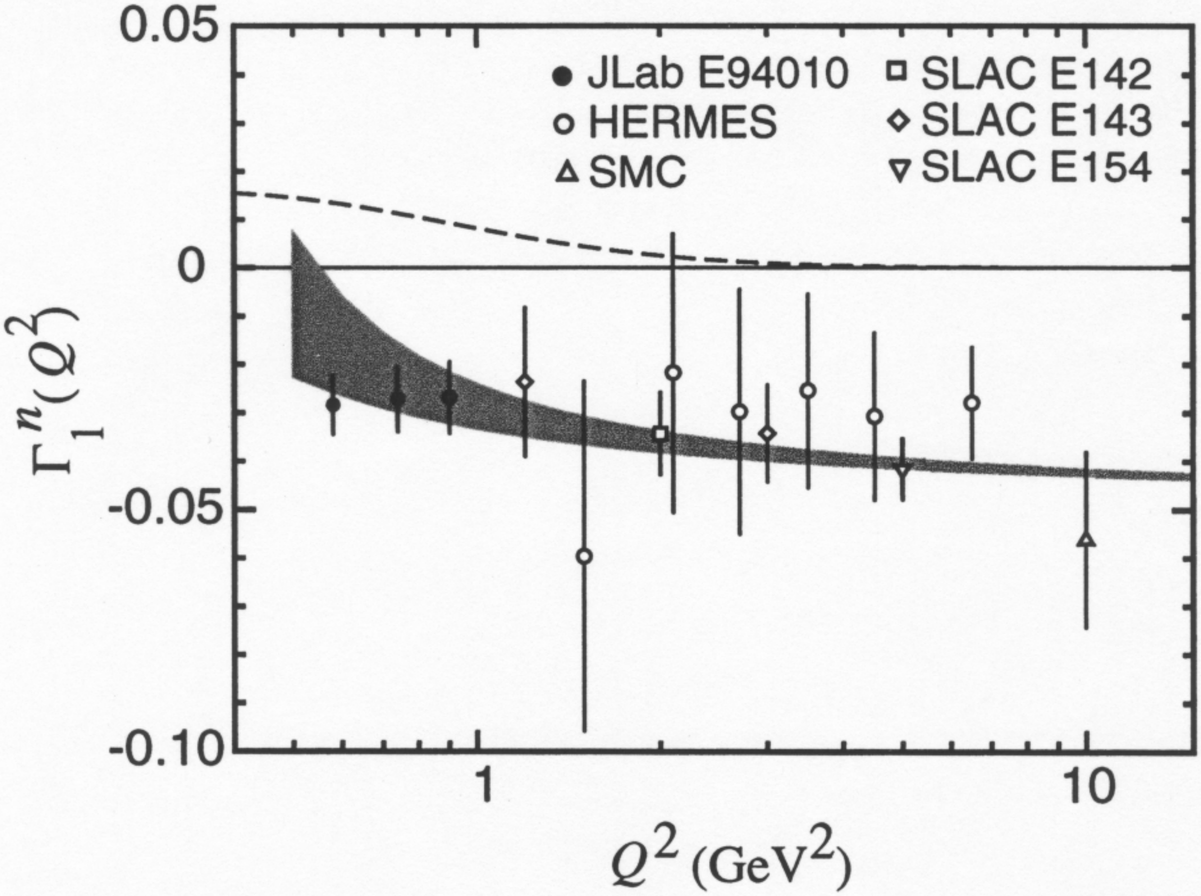
cf. MODELS

	SUM RULES	BAG	INSTANTON
χ_E^p	−0.07	0.10	−0.03
χ_B^p	0.02	0.06	0.02

LATTICE ?

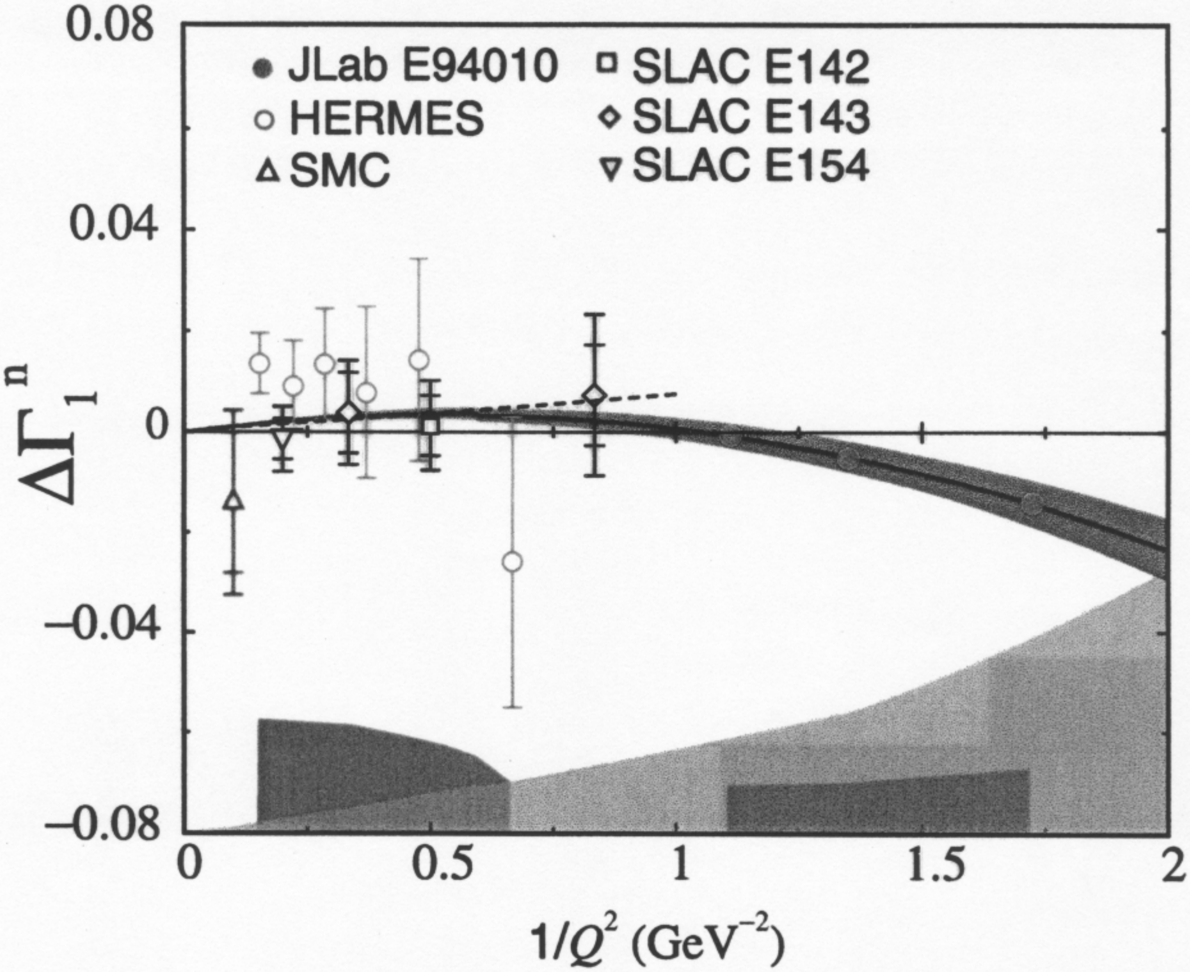


Duality in the neutron (^3He) g_1 structure function



Lowest moment of neutron g_1 structure function

Meziani et al., hep-ph/0404066



Higher twist contribution to Γ_1^n

$$\Delta\Gamma_1^n \equiv \Gamma_1^n - \Gamma_1^n(LT)$$

Meziani et al., hep-ph/0404066

- Extracted color polarizabilities

$$\chi_E^n = 0.033 \pm 0.029$$

$$\chi_B^n = -0.001 \pm 0.016$$

- Both χ_E^n and χ_B^n small, with χ_B^n consistent with zero

- Total higher twist *zero* at $Q^2 \approx 1$ GeV^2 and small down to $Q^2 \approx 0.5$ GeV^2

$\longrightarrow$ nonperturbative interactions between quarks and gluons not dominant at these scales

$\longrightarrow$ suggests *strong cancellations* betweem resonances, resulting in *dominance of* LT contribution

$\longrightarrow$ spectacular confirmation of duality in neutron!

Coherence vs. Incoherence

- Exclusive form factors

$\longrightarrow$ *coherent* scattering from quarks

$$d\sigma \sim \left(\sum_i e_i\right)^2$$

- Inclusive structure functions

$\longrightarrow$ *incoherent* scattering from quarks

$$d\sigma \sim \sum_i e_i^2$$

How does the square of a sum become the sum of squares?

Pedagogical Model

Two quarks bound in a harmonic oscillator potential $\longrightarrow$ *exactly solvable spectrum*

- Structure function given by sum of squares of transition form factors

$$F(\nu, \mathrm{q}^2) \sim \sum_n \left| \mathcal{F}_{0,n}(\mathbf{q}^2) \right|^2 \delta(E_n - E_0 - \nu)$$

- Charge operator $\sum_i e_i \exp(i\vec{q} \cdot \vec{r}_i)$ excites

even partial waves with amplitude $\propto (e_1 + e_2)^2$
odd partial waves with amplitude $\propto (e_1 - e_2)^2$

- Resulting structure function

$$F(\nu, \mathbf{q}^2) \sim \sum_n \left\{ (e_1 + e_2)^2 \mathcal{F}^2_{0,2n} + (e_1 - e_2)^2 \mathcal{F}^2_{0,2n+1} \right\}$$

- If states degenerate, cross terms ($\sim e_1 e_2$) cancel when averaged over nearby even and odd parity states

Minimum condition for duality:

$\longrightarrow$ *at least one complete set of* *__even__* *and* *__odd__* *parity resonances must be ~~summed over~~*

DON'T NEED $\sum$ OVER ALL RESON.CES

Close, Isgur (2001)

$\rightsquigarrow$ GET ID FROM SUBSET

Quark Model

- Symmetric and antisymmetric states generalize to $\mathbf{56}^{+}$ and $\mathbf{70}^{-}$ multiplets of spin-flavor SU(6)

- Scaling occurs if contributions from $\mathbf{56}^{+}$ and $\mathbf{70}^{-}$ have equal strengths

- Relative $N \to N^*$ transition probabilities

	$\mathbf{56}^{+}$		$\mathbf{70}^{-}$			
Repr^n.	$^2\mathbf{8}$	$^4\mathbf{10}$	$^2\mathbf{8}$	$^4\mathbf{8}$	$^2\mathbf{10}$	*sum*
F_1^p	9	8	9	0	1	27
F_1^n	4	8	1	4	1	18
g_1^p	9	−4	9	0	1	15
g_1^n	4	−4	1	−2	1	0

↑ S_{11}(1535)

Close, WM (2003)

Duality in SU(6) Quark Model

- Summing over all resonances in **56**$^+$ and **70**$^-$ multiplets

$$\Rightarrow \qquad R^{np} = \frac{F_1^n}{F_1^p} = \frac{2}{3}$$

$$A_1^p = \frac{g_1^p}{F_1^p} = \frac{5}{9}$$

$$A_1^n = \frac{g_1^n}{F_1^n} = 0$$

$\longrightarrow$ *as in quark-parton model* !!

- Expect duality to appear earlier for F_1^p than F_1^n

- Earlier onset for g_1^n than g_1^p

$\longrightarrow$ cancellations *within* multiplets for g_1^n

Summary & Outlook

- Remarkable confirmations of quark-hadron duality in structure functions

  $\longrightarrow$ *higher twists "small" down to low* Q^2 (~ 1 GeV^2)

- Local duality
  $\longrightarrow$ clues to origins of resonance cancellations from quark models
  $\longrightarrow$ challenge to understand from QCD

- Duality in other observables
  $\longrightarrow$ longitudinal (F_L) vs. transverse (F_1) structure functions
  $\longrightarrow$ spin structure
  $\longrightarrow$ nuclei (Fermi smearing $\rightarrow$ faster onset)
  $\longrightarrow$ other hadrons (pion)
  $\longrightarrow$ fragmentation functions

- More exciting discoveries ahead!