fastNLO

Thomas Kluge, DESY
Klaus Rabbertz, University Karlsruhe
Markus Wobisch, Fermilab

TeV4LHC workshop

October 20, 2005

- Motivation
- Concept
- The Product
- Status / Outlook

fastNLO

Motivation

- Interpretation of Experimental Data relies on:
  availability of Theory Calculations and ability to perform these reasonably **fast**
  Often need: repeated computations of the same cross section
  for different PDFs and/or $\alpha_s(M_z)$ values

- Examples for a specific analysis:
  - use various PDFs (CTEQ, MRST, Alekhin, Botje, H1, ZEUS, ...)
  - determine PDF uncertainties (PDF error sets)
  - use data set in fit of PDFs and/or α_s

- For some observables NLO predictions can be computed extremely fast
  (e.g.: DIS structure functions)

- ... but some are extremely slow: Drell-Yan and Jet Cross Sections
  (but very important in global PDF determinations)

$\Longrightarrow$ need new procedure for very fast repeated computations of NLO cross sections

fastNLO

New Concept

- Can be used for **any** observable in hadron-induced processes (hadron-hadron / DIS / photoproduction)
- Although labeled "fastNLO" $\rightarrow$ can be used in any order $\Rightarrow$ fastN^{n}LO
- Our concept does not include the theoretical calculation itself (leave this to theorists) $\rightarrow$ it requires existing flexible computer code — here: NLOJET++ (Zoltan Nagy)
- During the first computation no time is saved
  need full time of the original code: hours, days, weeks, months, ...
  to achieve high statistical precision
- Any further computation takes **a few seconds** (returning high statistical precision)
- This concept involves one single approximation (see later)
  But: precision of approximation can be quantified & arbitrarily improved (goal: 0.3%)

$\Rightarrow$ **here:** example for inclusive jet production in hadron-hadron collisions

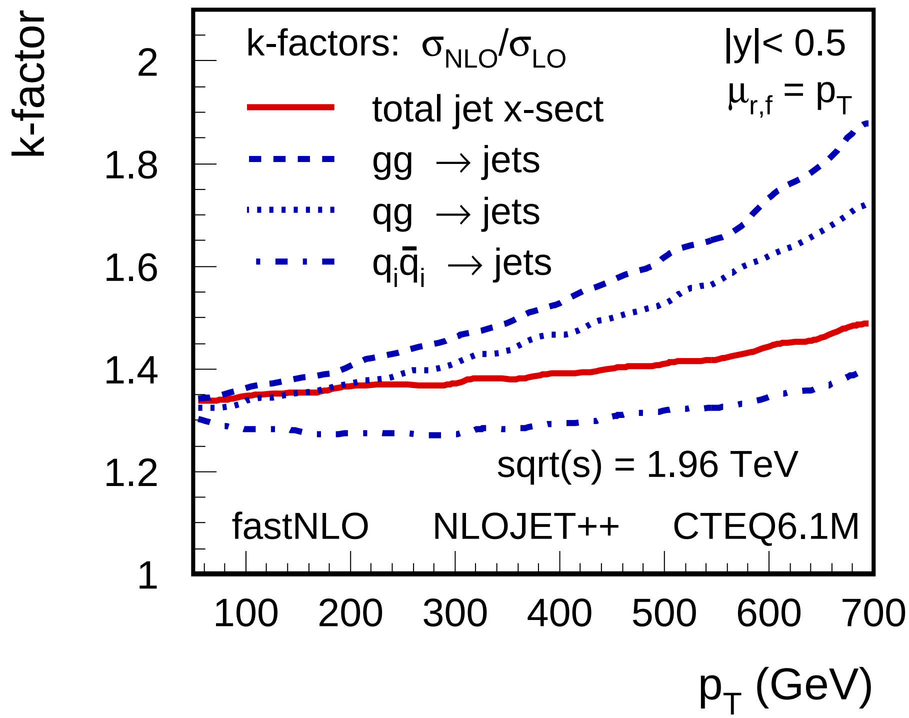
Current CTEQ Procedure

k-factor approximation:
- for a given PDF $\rightarrow$ compute k-factor for each bin: k = sigma(NLO)/sigma(LO)
- "relatively fast": compute LO cross section for arbitrary PDF
- multiply sigma(LO) with k-factor $\rightarrow$ get "NLO" prediction

problem:
- k-factor itself depends on the PDFs $\longrightarrow\longrightarrow\longrightarrow$
- higher for gluon induced subprocesses

reasons:
- different x-coverage in LO and NLO
- different k-factors for different subprocesses

limitations:
- even the LO computation is slow
- computing time depends on statistical precision



fastNLO
- as exact as you like (only a single high statistics computation is needed)
- much, much faster ($<$10 sec. – with high precision)

Jet Cross Section in hadron-hadron

General cross section formula for hadron-hadron collisions:

$$\sigma_{\rm hh} = \sum_n \alpha_s^n(\mu_r) \sum_{\text{PDFflavors } i} \sum_{\text{PDFflavors } j} c_{i,j,n}(\mu_r, \mu_f) \otimes f_i(x_1, \mu_f) \otimes f_j(x_2, \mu_f) \,.$$

- strong coupling constant α_s to the power n
- perturbative coefficients $c_{i,j,n}$
- parton density functions (PDFs) of the two hadrons $f_i(x),\ f_j(x)$
- renormalization scale μ_r, factorization scale μ_f, (ignore in the following $\Rightarrow \mu_{r,f} = p_T$)
- momentum fraction x

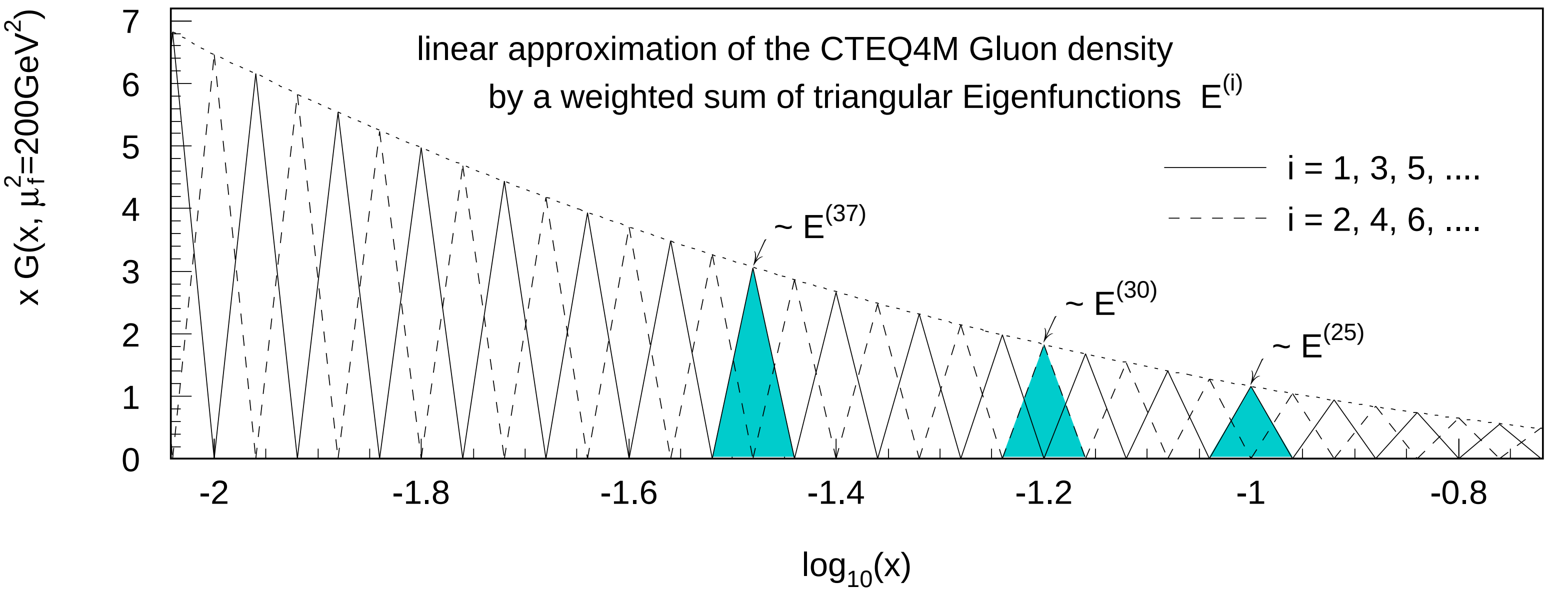
Standard procedure:

- integration over whole phase space (x_1, x_2) (usually Monte-Carlo method)
- at each MC integration point:
  - computation of observable (e.g. run jet algorithm, determine p_T, $|y|$ bin)
  - compute perturbative coefficient
  - get α_s and PDFs values

  $\Rightarrow$ add contribution to bin

goal: try to separate the PDFs from the integral

PDF Approximation



- introduce a set of discrete x-values labeled $x^{(i)}$ $(i = 0, 1, 2, \cdots, n)$
- with $x^{(n)} < x^{(n-1)} < x^{(n-2)} < \cdots < x^{(0)} = 1$
- around each $x^{(i)}$, define an eigenfunction $E^{(i)}(x)$
- with $E^{(i)}(x^{(i)}) = 1$, $E^{(i)}(x^{(j)}) = 0$ for $i \neq j$ and $\sum_i E^{(i)}(x) = 1$ for all x

- express a single PDF $f(x)$ by a linear combination of eigenfunctions $E^{(i)}(x)$ with coefficients given by the PDF values $f(x^{(i)})$ at the discrete points $x^{(i)}$

$$f(x) = \sum_i f(x^{(i)})\, E^{(i)}(x)$$

fastNLO

PDF Approximation (2)

processes with two hadrons – need Eigenfunctions in 2d-space (x_1, x_2)

- define $E^{(i,j)}(x_1, x_2) \equiv E^{(i)}(x_1) E^{(j)}(x_2)$
- product of two PDFs $f(x_1, x_2) \equiv f_1(x_1)\, f_2(x_2)$ is given by

$$f(x_1, x_2) = \sum_{i,j} f(x_1^{(i)}, x_2^{(j)})\; E^{(i,j)}(x_1, x_2)$$

note: this is an **approximation**!!
choice of triangular Eigenfunctions $\Longrightarrow$ linear interpolation of PDFs between adjacent $x^{(i)}$
this is the **only** approximation in fastNLO — precision can be arbitrarily improved!!
precision depends on:

- choice of set of $x^{(i)}$ — e.g. on $\log_{10}(1/x)$ or $\sqrt{\log_{10}(1/x)}$ (needs clever choice) ($\rightarrow$ higher $x^{(i)}$ density in x regions where PDFs have stronger curvature)
- number of x-bins (brute force) $\longrightarrow$ memory $\propto n^2$

$\Rightarrow$ **goal:** precision of 0.3% for all bins

Partonic Subprocesses in h-h

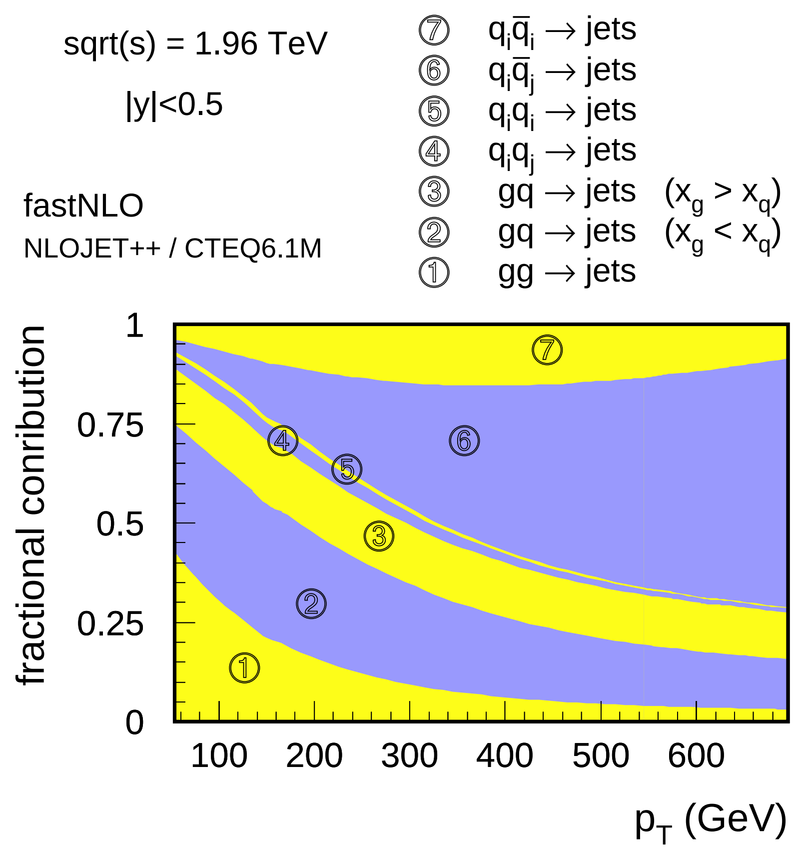
now: don't want to deal with 13×13 PDFs!!

For hadron-hadron $\longrightarrow$ jets there are **seven** relevant partonic subprocesses:

$$
\begin{array}{rlrl}
gg \longrightarrow \text{jets} & & & \propto H_1(x_1, x_2) \\
qg \longrightarrow \text{jets} & \text{plus} & \bar{q}g \longrightarrow \text{jets} & \propto H_2(x_1, x_2) \\
gq \longrightarrow \text{jets} & \text{plus} & g\bar{q} \longrightarrow \text{jets} & \propto H_3(x_1, x_2) \\
q_i q_j \longrightarrow \text{jets} & \text{plus} & \bar{q}_i \bar{q}_j \longrightarrow \text{jets} & \propto H_4(x_1, x_2) \\
q_i q_i \longrightarrow \text{jets} & \text{plus} & \bar{q}_i \bar{q}_i \longrightarrow \text{jets} & \propto H_5(x_1, x_2) \\
q_i \bar{q}_i \longrightarrow \text{jets} & \text{plus} & \bar{q}_i q_i \longrightarrow \text{jets} & \propto H_6(x_1, x_2) \\
q_i \bar{q}_j \longrightarrow \text{jets} & \text{plus} & \bar{q}_i q_j \longrightarrow \text{jets} & \propto H_7(x_1, x_2)
\end{array}
$$

The H_i are linear combinations of PDFs
$\longrightarrow$ reduced from 13×13 to seven!!

partonic subprocesses for $p\bar{p} \rightarrow$ jets



detail:
for hadron - anti-hadron collisions:
PDFs of the anti-hadron are expressed by the PDFs of the hadron (quarks ↔ anti-quarks)
here: swap $H_4 \leftrightarrow H_7$ and $H_5 \leftrightarrow H_6$

fastNLO

PDFs — Definitions

$$
\begin{aligned}
G(x,\mu_f) &= g(x,\mu_f)\\
Q(x,\mu_f) &= \sum_i q_i(x,\mu_f)\\
\bar{Q}(x,\mu_f) &= \sum_i \bar{q}_i(x,\mu_f)\\
S(x_1,x_2,\mu_f) &= \sum_i \left(q_i(x_1,\mu_f)\, q_i(x_2,\mu_f) + \bar{q}_i(x_1,\mu_f)\, \bar{q}_i(x_2,\mu_f)\right)\\
A(x_1,x_2,\mu_f) &= \sum_i \left(q_i(x_1,\mu_f)\, \bar{q}_i(x_2,\mu_f) + \bar{q}_i(x_1,\mu_f)\, q_i(x_2,\mu_f)\right)
\end{aligned}
$$

- $q_i(x)$ ($\bar{q}_i(x)$) — quark (anti-quark) density of flavor i
- $i = 1, \ldots, n_f$ — No. of flavors
- $G(x)$ — gluon density

fastNLO

Relevant Combinations of PDFs

$$
\begin{aligned}
H_1(x_1,x_2) &= G(x_1)\,G(x_2)\,, \\
H_2(x_1,x_2) &= \left(Q(x_1)+\bar{Q}(x_1)\right)\;G(x_2)\,, \\
H_3(x_1,x_2) &= G(x_1)\;\left(Q(x_2)+\bar{Q}(x_2)\right)\;, \\
H_4(x_1,x_2) &= Q(x_1)Q(x_2)+\bar{Q}(x_1)\bar{Q}(x_2)-S(x_1,x_2)\,, \\
H_5(x_1,x_2) &= S(x_1,x_2)\,, \\
H_6(x_1,x_2) &= A(x_1,x_2)\,, \\
H_7(x_1,x_2) &= Q(x_1)\bar{Q}(x_2)+\bar{Q}(x_1)Q(x_2)-A(x_1,x_2)\,.
\end{aligned}
$$

These are the seven combinations of PDFs, corresponding to the seven subprocesses

symmetries:
$H_n(x_1,x_2) = H_n(x_2,x_1)$ for $n = 1,4,5,6,7$ and $H_2(x_1,x_2) = H_3(x_2,x_1)$

$$H_k(x_1,x_2) = \sum_{(i,j)} H_k(x^{(i)},x^{(j)})\;E^{(i,j)}(x_1,x_2)$$

where $H_k(x^{(i)},x^{(j)})$ is a number $\leftrightarrow$ PDF information

Jet Cross Section in hadron-hadron

With these definitions of the seven $\boldsymbol{H_i}$ the cross section reads:

$$\boldsymbol{\sigma}_{\mathrm{hh}} = \sum_n \boldsymbol{\alpha}_s^n(\boldsymbol{\mu}_r) \sum_{k=1}^{7} \boldsymbol{c}_{k,n}(\boldsymbol{\mu}_r, \boldsymbol{\mu}_f) \otimes \boldsymbol{H}_k(\boldsymbol{x}_1, \boldsymbol{x}_2, \boldsymbol{\mu}_f)$$

Now: express $\boldsymbol{H_k}$ by linear combinations of the $\boldsymbol{E}^{(i,j)}(\boldsymbol{x}_1, \boldsymbol{x}_2)$

$$\boldsymbol{\sigma}_{\mathrm{hh}} = \sum_n \boldsymbol{\alpha}_s^n(\boldsymbol{\mu}_r) \sum_{k=1}^{7} \boldsymbol{c}_{k,n}(\boldsymbol{\mu}_r, \boldsymbol{\mu}_f) \otimes \left(\sum_{i,j} \boldsymbol{H}_k(\boldsymbol{x}^{(i)}, \boldsymbol{x}^{(j)}) \cdot \boldsymbol{E}^{(i,j)}(\boldsymbol{x}_1, \boldsymbol{x}_2) \right)$$

or, better:

$$\boldsymbol{\sigma}_{\mathrm{hh}} = \sum_n \boldsymbol{\alpha}_s^n(\boldsymbol{\mu}_r) \sum_{k=1}^{7} \sum_{i,j} \boldsymbol{H}_k(\boldsymbol{x}_1^{(i)}, \boldsymbol{x}_2^{(j)}) \left(\boldsymbol{c}_{k,n}(\boldsymbol{\mu}_r, \boldsymbol{\mu}_f) \otimes \boldsymbol{E}^{(i,j)}(\boldsymbol{x}_1, \boldsymbol{x}_2) \right)$$

important: integral is independent of PDFs!
the numbers $\boldsymbol{H}_k(\boldsymbol{x}^{(i)}, \boldsymbol{x}^{(j)})$ contain all information on the PDFs

$\Rightarrow$ **exactly what we wanted!!**

Last Step ...

define:

$$\tilde{\sigma}_{k,n}^{(i,j)} \equiv c_{k,n}(\mu_r, \mu_f) \otimes E^{(i,j)}(x_1, x_2)$$

$\Rightarrow$ the $\tilde{\sigma}_{k,n}^{(i,j)}$ contain all information on the observable

(the perturbative coefficients, the jet definition, and the phase space restrictions).

but: $\tilde{\sigma}_{k,n}^{(i,j)}$ is independent of the PDFs and α_s – needs to be computed only once!

The cross section is then given by the simple product **($\longrightarrow$ Master Formula!)**

$$\sigma_{\mathrm{hh}} = \sum_{i,j,k,n} \alpha_s^n(\mu_r)\; H_k(x_1^{(i)}, x_2^{(j)})\; \tilde{\sigma}_{k,n}^{(i,j)}$$

can be reevaluated **very** quickly for different PDFs and α_s values,

as e.g. required in the determination of PDF uncertainties or in global fits of PDFs

Implementation Steps

to implement a new observable in fastNLO:

- find theorist to provide flexible computer code
- identify elementary subprocesses & relevant PDF linear combinations
- define analysis bins (e.g. p_T, $|y|$)
- define Eigenfunctions $E(x)$, $E(x_1, x_2)$ (e.g. triangular) & the set of $x^{(i)}$
- to optimize x-range: find lower x-limit ($x_{\text{limit}} < x < 1$) (for each analysis bin)

<u>example:</u> DØ Run I measurement of Incl. Jet Cross Section, Phys. Rev. Lett.86, 1707 (2001)

- 90 analysis bins in (E_T, η)
- 2 orders of $\alpha_s(p_T)$ (LO & NLO)
- 7 partonic subprocesses
- No. of x-intervals for each bin: 50 (40?) $\longleftarrow$ (study precision of PDF approximation)
  $\Rightarrow (n^2 + n)/2 =$ 1275 (820?) Eigenfunctions $E^{(i,j)}(x_1, x_2)$

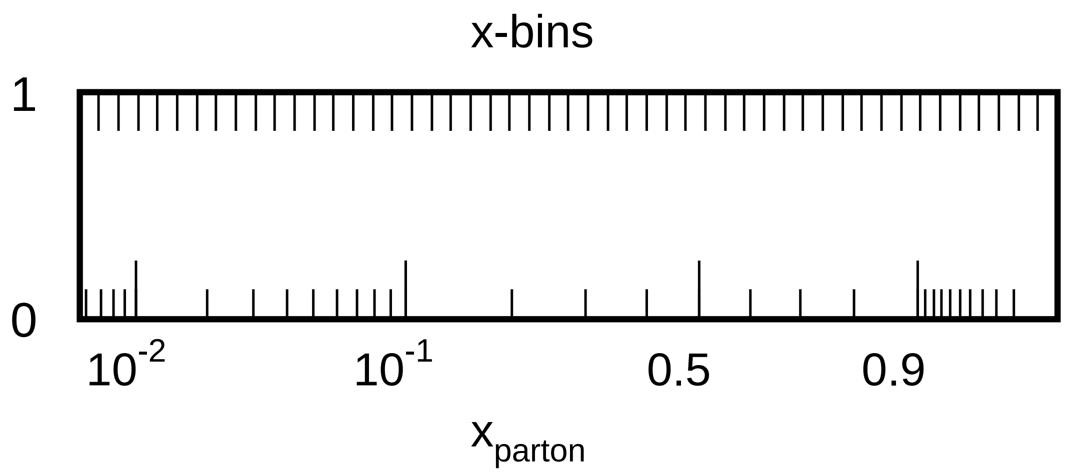
- compute 1.6M (1M?) variables $\tilde{\sigma}_{k,n}^{(i,j)}$ (times three, if scale variations are included)
  $\Rightarrow$ stored in huge table!!!

compute VERY long to achieve very high precision — (after all: needs to be done only once!)

Binning in x

How to choose the $x^{(i)}$??

- Goal: equidistant bins in some function $f(x)$
- simple choice: $f(x) = \log_{10}(x)$ gives too broad bins at high x $(x > 0.7)$
- possible choice: $f(x) = \log_{10}(x) + x$ $\Leftarrow$ but: no analytic inverse $f^{-1}(x)$
- good choice: $f(x) = -\sqrt{\log_{10}(1/x)}$



- example: 50 bins in x to cover $5.7 \cdot 10^{-3} < x < 1$
  (as used for Run I incl jets at $70 < E_T < 80$ GeV and $2 < |\eta| < 3$)

PDF Reweighting

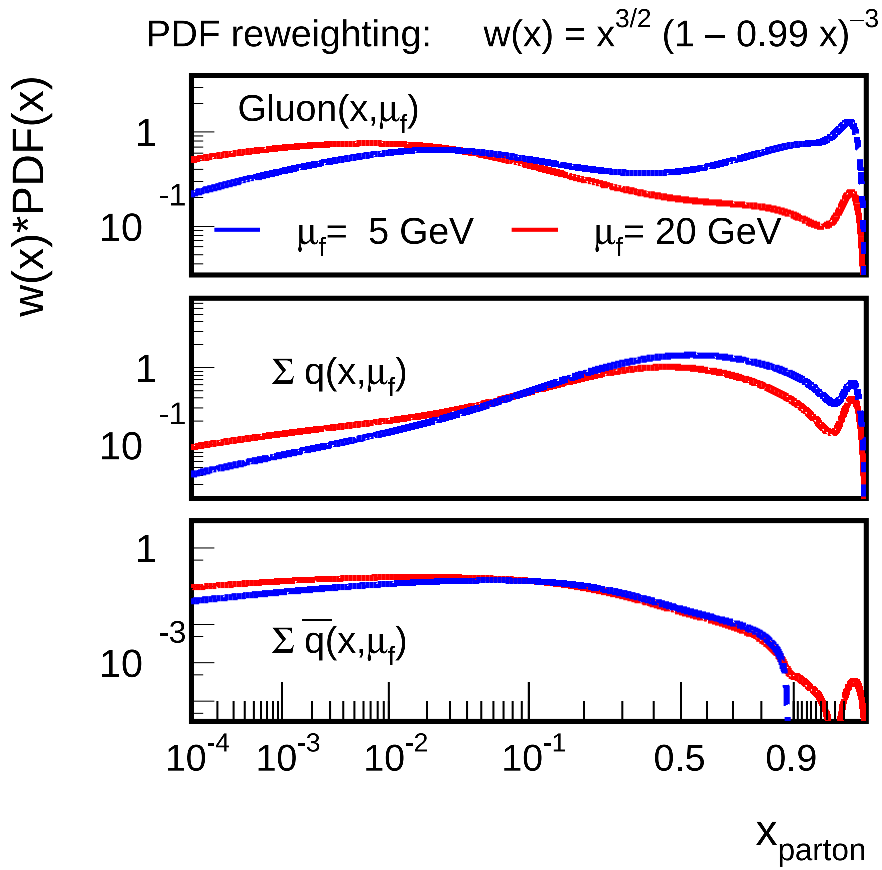
Approximation can be improved if PDFs are reweighted!!

- try to make PDFs more flat
  by reweighting with simple function $\boldsymbol{w(x)}$
  (independent of $\boldsymbol{\mu_f}$)

- choice:
  $\boldsymbol{w(x) = x^{3/2}\,(1 - 0.99x)^{-3}}$

- especially improved:
  behavior at high $\boldsymbol{x}$ (most critical)
  for all: Gluon, Quarks, Anti-Quarks
  at all scales $\boldsymbol{\mu_f}$

- absorb $\boldsymbol{w^{-1}(x)}$ into $\boldsymbol{E^{(i)}(x)}$

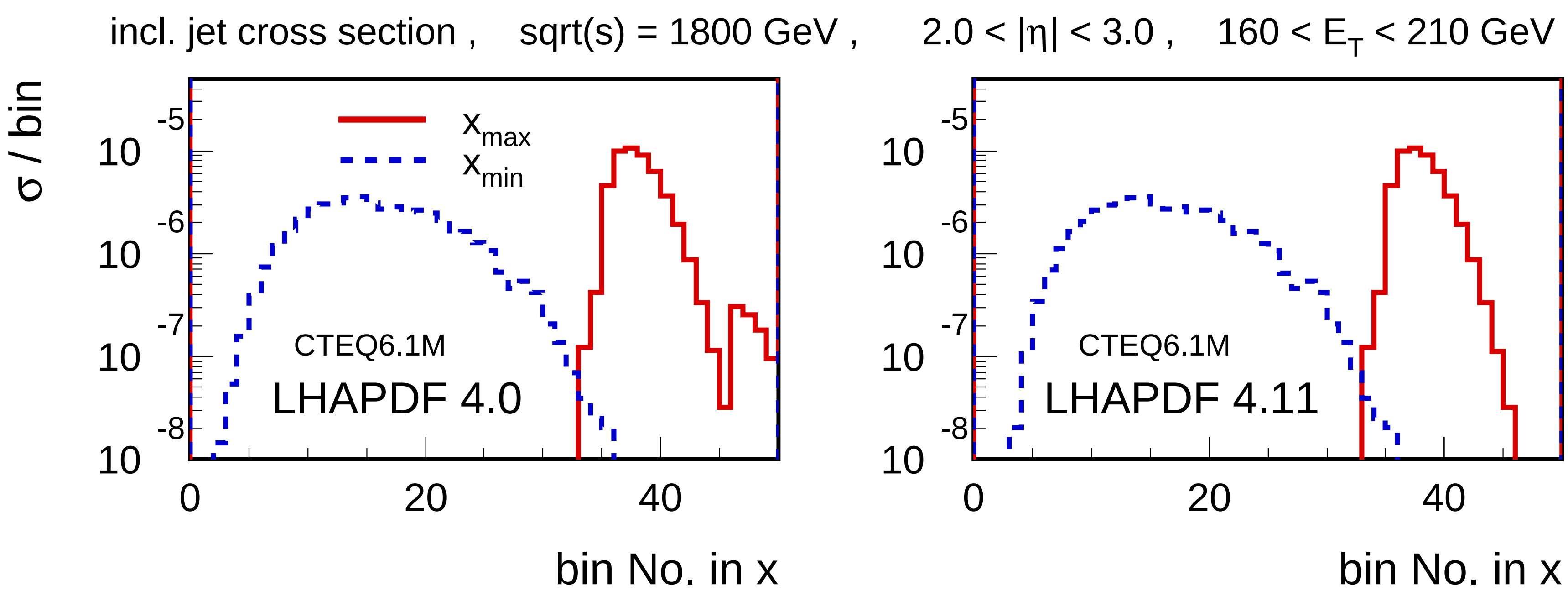


$\Longrightarrow$ reweighting reduces the PDF curvature for all factorization scales

Precision Study

High-Precision test showed 1.5% deviation in most critical region:
$\Rightarrow$ highest DØ E_T-bin in forward region

- one x is small and the other x is very large — x range: $0.028 < x < 1$



x-distribution showed unnatural shape – bug in LHAPDF for Cteq6(LHgrid) at $x > 0.9763$

With corrected PDF: fastNLO precision within required 0.3%

fastNLO

Beta Release

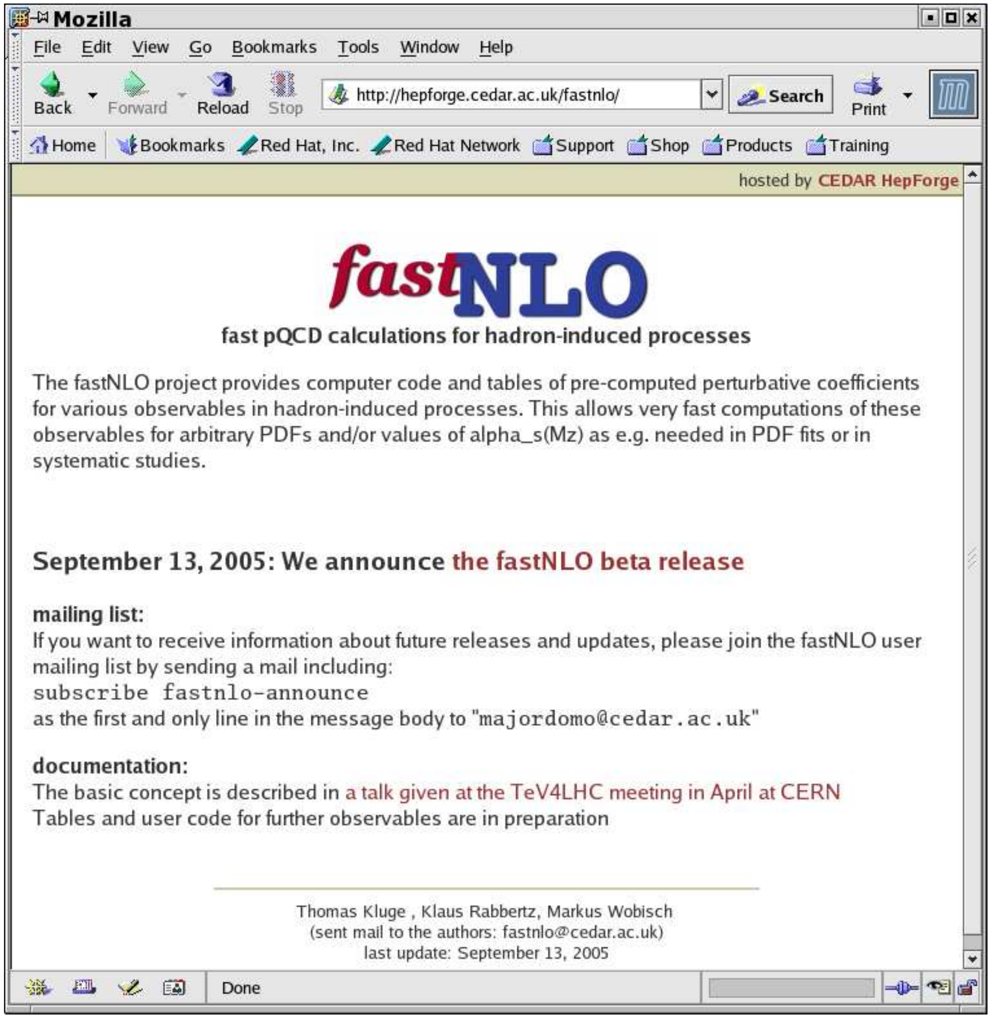
First Test-Release (Sept. 13, 2005):
Tevatron Run I Inclusive Jet Cross Section from CDF and DØ

code takes 8 seconds to compute results for CDF and DØ measurements

limited by CPU time for PDF access in LHAPDF

Calculations for LHC to be released soon!

now available at the CEDAR project:
hepforge.cedar.ac.uk/fastnlo/



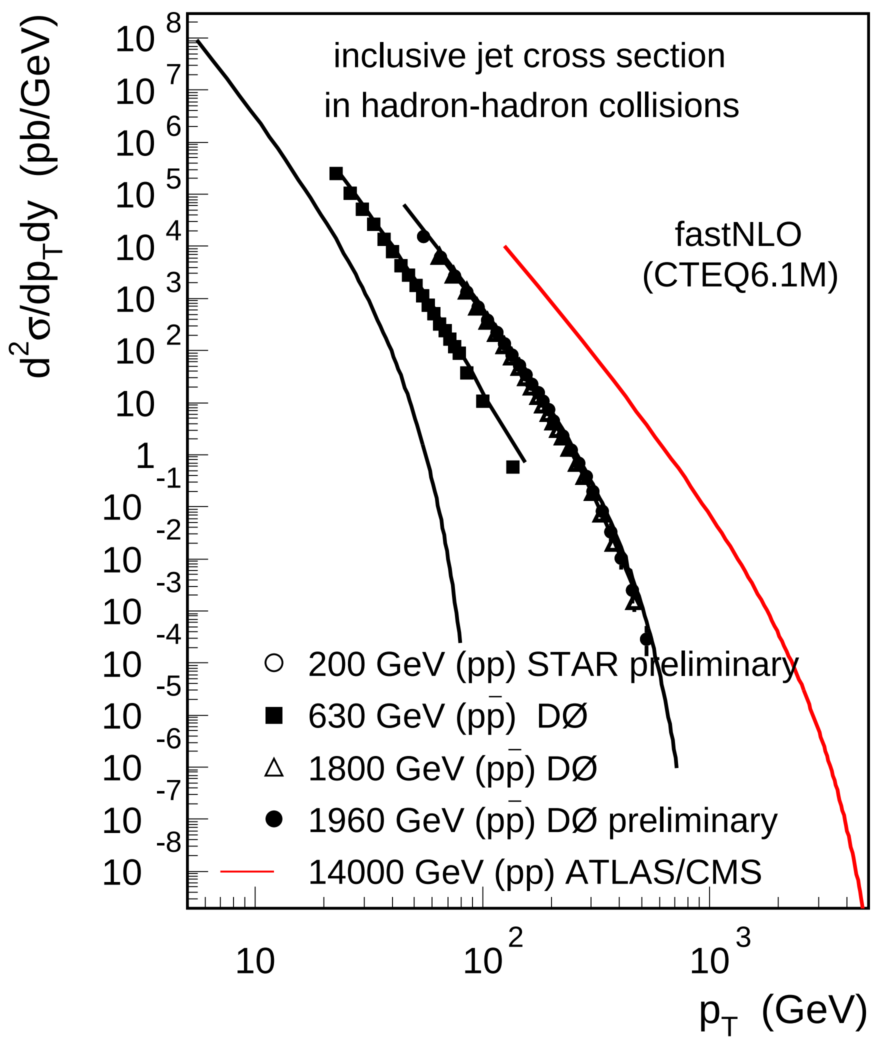
In Preparation

fastNLO Calculations already prepared for the inclusive jet cross section at:

- RHIC (200 GeV)
- Tevatron (630 GeV, 1800 GeV, 1960 GeV)
- LHC (14000 GeV)

further calculations in preparation:

- dijet cross section
- dijet azimuthal decorrelation
- three-jet production
- HERA jets!!!



we need your input! — what is important for you?

The Product

Everything is downloadable from the fastNLO Webpage (at CEDAR)
http://hepforge.cedar.ac.uk/fastnlo/

For each observable **fastNLO** will provide a package which includes:

- Tables of $\tilde{\boldsymbol{\sigma}}_{k,n}^{(i,j)}$ in different orders - for different ren. and fact. scales
- Stand-Alone Code to:
  - read tables
  - loop over PDFs (LHAPDF interface - or custom user interface for global fitters)
  - output cross section numbers as: array, ASCII, ROOT/HBOOK histograms
- Examples

Code computes NLO Predictions for a whole set of data points in the order of seconds (depends on speed of PDF interface)

Can easily be included into any user-specific analysis framework
- Tevatron & LHC experiments: make quick predictions
- MRST, CTEQ, Alekhin: use data in PDF fits
- HERA experiments: easily check jet predictions for their HERA-only PDF fits

fastNLO

Summary / Outlook

Status:

- concept for fastNLO is fully developed
- implementation of code for hadron-hadron jet cross section finished
- implementation of code for DIS jet cross sections almost finished
- test release for the Tevatron Run I jets is published

Outlook:

- publication is in preparation
- provide tables and user code for further observables
  $\Longrightarrow$ which jet algorithm(s) will be used at the LHC (?)
- later: include threshold resummation for hadron-hadron jets , Drell-Yan@NNLO / $\cdots$
  ???

We need feedback!!
CTEQ and MRST should use fastNLO for faster and more precise results!