NLO QCD corrections to $\mathrm{pp} \to \mathrm{t\bar{t}b\bar{b}}$ at the LHC

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based on

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JHEP **0808** (2008) 108 [arXiv:0807.1248]
PRL **103** (2009) 012002 [arXiv:0905.0110]
and new unpublished results

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Outline of the talk

(1) **Introduction** - NLO corrections to multi-leg processes, $t\bar{t}b\bar{b}$ production

(2) **Virtual corrections** - Feynman diagrams, tensor reduction, rational terms

(3) **Real corrections** - Dipole subtraction

(4) **Numerical results** - Predictions for the LHC, CPU performance

(1) Introduction

Six-particle processes of NLO priority list (2005/2007 Les Houches workshops)

$$\mathrm{pp} \to \quad \mathrm{t\bar{t}b\bar{b}}, \qquad \mathrm{t\bar{t}}jj, \qquad VV\mathrm{b\bar{b}}, \qquad VVjj, \qquad Vjjj, \qquad \mathrm{b\bar{b}b\bar{b}}$$

Importance of NLO for LHC phenomenology

- heavy SM particles + jets $\Rightarrow$ large backgrounds to many Higgs and BSM signals
- large powers of α_{S} $\Rightarrow$ huge QCD scale uncertainties
- many different scales $\Rightarrow$ scale-guess nontrivial

Technical challenges for $2 \to 4$ at NLO

- thousands of one-loop diagrams $\Rightarrow$ huge algebraic expressions
- computer codes slower than sec/point $\Rightarrow$ CPU-months for precise distributions
- spurious singularities (Gram determinants) $\Rightarrow$ serious numerical instabilities

The optimal NLO method(s) for multi-leg calculations?

Feynman diagrams and tensor reduction

- wide and successful experience up to $n = 5$ particles
  (pp $\to$ $\mathrm{t\bar{t}H}$, Hjj, VVj, VVV, $V\mathrm{b\bar{b}}$, $\mathrm{t\bar{t}j}$, $\mathrm{t\bar{t}Z}$,...)
- but complexity increases **faster than factorially for** $n \gg 1$

Methods of on-shell type

- less practical experience
- but complexity increases only **polynomially for** $n \gg 1$

What is the best method for realistic LHC applications?

- intermediate range of $n = 6, 7$ particles
- explicit NLO calculations can tell us more than $n \gg 1$ asymptotic scaling ...

Completion of the first $2 \to 4$ calculations of the priority list

Within the last few months—four years after Les Houches wish list—four groups, using different methods, have completed two wish-list processes

- **Two calculations for $\mathrm{pp} \to \mathrm{t\bar{t}b\bar{b}}$ with permille agreement**
  - arXiv:0905.0110 by Bredenstein, Denner, Dittmaier and S. P. based on Feynman diagrams and tensor integrals
  - arXiv:0907.4723 by Bevilacqua, Czakon, Papadopoulos, Pittau and Worek based on OPP reduction and HELAC
- **Two calculations for $\mathrm{pp} \to \mathrm{W}jjj$ (leading-colour and full results)**
  - arXiv:0906.1445 by Ellis, Melnikov and Zanderighi based on D-dimensional unitarity (leading-colour approximation)
  - arXiv:0907.1984 by Berger, Bern, Dixon, Febres Cordero, Forde, Gleisberg, Ita, Kosower and Maitre based on generalized unitarity (full colour)

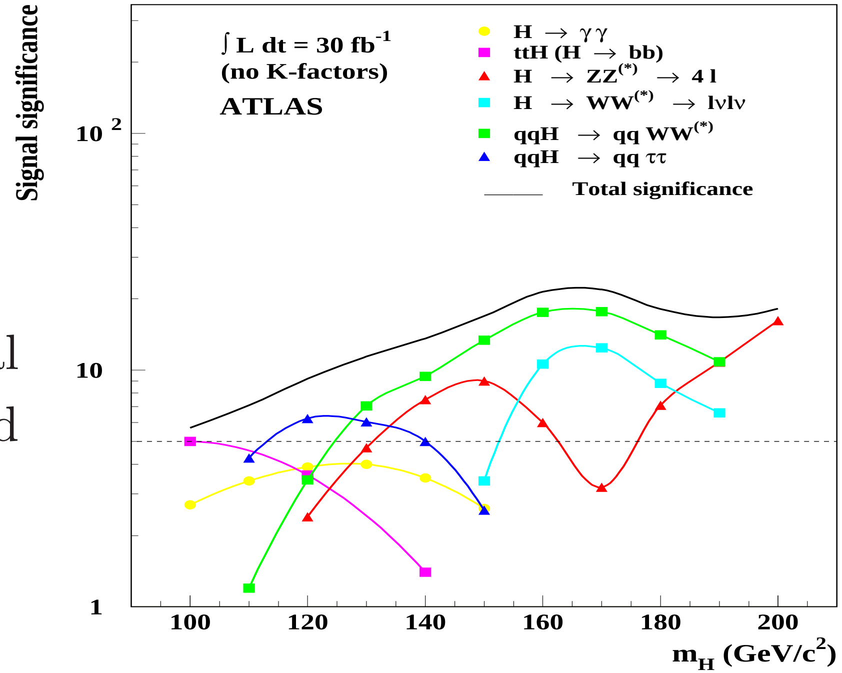
None of the methods seems to be in bad shape . . . and the old good Feynman diagrams are actually in excellent shape

Phenomenological motivation for $t\bar{t}b\bar{b}$: irreducible background to $t\bar{t}H(H \to b\bar{b})$

Associated $t\bar{t}H(H \to b\bar{b})$ production

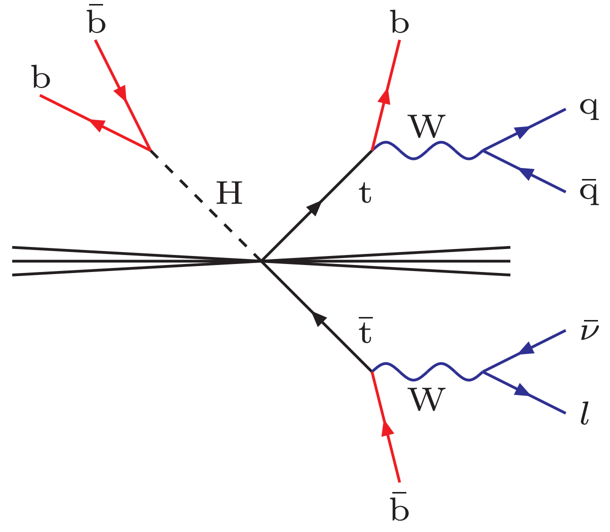
- opportunity to observe $H \to b\bar{b}$ channel and exploit dominance of its branching ratio for $M_H < 135\,\text{GeV}$
- measurement of top Yukawa coupling
- ATLAS TDR indicated discovery potential (disappeared after more reliable background estimates)
- the background has a dramatic impact

Early ATLAS studies of Higgs discovery potential (ATLAS '03)



Idea of $t\bar{t}H(H \to b\bar{b})$ analysis

- consider semileptonic decay channel: $b\bar{b}b\bar{b}jjl\nu$ final state with **four b-quarks**!
- **identify** $b\bar{b}$ **pair from Higgs decay**
- observe resonance in $m_{b\bar{b}}$ distribution



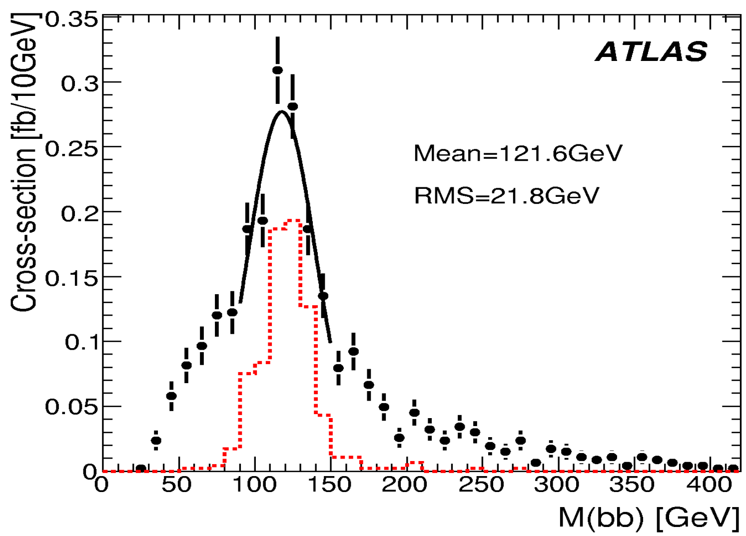
Main problem: b-quark combinatorics

- perform full $t, \bar{t}$ reconstruction to identify b-quarks from top (and Higgs) decay
- very difficult due to presence of ≥ 6 jets
- **rate of correct b-pairings only 1/3!**

Consequences

- **dilution of Higgs resonance**
- **increase of background** in resonance region

dilution of Higgs resonance in $t\bar{t}H$

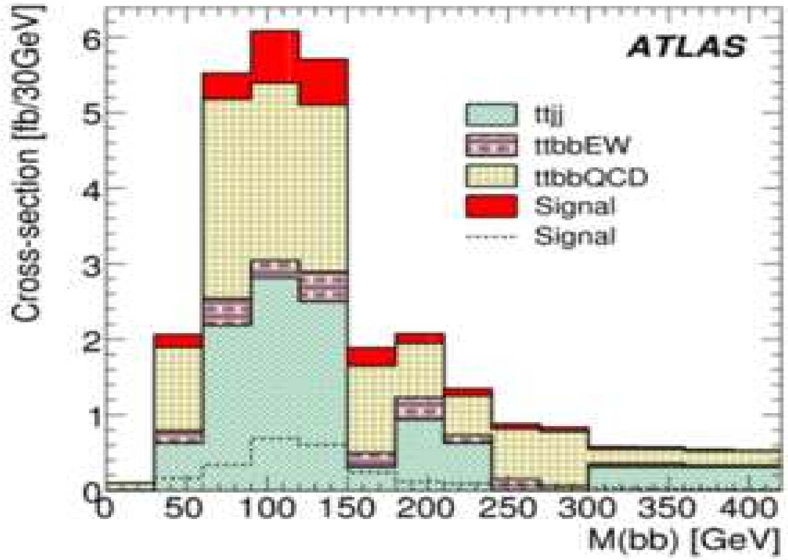


ATLAS CSC-note,CERN-OPEN-2008-020

Background and systematic uncertainty

Backgrounds (ATLAS analysis)

- $t\bar{t}b\bar{b}$ (AcerMC, $\mu_{QCD} = m_t + m_{\bar{b}b}/2$)
- $t\bar{t}jj$ (MC@NLO, $\mu^2_{QCD} = m_t^2 + < p^2_{T,t} >$)



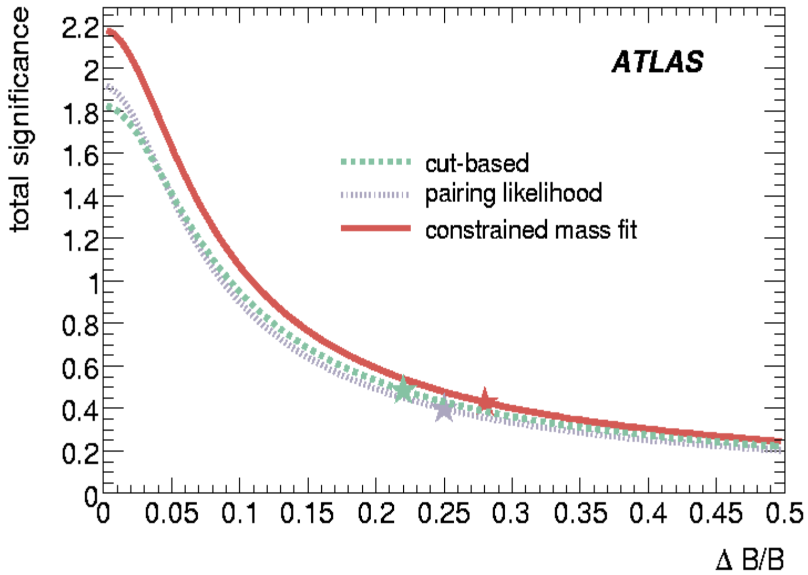
Statistics and systematics (30 fb^{-1})

- $S/\sqrt{B} \simeq 2$ sufficient for measurement
- $S/B \simeq 0.1$ implies that $\Delta B/B$ systematic uncertainty of $\mathcal{O}(10\%)$ kills measurement!

Strategy for precise determination of B

- measure B normalization in signal-free region
- extrapolate to signal-rich region using precise shape predictions

Impossible without $t\bar{t}b\bar{b}$ and $t\bar{t}jj$ at NLO!



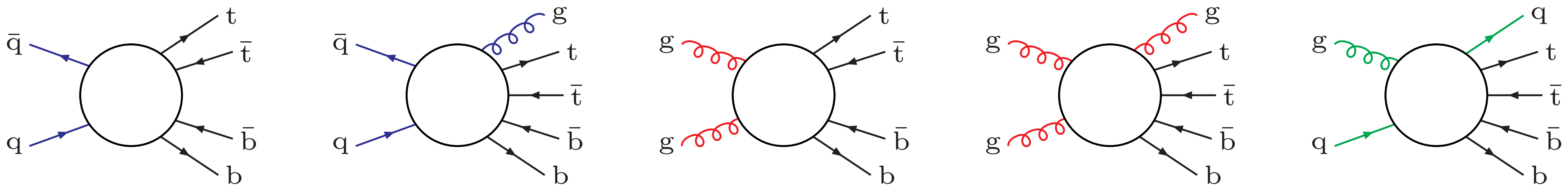
ATLAS CSC note, CERN-OPEN-2008-020

Lesson from NLO calculations for $\mathrm{pp} \to \mathrm{t\bar{t}H}$ signal and two minor backgrounds

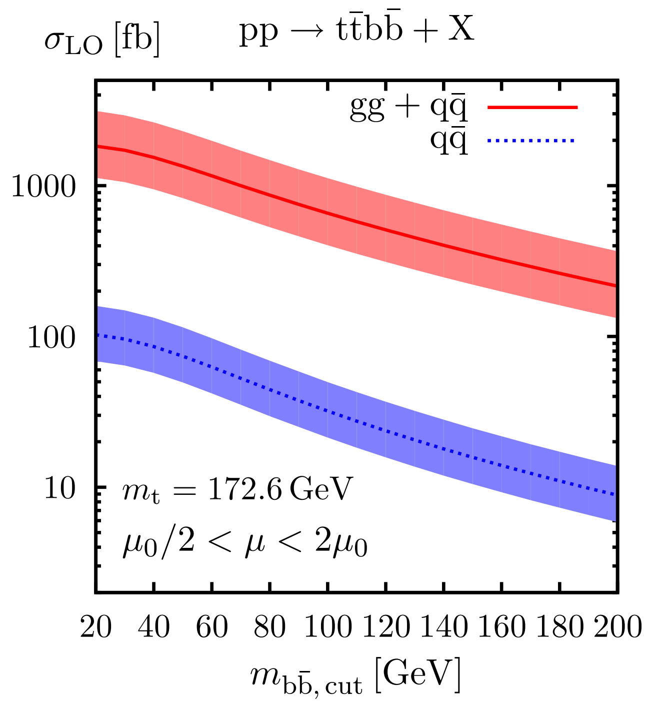
Scale choice $\mu_{\mathrm{QCD}} = E_{\mathrm{thr}}/2 \quad \Rightarrow \quad$ moderate K-factors

Process	QCD scale	K-factor	Reference
$\mathrm{pp} \to \mathrm{t\bar{t}H}$	$m_{\mathrm{t}} + M_{\mathrm{H}}/2$	1.2	Beenakker/Dittmaier/Krämer/Plümper/Spira/Zerwas (2001) Dawson/Reina/Wackeroth/Orr/Jackson (2001) Peng/Wen-Gan/Hong-Shen/Ren-You/Yi (2005)
$\mathrm{pp} \to \mathrm{t\bar{t}j}$ ($p_{\mathrm{T,jet}} > 20$–$50\,\mathrm{GeV}$)	m_{t}	1.0–1.15	Dittmaier/Uwer/Weinzierl (2007)
$\mathrm{pp} \to \mathrm{t\bar{t}Z}$	$m_{\mathrm{t}} + M_{\mathrm{Z}}/2$	1.35	Lazopoulus/McElmurry/Melnikov/Petriello (2007)

Partonic channels contributing to $pp \to t\bar{t}b\bar{b}$ at NLO



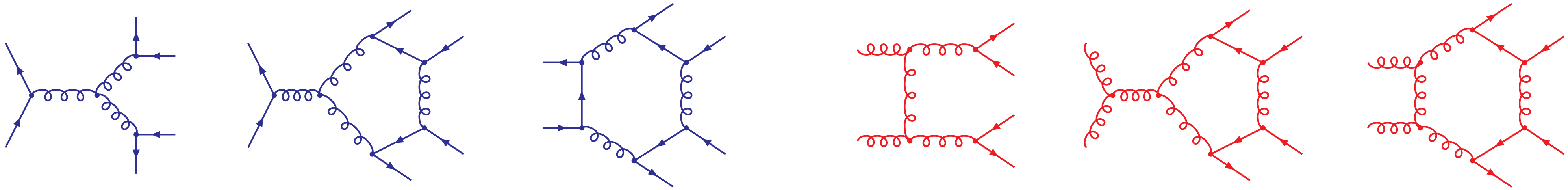
Relative weights and number of Feynman diagrams



	$q\bar{q}$	gg	qg
# of **LO** diags.	7	36	
# of **one-loop** diags	188	1003	
# of **real** diags.	64	341	64
$(\sigma/\sigma_{\mathrm{tot}})_{\mathrm{NLO}}$	3%	92%	5%

(2) Tree and one-loop contributions to $q\bar{q}/gg \to t\bar{t}b\bar{b}$

Tree and one-loop sample diagrams in the $q\bar{q}$ and gg channels



7 trees | 24 pentagons | 8 hexagons | 36 trees | 114 pentagons | 40 hexagons

Two independent calculations

- diagrams generated with `FeynArts 1.0 / 3.2` [Külbeck/Böhm/Denner '90; Hahn '01]
- one calculation uses `FormCalc 5.2` [Hahn '06] for preliminary algebraic manipualations (Dirac algebra, covariant decomposition)
- bulk of reduction with two in-house `MATHEMATICA` programs
- numerics with two independent `Fortran77` codes (two libraries for tensor integrals)

Top quarks massive and bottom quarks massless

Structure of the one-loop calculation

(a) Diagram-by-diagram approach

(b) Colour factorization

(c) Covariant decomposition of tensor integrals

(d) Numerical reduction of tensor integrals to scalar integrals

(e) Rational parts

(f) Algebraic reduction of helicity-dependent parts

(a) Diagram-by-diagram approach

$$\sum_{\text{col.,pol.}} \mathcal{A}_{\text{loop}}\, \mathcal{A}^*_{\text{tree}} = \sum_{i=1}^{N_{\text{diag}}} \left(\sum_{\text{col.,pol.}} \mathcal{D}^{(i)}_{\text{loop}}\, \mathcal{A}^*_{\text{tree}} \right)$$

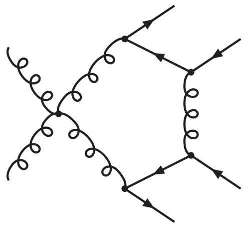
The one-loop–tree interference is computed diagram-by-diagram

- the contributions of $N_{\text{diag}} \sim 1000$ loop diagrams are computed *each by a separate* `Fortran` *routine* and added
- the large-N_{diag} cost is strongly reduced by *recycling a multitude of common substructures* (tensor integrals, helicity structures, ...)

(b) Colour factorization

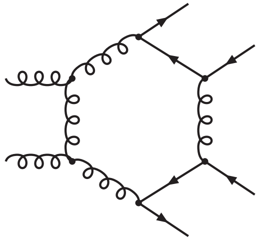
Advantage of using individual Feynman diagrams

- apart from the (few) diagrms involving 4-gluon vertices



$$= \mathcal{C}_1^{(i)}\mathcal{D}_1^{(i)} + \mathcal{C}_2^{(i)}\mathcal{D}_2^{(i)} + \mathcal{C}_3^{(i)}\mathcal{D}_3^{(i)}$$

- for most diagrams all colour matrices factorize in a single colour structure $\mathcal{C}^{(i)}$



$$= \mathcal{C}^{(i)}\mathcal{D}^{(i)}$$

The cost of colour sums is reduced to zero

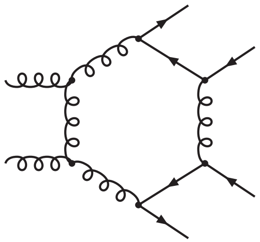
- one computes only one (few) time-expensive colour-less part(s) $\mathcal{D}^{(i)}$ per diagram
- the factorized and trivial $\mathcal{C}^{(i)}$ provide full colour information

(c) Covariant decomposition of tensor integrals

N-point tensor integrals are expressed in terms of **covariant structures** consisting of **metric tensors** $g^{\mu\nu}$ and **external momenta** $p_1^{\mu}, \ldots, p_{N-1}^{\mu}$

$$\frac{(2\pi\mu)^{4-D}}{\mathrm{i}\pi^2} \int \mathrm{d}^D q \, \frac{q^{\mu_1} \ldots q^{\mu_P}}{\prod_{i=0}^{N-1} [(q+p_i)^2 - m_i^2]} \quad = \sum_{i_1 \leq \ldots \leq i_P = 0}^{N-1} T^{(N)}_{i_1 \ldots i_P} \; \{g \ldots g p \ldots p\}^{\mu_1 \ldots \mu_P}_{i_1 \ldots i_P}$$

Each loop diagram becomes a linear combination



$$= \sum_P \sum_{i_1 \leq \ldots \leq i_P = 0}^{N-1} T^{(N)}_{i_1 \ldots i_P} \; K_{i_1 \ldots i_P}(D)$$

The two ingredients are handled in completely different ways

(d) The covariant tensor integrals $T^{(N)}_{i_1 \ldots i_P}$ are evaluated by a *numerical code* that reduces them to scalar integrals (process-independent)

(e–f) The loop-independent parts $K_{i_1 \ldots i_P}(D)$, which contain spinor chains, etc., undergo heavy *algebraic manipulations* (process-dependent)

(d) Numerical reduction of tensor integrals to scalar integrals

Collection of methods developed for $e^+e^- \to 4f$ [Denner/Dittmaier '05]

- For $N \geq 5$, exploiting **space-time 4-dim.**, one can simultaneously reduce tensor rank and # of propagators w.o. Gram-determinant instabilities

  Melrose '65; Denner/Dittmaier '02 & '05; Binoth/Guillet/Heinrich/Pilon/Schubert '05

- For $N = 3, 4$ depending on the presence of **Gram-determinant instabilities** one employs **different reductions**
  - in phase-space regions w.o. instabilities one can use **PV** Passarino/Veltman '79
  - otherwise **instabilities are avoided with various alternative reductions**: modified set of master integrals, solutions of PV identities w.o. Gram det., expansions in small Gram det.,... Denner/Dittmaier '05

  (see also analogous methods by Ferroglia/Passera/Passarino/Uccirati '03; Binoth/Guillet/Heinrich/Pilon/Schubert '05; Ellis/Giele/Zanderighi '06)

- For $N = 1, 2$ explicit **analytic expressions** are employed (no reduction)

  Passarino/Veltman '79; Denner/Dittmaier '05

(e) Rational parts

$$K_{i_1\ldots i_P}(D) \quad \underbrace{T^{(N)}_{i_1\ldots i_P}}_{\frac{R_1}{(D-4)} + \frac{R_1}{(D-4)} + \frac{R_2}{(D-4)^2} + \text{finite part}} \quad \Rightarrow K'_{i_1\ldots i_P}(4)\,(R_1 + R_1) + \frac{1}{2}K''_{i_1\ldots i_P}(4)\,R_2 + \ldots$$

When tensor integrals are combined with their D-dimensional coefficients

- UV and IR poles require $(D-4)$ expansions (performed algebraically)
- this produces **rational terms** proportional to the pole residues

Rational terms of IR origin

- require the heaviest algebraic work but **cancel in any** *unrenormalized* **QCD amplitude** (proven in App. A of arXiv:0807.1248)
- can thus be neglected from the beginning

Rational terms of UV origin

- extracted automatically by means of a catalogue of UV residues R_1
- after the relevant $(D-4)$-expansions we can continue the calculation in $D = 4$

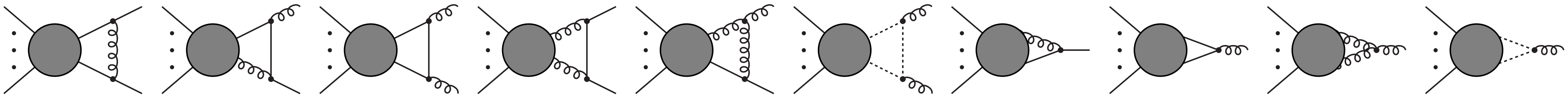
Cancellation of rational terms of IR origin (sketch of the proof)

Rational terms originate from **D-dependent $g^{\mu\nu}$-contractions** of type $g_{\nu\lambda}\Gamma^{\nu\lambda}$

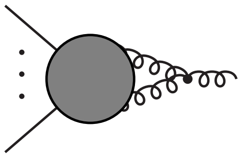
$$g_{\nu\lambda}\, g^{\nu\lambda} = D, \qquad g_{\nu\lambda}\, \gamma^{\nu} \not{p} \gamma^{\lambda} = (2-D)\not{p}, \ldots$$

(1) **The tensor reduction is free from IR rational terms** since in the soft and collinear regions ($q^\mu \to x p^\mu$) the tensor integrals cannot produce $g^{\mu\nu}$

(2) **All possible diagrams involving IR-divergent integrals**



can be cast into a form where $g_{\nu\lambda}\Gamma^{\nu\lambda}$ **contractions cancel in IR regions**



$$= \int \frac{\mathrm{d}^D q}{q^2(q+p)^2} \underbrace{\epsilon^{\mu *}(p)\,(2q+p)_\mu}_{\to\, 0 \text{ in soft/coll. regions}}\; g_{\nu\lambda}\,\Gamma^{\nu\lambda}(q) + \ldots$$

(f) Reduction of the helicity-dependent parts of the diagrams

$$K_{i_1 \dots i_P} = \sum_{n=1}^{N_{\mathrm{SME}}} \mathcal{S}_n \, K^{(n)}_{i_1 \dots i_P}$$

The last and most involved part of the algebraic manipulation

- reduce helicity-dependent parts of all Feynman diagrams to a *common* and *minimal set* of Standard Matrix Elements (SMEs)

- isolating helicity information into compact spinor chains $\mathcal{S}_n$ renders helicity sums *diagram-independent* and *extremely fast*

Six-fermion channel ($q\bar{q} \to t\bar{t}b\bar{b}$)

$$\underbrace{\left[\bar{v}(p_1)\dots\gamma_\mu\gamma_\nu\not{p}_3\dots u(p_2)\right]}_{q\bar{q}\text{ chain}}\underbrace{\left[\bar{v}(p_3)\dots\gamma^\mu\gamma^\nu\gamma^\rho\not{p}_6\dots u(p_4)\right]}_{t\bar{t}\text{ chain}}\underbrace{\left[\bar{v}(p_5)\dots\gamma_\rho\not{p}_2\not{p}_3\dots u(p_6)\right]}_{b\bar{b}\text{ chain}}$$

(1) **Process-independent identities in D dimensions**

- Dirac equation, Dirac algebra, momentum conservation, standard ordering
- yields $\mathcal{O}(10^3)$ SMEs: many $\gamma^\mu \otimes \gamma_\mu$ contractions between different chains

(2) **Process-dependent identities in $D = 4$** (avoid unstable denominators!)

- we introduce chiral projectors in each fermion chain

$$\omega_\pm = \frac{1}{2}(1 \pm \gamma^5), \qquad u(p_j) \Rightarrow [\omega_+ + \omega_-]\, u(p_j)$$

- then we can exploit various **identities of Chisholm-type**

$$\left(\gamma^\mu\gamma^\alpha\gamma^\beta\,\omega_\pm\right) \otimes \left(\gamma_\mu\,\omega_\mp\right) = \left(\gamma^\mu\,\omega_\pm\right) \otimes \left(\gamma^\alpha\gamma^\beta\gamma_\mu\,\omega_\mp\right) \quad \text{etc.}$$

that permit to exchange Dirac matrices between different fermion chains

- many combinations of identities ⇒ **fairly sophisticated and powerful reduction algorithm**
- at the end of the day **200 SMEs** for the $q\bar{q}$ channel
  - 10×8 of "massless" type: one Dirac matrix per chain

$$\left[\bar{v}(p_1)\not{p}_i\omega_\alpha u(p_2)\right]\ \left[\bar{v}(p_3)\gamma^\mu\omega_\beta u(p_4)\right]\ \left[\bar{v}(p_5)\gamma^\mu\omega_\rho u(p_6)\right]$$
$$\left[\bar{v}(p_1)\not{p}_i\omega_\alpha u(p_2)\right]\ \left[\bar{v}(p_3)\not{p}_j\omega_\beta u(p_4)\right]\ \left[\bar{v}(p_5)\not{p}_k\omega_\rho u(p_6)\right]$$

  - 15×8 of "massive" type: 2/0 Dirac matrices inside the $t\bar{t}$ chain

$$\left[\bar{v}(p_1)\not{p}_i\omega_\alpha u(p_2)\right]\ \left[\bar{v}(p_3)\not{p}_j\gamma^\mu\omega_\beta u(p_4)\right]\ \left[\bar{v}(p_5)\not{p}_k\omega_\rho u(p_6)\right]$$
$$\left[\bar{v}(p_1)\gamma^\mu\omega_\alpha u(p_2)\right]\ \left[\bar{v}(p_3)\not{p}_j\not{p}_j\omega_\beta u(p_4)\right]\ \left[\bar{v}(p_5)\gamma^\mu\omega_\rho u(p_6)\right]$$
$$\left[\bar{v}(p_1)\gamma^\mu\omega_\alpha u(p_2)\right]\ \left[\bar{v}(p_3)\gamma^\mu\gamma^\nu\omega_\beta u(p_4)\right]\ \left[\bar{v}(p_5)\gamma^\nu\omega_\rho u(p_6)\right]$$
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$$\left[\bar{v}(p_1)\not{p}_i\omega_\alpha u(p_2)\right]\ \left[\bar{v}(p_3)\omega_\beta u(p_4)\right]\ \left[\bar{v}(p_5)\not{p}_k\omega_\rho u(p_6)\right]$$

- **Price to pay: process-dependent and most time-consuming part of the algebraic reduction ⇒ really needed?!**

Four-fermion channel ($gg \to \mathrm{t\bar{t}b\bar{b}}$)

$$\underbrace{\left\{\epsilon_1^\mu \epsilon_2^\nu,\, (\epsilon_1\epsilon_2) p_2^\mu p_4^\nu,\, (\epsilon_1 p_4)(\epsilon_2 p_3) g^{\mu\nu}, \dots\right\}}_{\text{gluon polarization vectors}} \underbrace{\left[\bar{v}(p_3)\dots\gamma_\mu\gamma_\rho \not{p}_6 \dots u(p_4)\right]}_{\mathrm{t\bar{t}}\text{ chain}} \underbrace{\left[\bar{v}(p_5)\dots\gamma^\rho\gamma_\nu \not{p}_2 \not{p}_3 \dots u(p_6)\right]}_{\mathrm{b\bar{b}}\text{ chain}}$$

Process-independent identities in D dimensions

- $\epsilon_i p_{1,2} = 0$, Dirac eq., Dirac algebra, momentum conservation, standard ordering

Two alternative reductions in $D = 4$

(A) sophisticated method similarly as for six-fermion channel $\Rightarrow$ **502 SMEs**

(B) less-sophisticated and process-independent reduction

$$\gamma^{\mu_1}\gamma^{\mu_2}\gamma^{\mu_3}\gamma^{\mu_4}\gamma^{\mu_5} = g^{\mu_1\mu_2}\gamma^{\mu_3}\gamma^{\mu_4}\gamma^{\mu_5} + \dots + g^{\mu_1\mu_2}g^{\mu_3\mu_4}\gamma^{\mu_5} + \dots$$

Chisolm-based identitiy w.o. chiral projectors $\Rightarrow$ **970 SMEs**

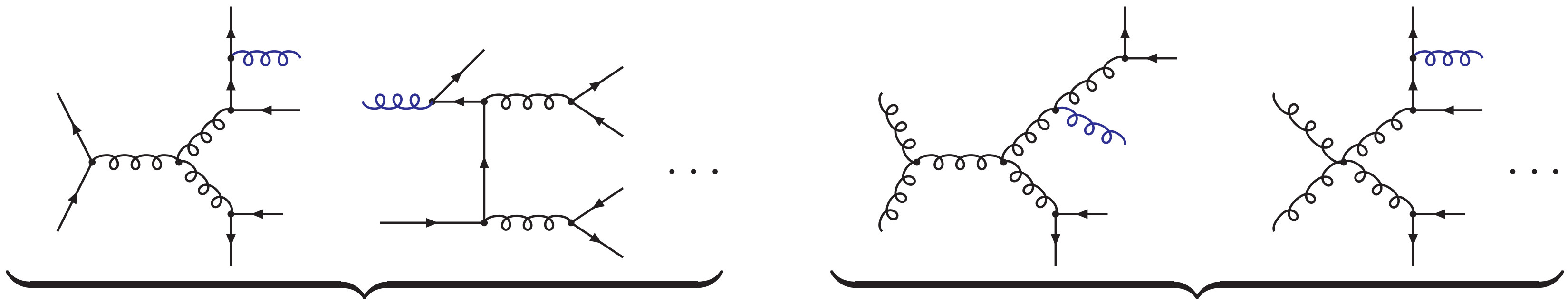
Surprising result

Speed of codes based on reduction A and B almost identical: **CPU efficiency not due to highly sophisticated process-dependent manipulations!**

(3) Real corrections (qq/gg/qg channels)

- Also for the real corrections: 2 independent calculations

Two types of matrix elements (Six- and four-fermion amplitudes)



$\bar{q}q \to t\bar{t}b\bar{b}g$ and $qg \to t\bar{t}b\bar{b}q$ (64 diagrams) $gg \to t\bar{t}b\bar{b}g$ (341 diagrams)

- Madgraph 4.1.33 [Alwall/Demin/deVisscher/Frederix/Herquet/Maltoni/Plehn/Rainwater/Stelzer'07] for all channels

- analytical calculation with Weyl–van der Waerden spinors [Dittmaier '98] for qq/qg channels

- in-house numerical algortihm based on off-shell recursions [Berends/Giele '88; Caravaglios/Moretti '95; Draggiotis/Kleiss/Papadopoulos '98] for gg channel

Treatment of soft and collinear singularities with dipole subtraction

Catani/Seymour '96; Dittmaier '99; Catani/Dittmaier/Seymour/Trócsányi '02

$$\int \mathrm{d}\sigma_{2\to 5} = \int \left[\mathrm{d}\sigma_{2\to 5} - \sum_{\substack{i,j=1\\ i\neq j}}^{6} \mathrm{d}\sigma_{2\to 5}^{\mathrm{dipole},ij} \right] + \sum_{\substack{i,j=1\\ i\neq j}}^{6} \mathcal{F}_{ij} \otimes \mathrm{d}\sigma_{2\to 4}$$

- numerically stable/efficient but non-trivial: 30 qq/gg (10 qg) subtraction terms
- in-house dipoles checked against `MadDipole`[Frederix/Gehrmann/Greiner '08] (gg/qg) and PS slicing[Giele/Glover '92; Giele et al. '93; Keller/Laenen '98; Harris/Owens '01] (qq)
- initial-state collinear singularities cancelled by $\overline{\mathrm{MS}}$-redefinition of PDFs

Phase-space integration

- adaptive multi-channel Monte Carlo [Berends/Kleiss/Pittau '94; Kleiss/Pittau '94] as in `RACOONWW`[Denner/Dittmaier/Roth/Wackeroth'99]/`PROFECY4f`[Bredenstein/Denner/Dittmaier/Weber'06]
- $\mathcal{O}(1400)$ channels to map all peaks from propagators (300) and dipoles (1100)

11-dimensional phase space, many channels and dipoles $\Rightarrow$ CPU-time! (see later)

Numerical checks

(A) LO checked against SHERPA [Gleisberg/Hoche/Krauss/Schalicke/Schumann/Winter '03]

(B) Precision checks for individual NLO components in single PS points
(typical precision: 10 to 14 digits)

Virtual corrections

- UV, soft and collinear cancellations
- agreement between 2 independent implementations

Real emission

- agreement of $2 \to 5$ matrix elements
- agreement between two dipole implementations
- cancellations in soft and collinear regions

(C) Integrated NLO cross section

- **two independent calculations agree at 1-2 sigma level with** $10^{-3} \times \sigma_{\text{NLO}}$ **statistical accuracy**

(4) NLO results for the LHC

Parton masses

- $m_{\rm t} = 172.6\,{\rm GeV}$ and $m_{\rm b} = 0$ (massless approximation better than 3% at LO)

Recombination of collinear $\rm b\bar{b}$, $\rm b\mathit{g}$, $\rm \bar{b}\mathit{g}$, **with** $k_{\rm T}$**-Jet-Algorithm** hep-ex/0005012

- partons with $|\eta| < 5$ $\Rightarrow$ b-jets with $\sqrt{\Delta\phi^2 + \Delta y^2} > D = 0.4$

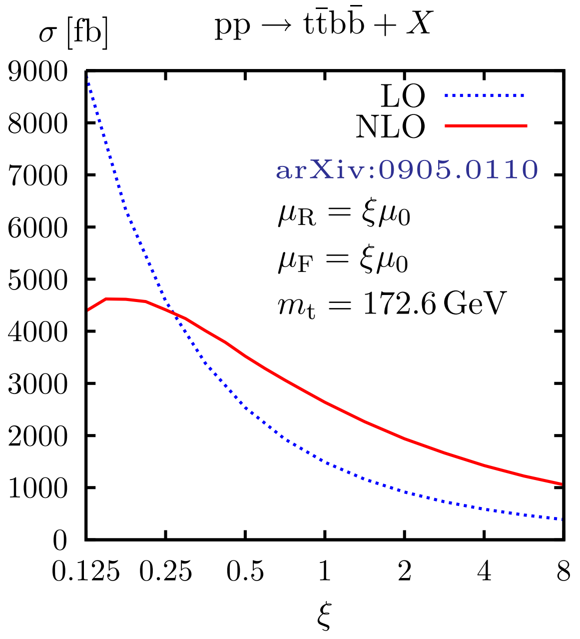
Cuts for b-jets (motivated by $\rm t\bar{t}H$ analysis)

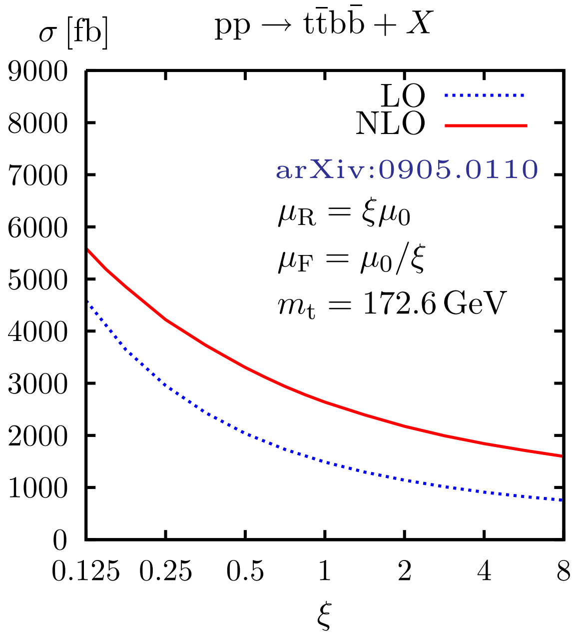
- require two b-jets with $p_{{\rm T},j} > 20\,{\rm GeV}$, $y_j < 2.5$, $m_{\rm b\bar{b}} > 100\,{\rm GeV}$
- top quarks fully inclusive (no decays and no cuts)

PDFs, scale variations and central scale

- CTEQ6M with $\alpha_{\rm S}(M_{\rm Z}) = 0.118$
- LO and NLO uncertainty estimated with factor-2 scale variations
- old scale choice $\mu_0 = m_{\rm t} + m_{\rm b\bar{b}}/2$ $\Rightarrow$ new scale choice $\mu_0^2 = m_{\rm t}\sqrt{p_{\rm T,b}\,p_{\rm T,\bar{b}}}$

LO and NLO scale dependence of $\sigma_{\rm tot}$





High sensitivity to scale choice

- LO proportional to $\alpha_{\rm S}(\mu_{\rm R})^4 \Rightarrow 78\%$ uncertainty

Original scale choice based on $\rm t\bar{t}H$ signal $(K \simeq 1.2)$

$$\mu_0 = E_{\rm thr}/2 = m_{\rm t} + m_{\rm b\bar{b}}/2$$

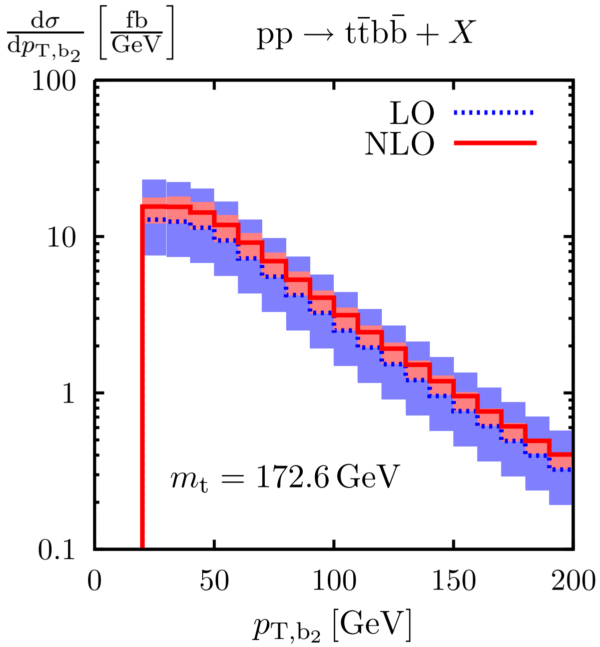
- used by ATLAS assuming $\rm t\bar{t}H \simeq t\bar{t}b\bar{b}$
- but at NLO we found large K-factor (1.8) and scale dependence (34%) [arXiv:0905.0110] $(D = 0.8,\ m_{\rm b\bar{b},cut} = 0)$

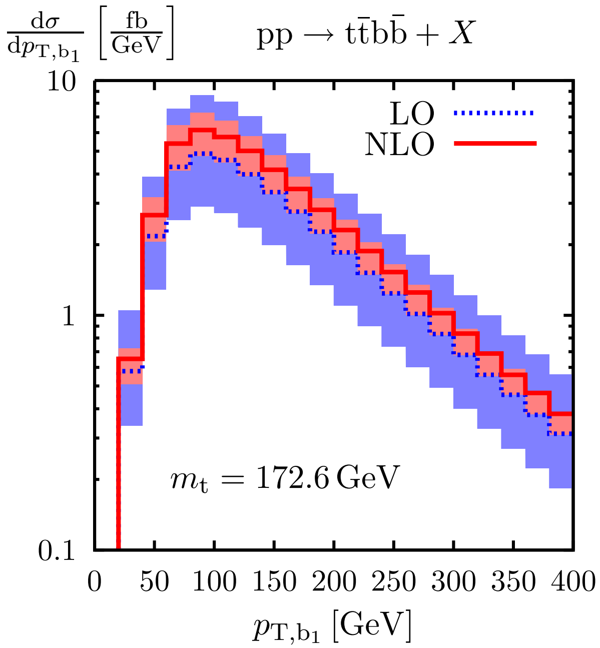
QCD dynamics of $\rm t\bar{t}H/t\bar{t}b\bar{b}$ completely different

- $(gg \to \rm t\bar{t}H) \times (H \to b\bar{b}) = \mathcal{O}(\alpha_s^2)$
- $(gg \to {\rm t\bar{t}}g) \times (g \to \rm b\bar{b}) = \mathcal{O}(\alpha_s^4)$

Several $\rm t\bar{t}b\bar{b}$ channels (b can be emitted from IS gluons!)

- no simple (factorized) mechanism that dictates unique scale choice





New (pragmatic) scale choice

Combine different scales observed in $t\bar{t}b\bar{b}$ distributions

$$\mu_0^2 = m_t\sqrt{p_{T,b}\,p_{T,\bar{b}}}$$

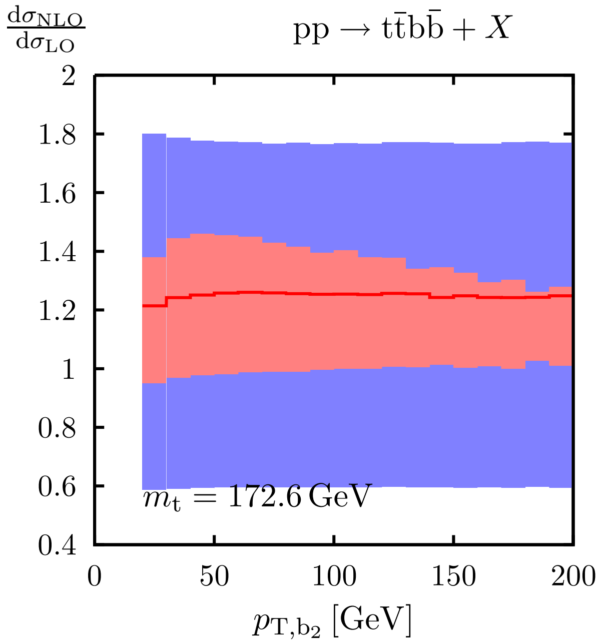
p_T-distributions of individual b-jets

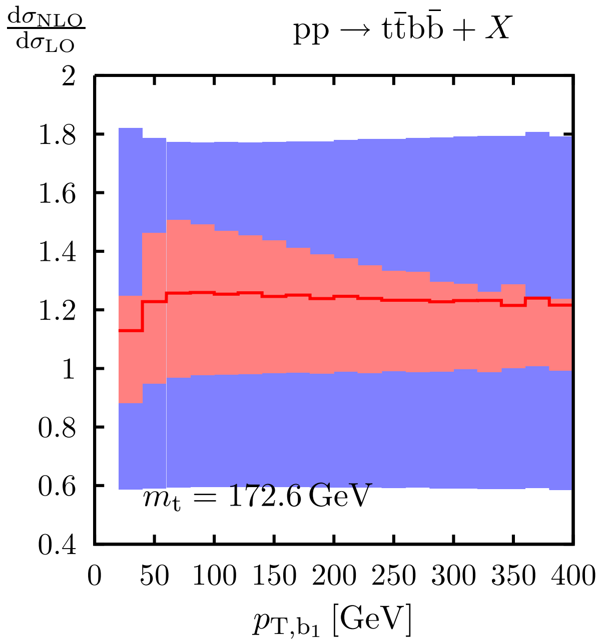
The two b-jets have typically

$$p_{T,b} \ll m_t$$

and rather different distributions

- softest b-jet (upper plot) tends to saturate the cut at 20 GeV
- hardest b-jet (lower plot) has $p_T \sim 100\,GeV$ and extends over wider p_T-range

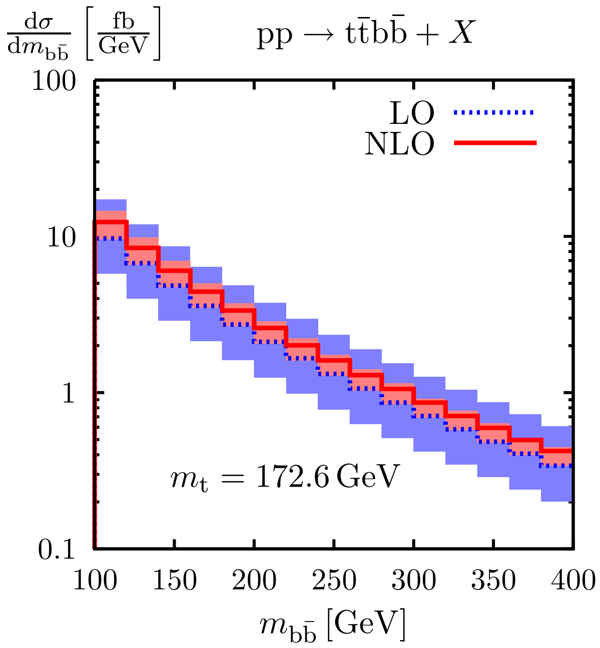




p_T-distribution of individual b-jets

Relative NLO/LO corrections show that **new scale choice clearly improves convergence**

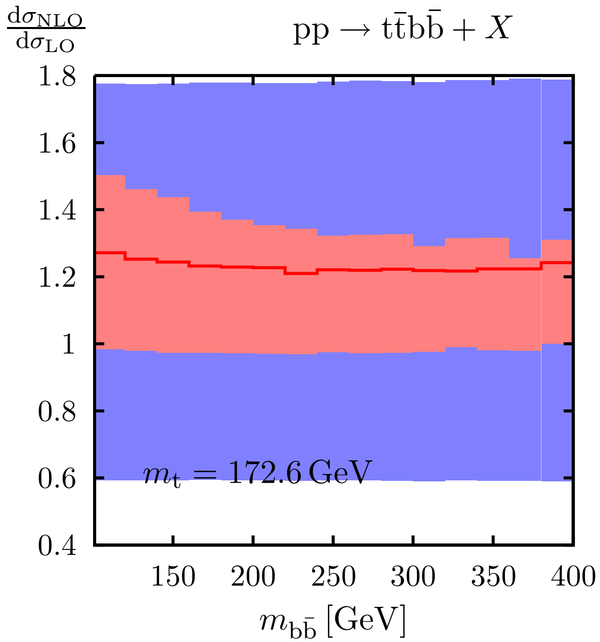
- NLO band perfectly fits within LO band: much smaller NLO correction ($K \simeq 1.25$)
- K-factor almost constant over wide p_T-range both for soft-b (upper plot) and hard-b (lower plot) distributions
- NLO scale uncertainty reduced to about 20%

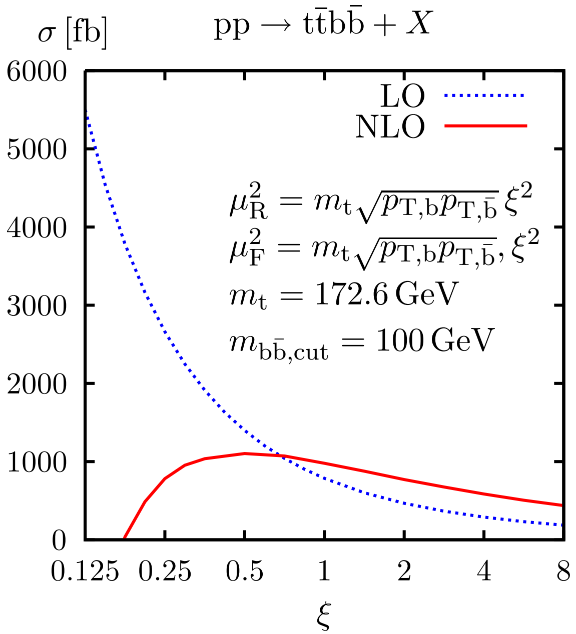


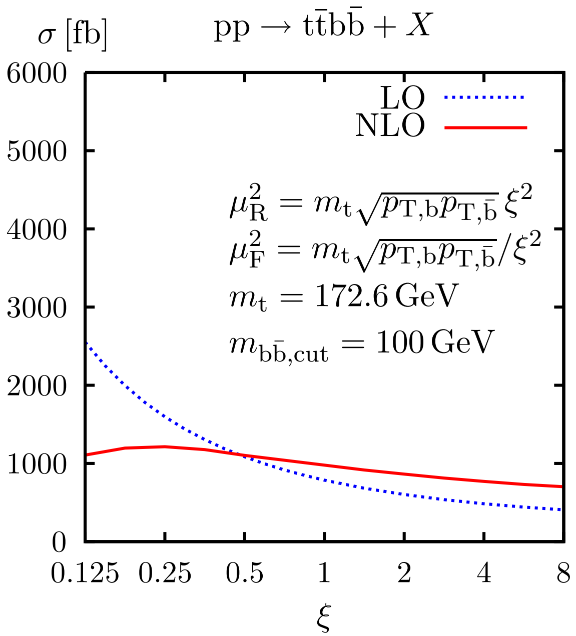
$b\bar{b}$ invariant-mass distribution

Crucial observable for $t\bar{t}H$ production

- small NLO correction ($K \simeq 1.25$)
- dynamical scale choice permits to approximate NLO effects by constant K-factor
- NLO scale uncertainty $\sim 20\%$







LO and NLO scale dependence of σ_{tot}

Uniform (upper plot) and antipodal (lower plot) variations around new central scale

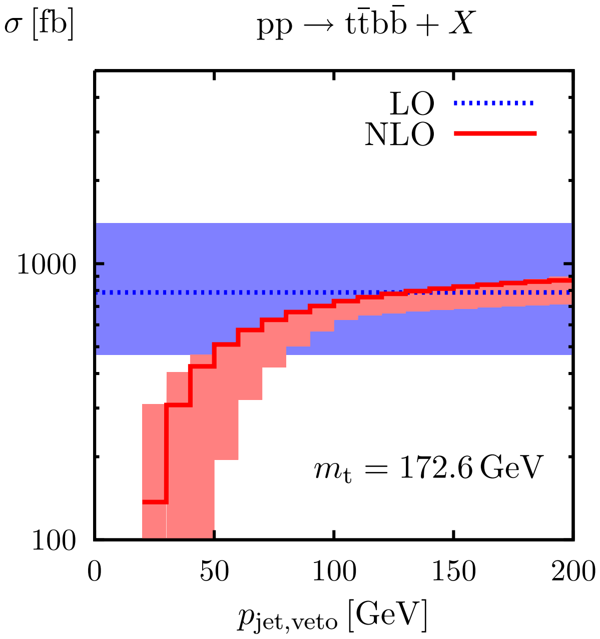
$$\mu_0^2 = m_t\sqrt{p_{T,b}p_{T,\bar{b}}}$$

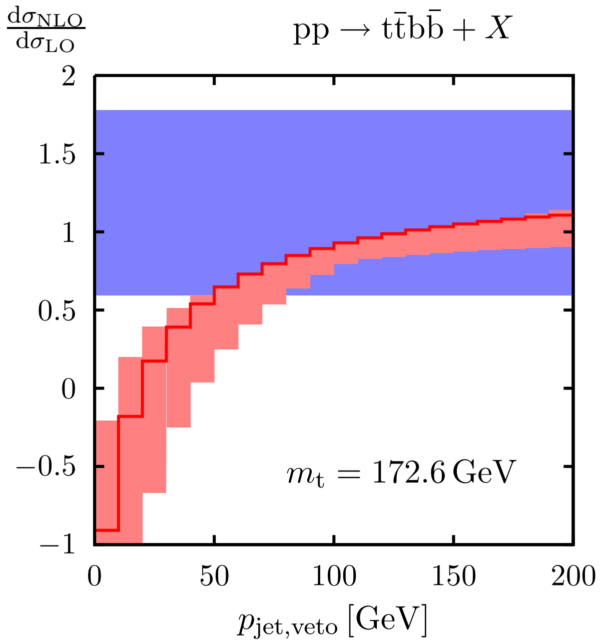
Good news for theory: improved convergence

- small correction & uncertainty ($K = 1.25 \pm 21\%$)
- evident from shape of NLO curves: central scale close to a maximum

Bad news for experiment: enhancement of $t\bar{t}b\bar{b}$ background

- was already dramatic with $\mu_{\mathrm{old}} = E_{\mathrm{thr}}/2$ ($K \simeq 1.8$)
- becomes even worse with $\mu_0 \simeq 0.5\mu_{\mathrm{old}}$ (in spite of smaller K-factor)





Effect of a jet veto

Reduction of large $t\bar{t}b\bar{b}$ background [arXiv:0905.0110]

- $p_{\text{jet,veto}} \sim 50\,\text{GeV} \quad \Rightarrow \quad$ sizable suppression
- perturbative stability must be investigated in detail!

Perturbative instability for small $p_{\text{jet,veto}}$

- veto $\Rightarrow$ negative contribution $-\alpha_s^5 \ln^2(Q_0/p_{\text{jet,veto}})$
- IR log dramatically enhances NLO uncertainty
- $p_{\text{jet,veto}} < 40\,\text{GeV} \Rightarrow$ NLO-band enters $K < 0$ range
  NLO prediction completely unrealiable!

Safe jet-veto values: $p_{\text{jet,veto}} \simeq 100\,\text{GeV}$

- NLO effect reduced from $K = 1.25$ to $K \simeq 0.9$
- NLO predictions as stable as for σ_{tot}
  (19% scale uncertainty)

Statistical precision and speed of the calculation

Single 3GHz Intel Xeon processor & `pgf77` Portland compiler

	$\sigma/\sigma_{\mathrm{LO}}$	# events (after cuts)	$(\Delta\sigma)_{\mathrm{stat}}/\sigma$	runtime	time/event
NLOtree (gg)	85%	5.8×10^6	0.4×10^{-3}	2h	< 1.4ms
virtual (gg)	10%	0.46×10^6	0.7×10^{-3}	20h	160ms
real + dipoles (gg/qg)	87%	16.5×10^6	2.6×10^{-3}	47h	10ms

- **2–3 CPU-days** $\Rightarrow$ $\mathcal{O}(10^7)$ events and $\mathcal{O}(10^{-3})$ **stat. accuracy** for σ_{tot}

  (distributions obtained with $\sim 5 \times 10^8$ events after cuts)

- **speed of virtual corrections** is remarkably high: 160 ms/event
  (including colour and polarization sums!)

Some (process-dependent) remarks about CPU efficiency

- Speed of one-loop Feynman diagrms **in striking contrast to pessimistic expectations** based on factorial complexity

- **Is it possible to beat 160ms/event?**

  Looking at CPU-cost of *method-independent* and *minimal* ingredient

  Master (scalar) Integrals $\sim$ 10 ms/event

  suggests that there is not much room for further dramatic improvement

Conclusions

NLO QCD calculation for $\mathrm{pp} \to \mathrm{t\bar{t}b\bar{b}}$ **at the LHC**

- $2 \to 4$ reaction with highest priority in the 2005 Les Houches wish list
- very important for $\mathrm{t\bar{t}H}$ measurement

QCD scale used by ATLAS not adequate $\Rightarrow$ **replaced by new scale**

- this stabilizes QCD predictions ($K \simeq 1.8 \Rightarrow 1.25$)
- but doubles $\mathrm{pp} \to \mathrm{t\bar{t}b\bar{b}}$ cross section wrt ATLAS studies

Technical test of diagrammatic tensor-reduction approach

- remarkably high CPU efficiency
- obtained with process-independent techniques
- very good perspectives to study other six-particle processes!