Exercises to Lecture 2: Electroweak Sector of the SM

Exercise 4 (Projection properties of γ^5):

In the lecture the projection properties of

$$\psi_L = \frac{1}{2}\left(1-\gamma^5\right)\psi$$
$$\psi_R = \frac{1}{2}\left(1+\gamma^5\right)\psi$$

have been discussed. It is obvious that $\psi = \psi_L + \psi_R$.

a)
Proof the following relation:

$$\left(\frac{1}{2}\left(1\pm\gamma^5\right)\right)^2 = \frac{1}{2}\left(1\pm\gamma^5\right)$$

i.e. the corresponding operators are projectors.

b)
Proof the following relation:

$$\frac{1}{2}\left(1+\gamma^5\right)\cdot\frac{1}{2}\left(1-\gamma^5\right) = 0$$

i.e. the two operators are orthogonal to each other.

c)
Proof the relation

$$\overline{e}\gamma^\mu\left(\frac{1-\gamma^5}{2}\right)\nu = \overline{e}_L\gamma^\mu\nu_L$$

i.e. the projector acts on the spinors in both directions. For this proof make use of the projector property that you have shown in **a)**.

d)
Proof that

$$\overline{e}\gamma^\mu\partial_\mu e + \overline{\nu}\gamma^\mu\partial_\mu\nu = \overline{e}_L\gamma^\mu\partial_\mu e_L + \overline{e}_R\gamma^\mu\partial_\mu e_R + \overline{\nu}\gamma^\mu\partial_\mu\nu$$

even though $e = e_L + e_R$. For this proof make use of the orthogonality that you have shown in **b)**.

Exercise 5 (Chiral transformation):

The transformation $\chi : \psi \to \gamma^5 \psi$ is called *chiral transformation*. It turns for instance axial vectors into vectors and vice versa.

a)
Check how the *chiral transformation* acts on the adjoint spinor $\overline{\psi}$.

b)
Proof that $e_L (e_R)$ are *eigenstates* to the *chiral transformation* with *eigenvalues* $-1(+1)$.

c)
Proof that terms of type $\overline{\psi}\gamma^\mu \partial_\mu \psi$ are covariant under chiral transformations, but terms of type $\overline{\psi} m \psi$ are not.