Exercises to Lecture 4: From Lagrangian to Observable

Exercise 9 (Time evolution of *Green's* function):

In the lecture you have discussed that there are two Green's functions as solutions to the Dirac equation depending on the evolution behavior, backwards or forwards in time. Assume a spinor

$$\phi(x') = u(k)e^{-ik_0 t' + i\vec{k}\vec{x}'}$$

and proof that for the Green's function for $t > t'$ you obtain

$$\phi(x) = i \int \mathrm{d}^3 x' K(x - x')\gamma^0 \phi(t', \vec{x}')$$

Note: make use of he fact that $u(k)$ fulfills the Dirac equation. Show that using the Green's function for $t < t'$ leads to 0. This proofs that particles with positive energy evolve forward in time. You could show the corresponding for particles with negative energy that evolve backwards in time.

Exercise 10 (Triviality bound on Higgs boson mass):

In the lecture you have discussed the solution of the renormalization group equation for the Higgs self-coupling λ for $Q^2 \ll m_H$:

$$\lambda(Q^2) = \frac{\lambda(\mathrm{v}^2)}{1 - \frac{3}{4\pi^2}\lambda(\mathrm{v}^2)\log(Q^2 \mathrm{v}^2)}$$

Note that $\mathrm{v} = \sqrt{\frac{\mu^2}{\lambda}} = 246\,\mathrm{GeV}$. Calculate the Landau pole, where the denominator turns to zero. Obtain a form for m_H from this equation from which you can derive upper bounds on m_H. Calculate the upper bounds for the assumption that the SM is a valid and perturbative theory up to scales of $Q = 10^3\,\mathrm{GeV}$ ($Q = 10^{16}\,\mathrm{GeV}$).

Exercise 11 (Stability bound on Higgs boson mass):

In the lecture you have also seen the solution of the renormalization group equation for λ for the case that $m_H \ll m_t$:

$$\lambda(Q^2) = \lambda(\mathrm{v}^2) - \frac{3}{16\pi^2}\frac{m_t^4}{\mathrm{v}^4}\log\left(Q^2/\mathrm{v}^2\right)$$

derive the equation this time to set lower limits on m_H. Calculate the upper bounds for the assumption that the SM is a valid and perturbative theory up to scales of $Q = 10^3\,\mathrm{GeV}$ ($Q = 10^{16}\,\mathrm{GeV}$).