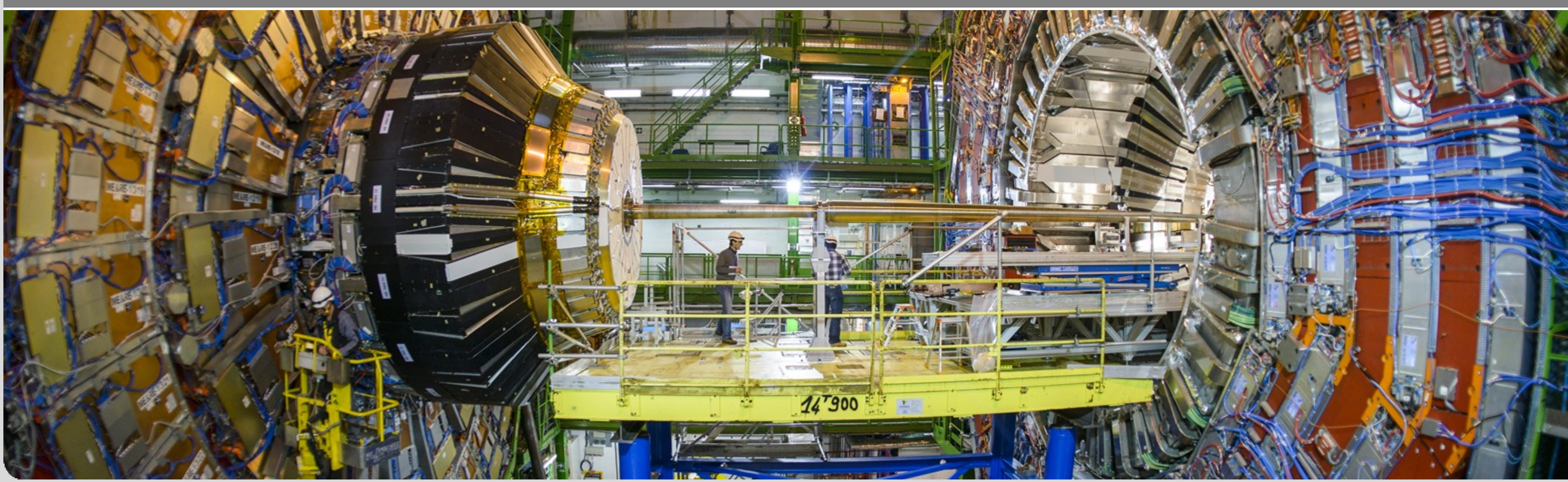
Karlsruhe Institute of Technology

Electroweak Sector of the SM

Roger Wolf

29. April 2014

INSTITUTE OF EXPERIMENTAL PARTICLE PHYSICS (IEKP) – PHYSICS FACULTY



Recap from Last Time

Gauge Field Theories:

$$\psi(\vec{x},t) \to \psi'(\vec{x},t) = e^{i\vartheta}\psi(\vec{x},t)$$
$$\overline{\psi}(\vec{x},t) \to \overline{\psi}'(\vec{x},t) = \overline{\psi}(\vec{x},t)e^{-i\vartheta}$$

→ *(Local Gauge Invariance)*

$$\partial_\mu \to D_\mu = \partial_\mu - ieA_\mu$$
$$D_\mu \to D'_\mu = D_\mu - i\partial_\mu\vartheta$$
$$A_\mu \to A'_\mu = A_\mu + \frac{1}{e}\partial_\mu\vartheta$$

→ *(Covariant Derivative)*

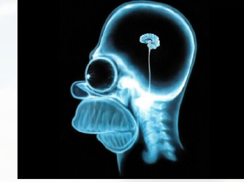
$$F_{\mu\nu} \equiv [D_\mu, D_\nu] = \partial_\mu A_\nu - \partial_\nu A_\mu$$
$$F_{\mu\nu} \to F'_{\mu\nu} = F_{\mu\nu}$$

→ *(Field Strength Tensor)*

$$\mathcal{L} = \overline{\psi}\,(i\gamma^\mu D_\mu - m)\,\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

→ *(Lagrange Density)*

Quiz of the Day



- What is the main characteristic of a Lie group?

- Intuitive experimental evidence for parity violation & propagator structure of the weak IA.

- The W boson only couples to left-handed particles! Does the Z boson also couple only to left-handed particles?

Schedule for Today

3

Sketch of the Electroweak Sector of the SM:

- Left (Right)-handed States
- Local $SU(2) \times U(1)$ Symmetry
- Weinberg Rotation

2

Phenomenology of Weak Interaction

1

Review of Lie-Groups:

- $U(1)$ & $SU(2)$
- (Non-) Abelian Gauge theories

Review of Lie-Groups

Marius Sophus Lie
(*17. December 1842, † 18. February 1899)

Unitary Transformations

$U(1)$ phase transformation

$$\psi(\vec{x},t) \rightarrow \psi'(\vec{x},t) = e^{i\vartheta}\psi(\vec{x},t)$$

- $U(1)$ is a group of unitary transformations in $\mathbb{R}^n$ with the following properties:

$$\mathbf{G} \in U(n) \qquad \mathbf{G}^{+}\mathbf{G} = \mathbb{I}_n \qquad \det \mathbf{G} = \pm 1$$

- Splitting an additional phase from $\mathbf{G}$ one can reach that $\det\mathbf{G} = 1$:

$$U(n) = U(1) \times SU(n)$$

$\det \mathbf{G} = \pm 1$ $\det \mathbf{G} = +1$

(*Unitary Transformations*) (*Special Unitary Transformations*)

Infinitesimal → Finite Transformations

- The $SU(n)$ can be composed from infinitesimal transformations with a continuous parameter $\vartheta \in \mathbb{R}$:

$$\mathbf{G}|_{\mathrm{inf}} = \mathbb{I} + i\vartheta_{\mathrm{inf}}\mathbf{t} \qquad (\,\vartheta_{\mathrm{fin}} \in \mathbb{R}\,,\;\; \mathbf{t} \in \mathcal{M}(\mathrm{n} \times \mathrm{n})\,)$$

$$\mathbf{G}|_{\mathrm{fin}} = \left(\mathbb{I}_n + i\,\frac{\vartheta_{\mathrm{fin}}}{m}\mathbf{t}\right)^m \xrightarrow{\quad n \to \infty \quad} e^{i\vartheta_{\mathrm{fin}}\cdot\mathbf{t}}$$

→ t generators of G.

→ t define structure of G.

- The set of G forms a *Lie-Group*.
- The set of t forms the tangential-space or *Lie-Algebra*.

Properties of $\mathbf{t}$

- Hermitian:

$$\mathbf{G}^{+}\mathbf{G} = \mathbb{I}_n$$
$$= \left(\mathbb{I}_n - i\vartheta\mathbf{t}^{+}\right)\left(\mathbb{I}_n + i\vartheta\mathbf{t}\right) = \mathbb{I}_n + i\vartheta\underbrace{\left(\mathbf{t} - \mathbf{t}^{+}\right)}_{\boxed{\mathbf{t} = \mathbf{t}^{+}}} + O(\vartheta^2)$$

- Traceless (example $SU(n)$):

$$\det \mathbf{G} = \det\left(\mathbb{I}_n + i\vartheta\mathbf{t}\right)$$
$$= 1 + i\vartheta\mathrm{Tr}(\mathbf{t}) + O(\vartheta^2) \stackrel{!}{=} 1$$

$$\boxed{Tr(\mathbf{t}) = 0}$$

- Dimension of tangential space:

$$\begin{pmatrix} * & * & * & * & * \\ * & * & * & * & \\ * & * & * & & \\ * & * & & & \\ * & & & & \end{pmatrix}$$

- n real entries in diagonal.
- $1/2 \cdot n(n-1)$ complex entries in off-diagonal.
- -1 for $SU(n)$ for trace req.

- $U(n)$ has n^2 generators.
- $SU(n)$ has $(n^2 - 1)$ generators.

Examples that appear in the SM ($U(1)$)

- $U(1)$ Transformations (equivalent to $O(2)$):
  - Number of generators: $1^2 = 1$ **NB:** what is the Generator?

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  - Number of generators: $1^2 \quad = 1$ **NB:** what is the Generator? → The generator is 1.

Examples that appear in the SM ($SU(2)$)

- $SU(2)$ Transformations (equivalent to $O(3)$):
  - Number of generators: $(2^2 - 1) = 3$ i.e. there are 3 matrices $\{\mathbf{t}_j\}$, which form a basis of traceless hermitian matrices, for which the following relation holds:

$$\mathbf{G} = e^{i \sum_{j=1}^{3} \vartheta_j \mathbf{t}_j}$$

  - Explicit representation:

$$\mathbf{t}_j = \frac{1}{2}\sigma_j \qquad (j = 1 \ldots 3)$$

(*3 Pauli Matrices*)

$$[\mathbf{t}_i, \mathbf{t}_j] = i\epsilon_{ijk}\mathbf{t}_k$$

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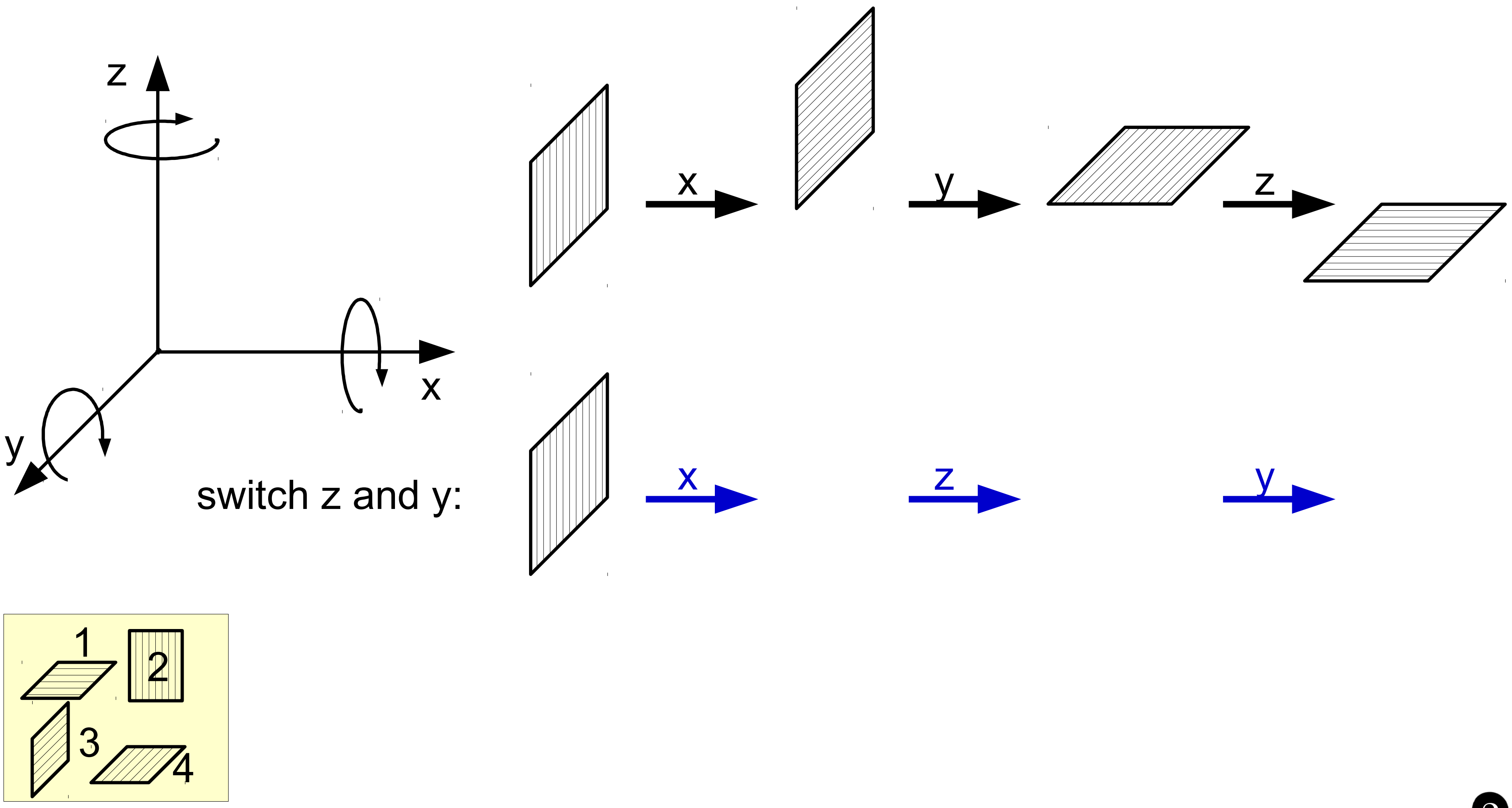
- algebra closes.
- structure constants of $SU(2)$.

Non-Abelian Symmetry Transformations

- Example $O(3)$ (90° rotations in $\mathbb{R}^3$):



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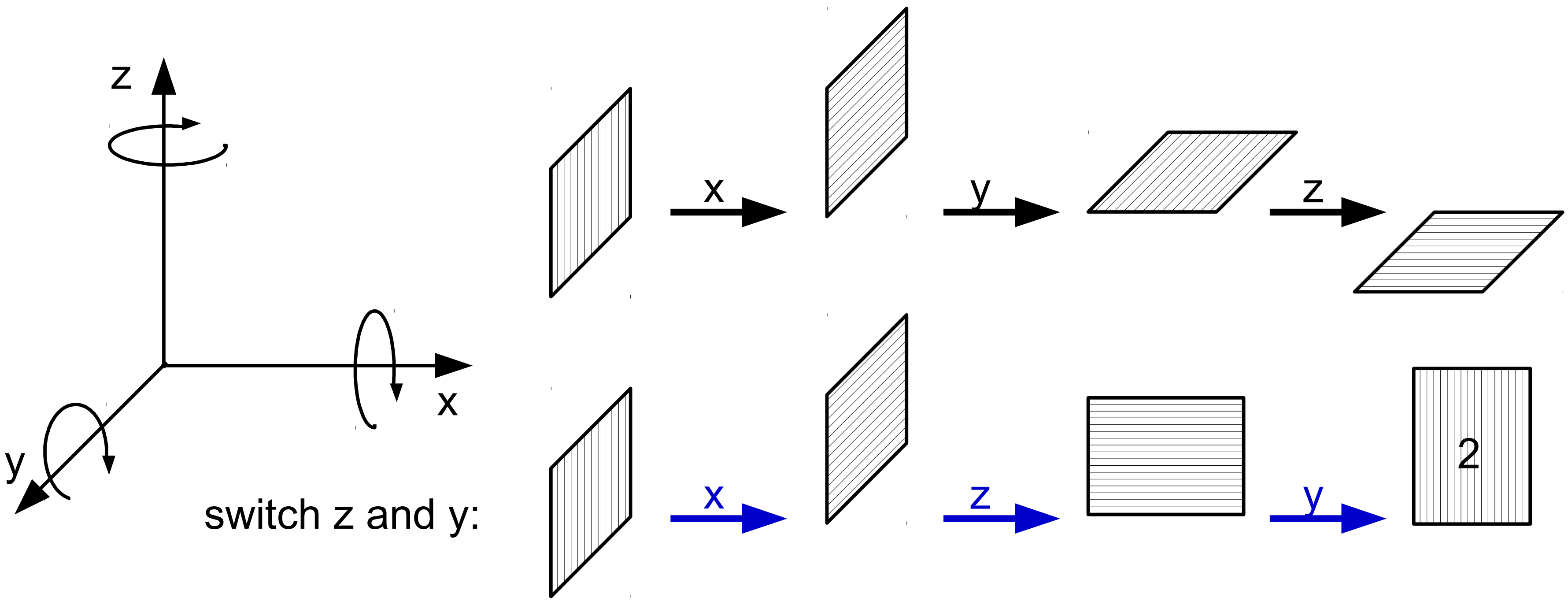

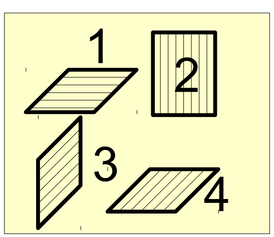

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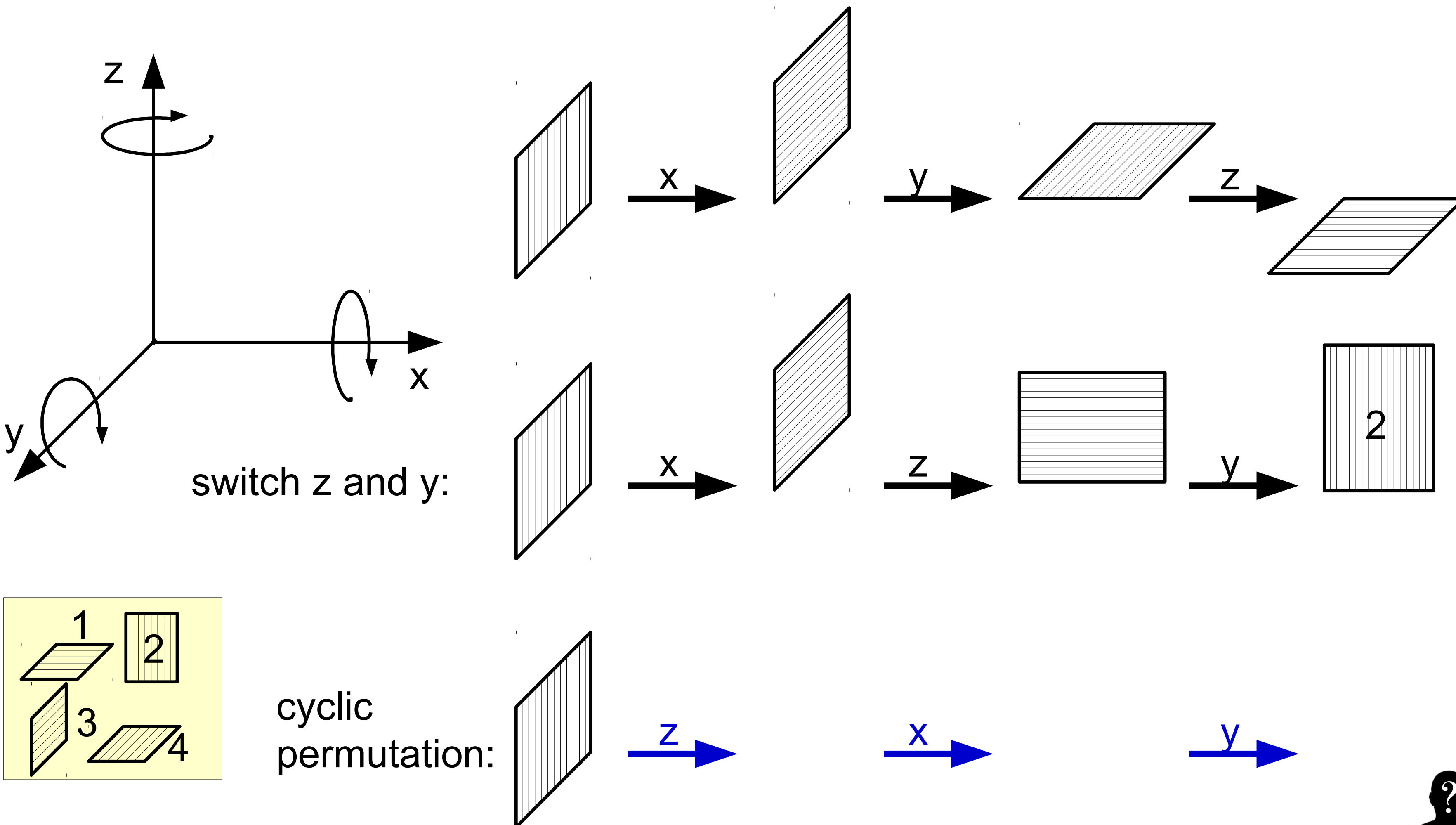

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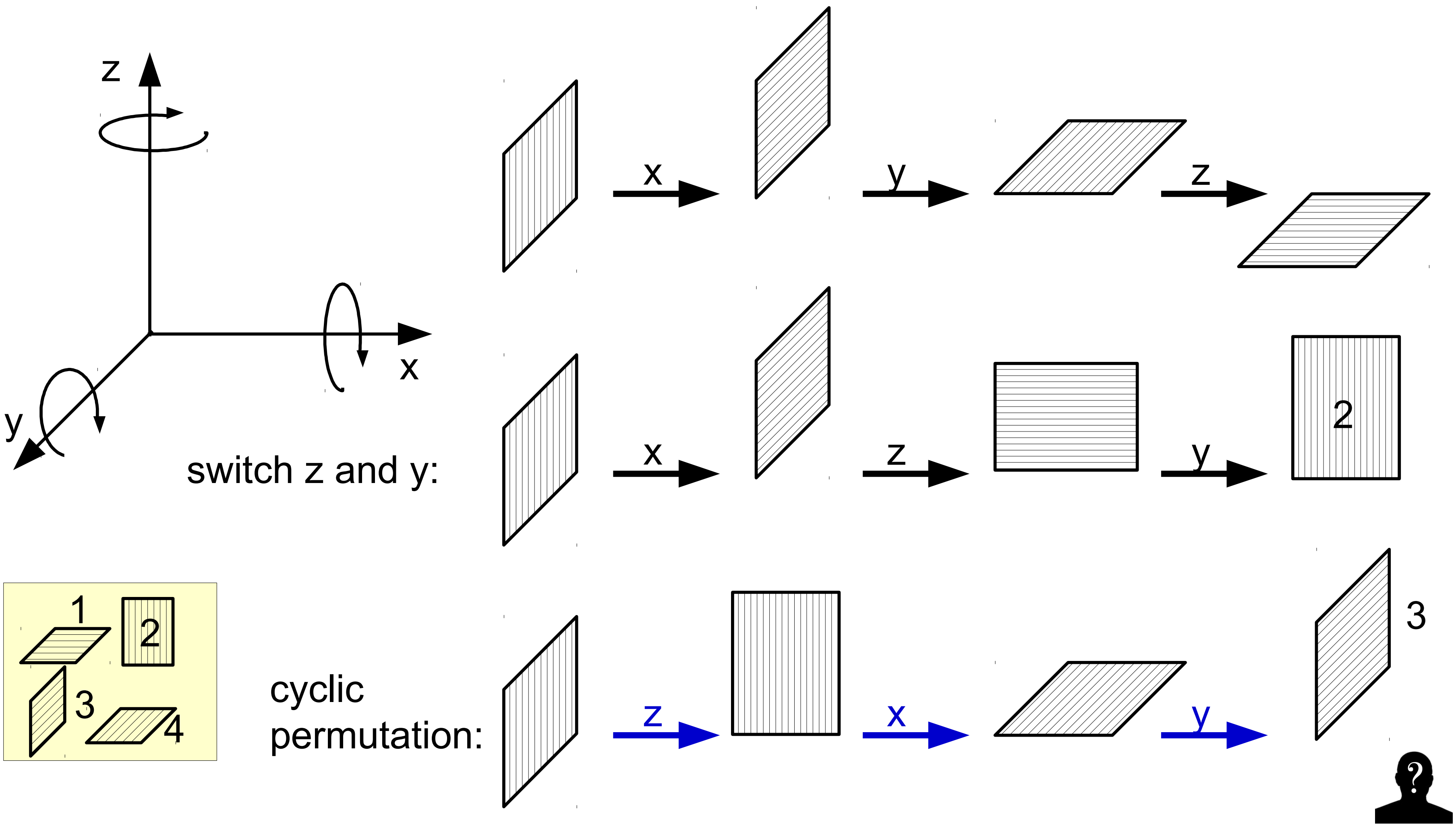

Examples that appear in the SM ($SU(3)$)

- $SU(3)$Transformations (equivalent to $O(4)$):
  - Number of generators: $(3^2 - 1) = 8$ ($\rightarrow$ *8 Gell-Mann Matrices*)

Abelian vs. Non-Abelian Gauge Theories

Abelian:

$$\psi(\vec{x},t) \rightarrow \psi'(\vec{x},t) = e^{i\vartheta}\psi(\vec{x},t)$$
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$$\mathcal{L} = \overline{\psi}\,(i\gamma^\mu D_\mu - m)\,\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

Non-Abelian:

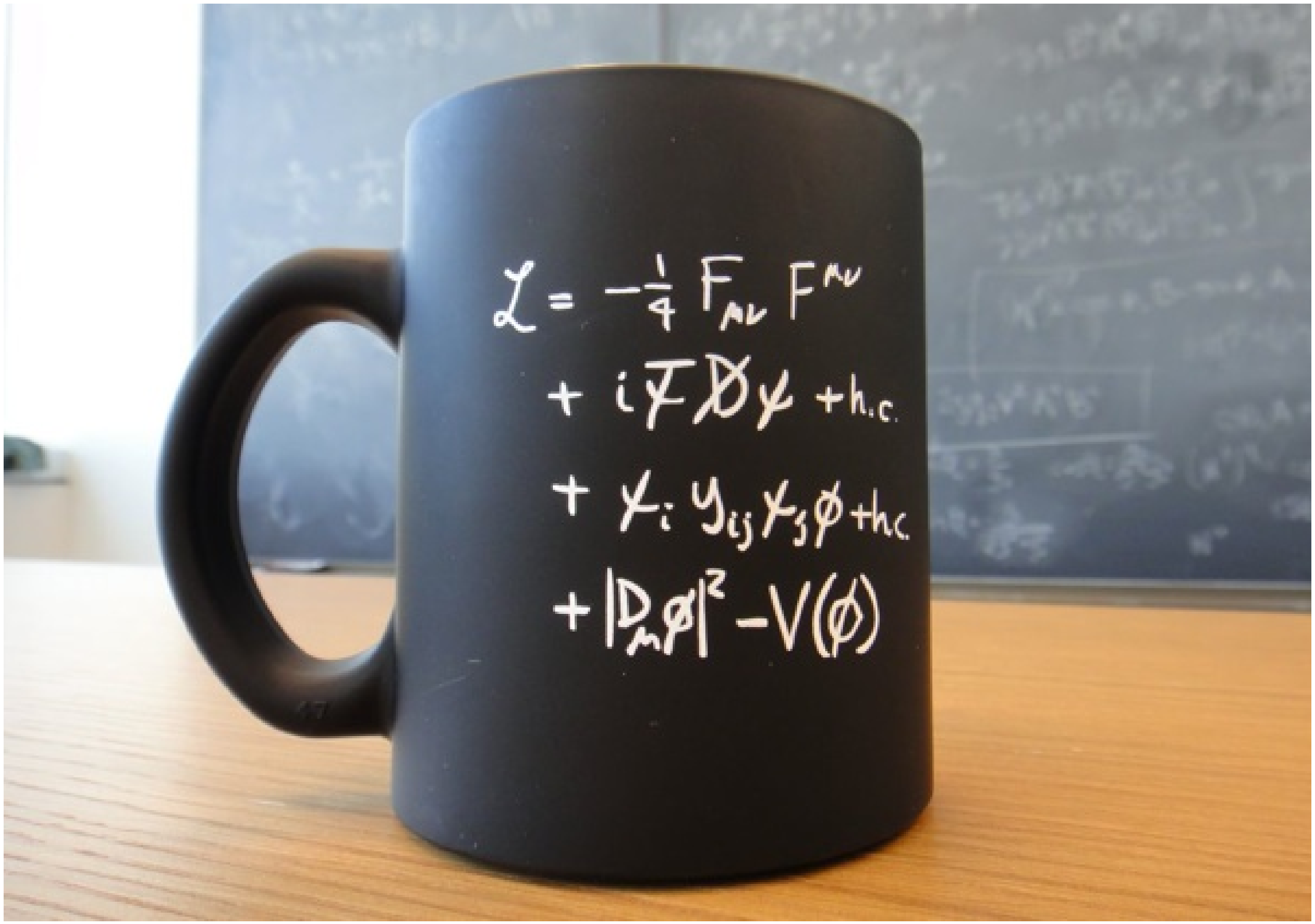
$$\psi(\vec{x},t) \rightarrow \psi'(\vec{x},t) = e^{i\vartheta_\mathrm{a}\mathbf{t}_\mathrm{a}}\psi(\vec{x},t)$$
$$\overline{\psi}(\vec{x},t) \rightarrow \overline{\psi}'(\vec{x},t) = \overline{\psi}(\vec{x},t)e^{-i\vartheta_\mathrm{a}\mathbf{t}_\mathrm{a}}$$

$$\partial_\mu \rightarrow D_\mu = \partial_\mu - igW_{\mu,\mathrm{a}}\mathbf{t}_\mathrm{a}$$
$$D_\mu \rightarrow D'_\mu = D_\mu - i\left[\vartheta_\mathrm{a}\mathbf{t}_\mathbf{a}, D_\mu\right]$$
$$W_\mu \rightarrow W'_\mu = W_\mu + i\left[\vartheta_\mathrm{a}\mathbf{t}_\mathrm{a}, W_{\mu,\mathrm{a}}\mathbf{t}_\mathrm{a}\right] + \frac{1}{g}\partial_\mu\left(\vartheta_\mathrm{a}\mathbf{t}_\mathrm{a}\right)$$
$$F_{\mu\nu} \equiv [D_\mu, D_\nu] = \partial_\mu W_\nu - \partial_\nu W_\mu - ig\left[W_\mu, W_\nu\right]$$

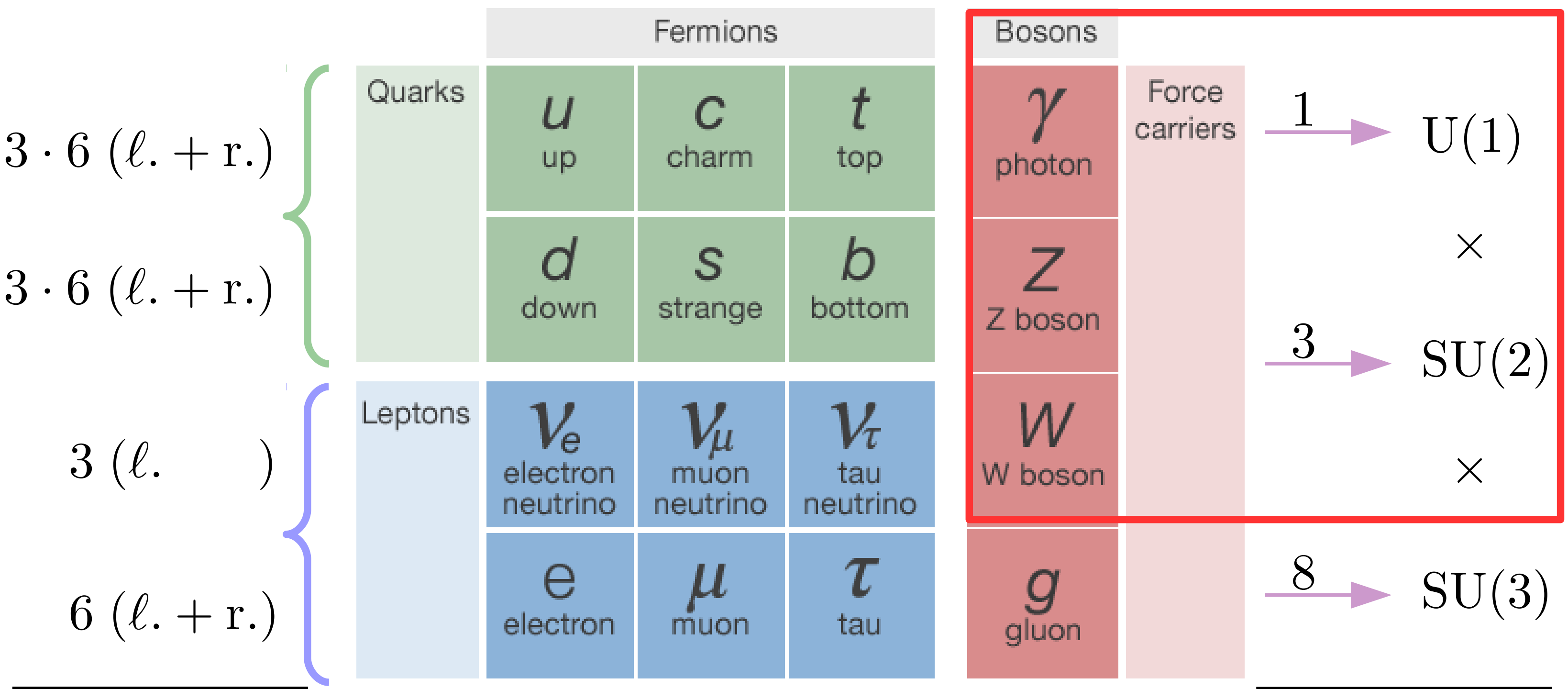
$$F_{\mu\nu} \rightarrow F'_{\mu\nu} = F_{\mu\nu} - i\left[\vartheta_\mathrm{a}\mathbf{t}_\mathrm{a}, F_{\mu\nu}\right]$$

$$\mathcal{L} = \overline{\psi}\,(i\gamma^\mu D_\mu - m)\,\psi - \frac{1}{4}F_{\mathrm{a}\mu\nu}F^{\mathrm{a}\mu\nu}$$

The SM of Particle Physics



Constituents and Interactions of the SM



45 (Fermion fields)

12 (Gauge fields)

Constituents and Interactions of the SM

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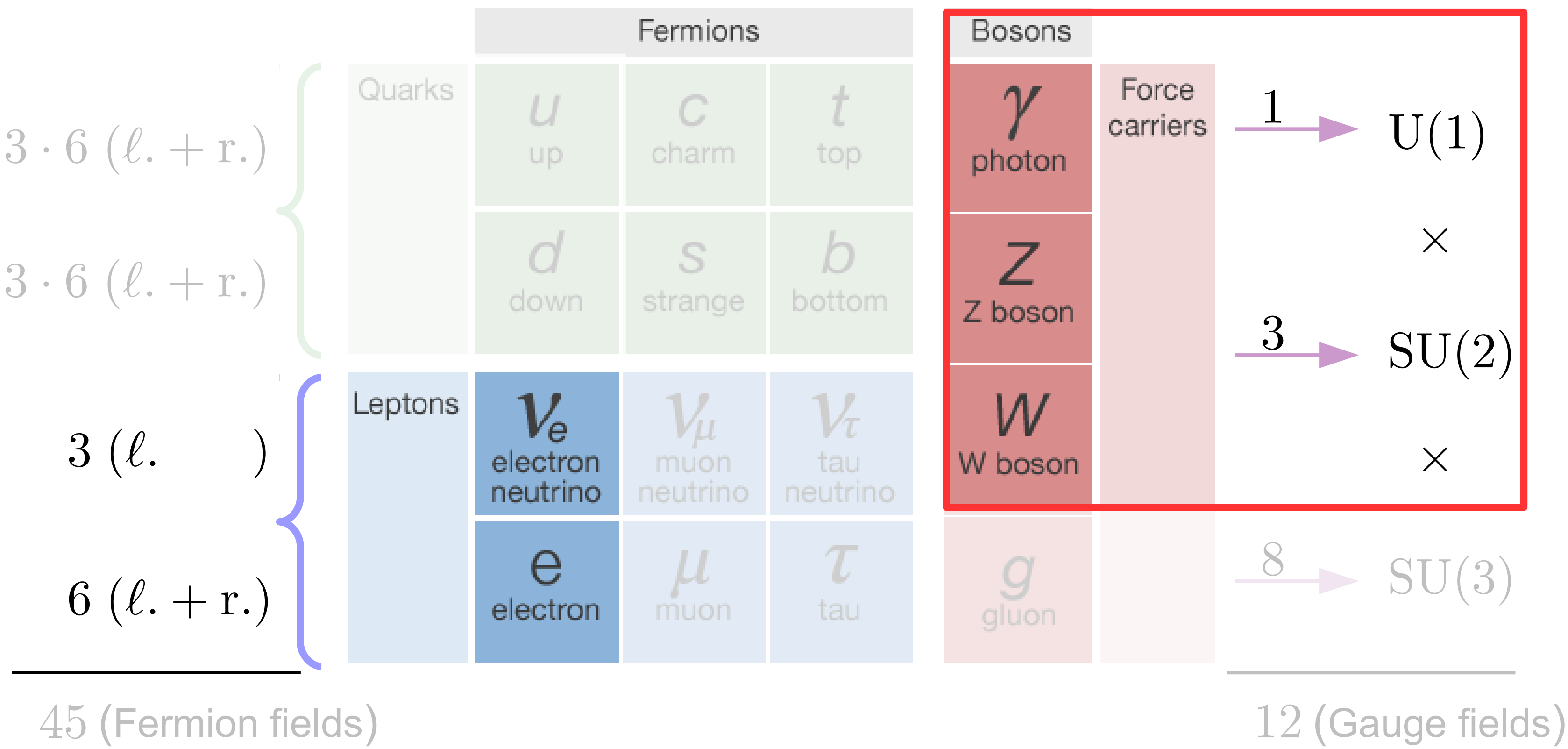
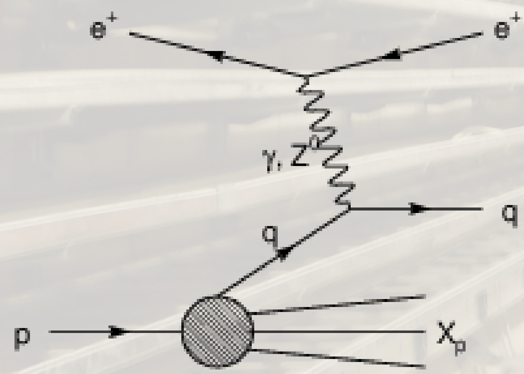
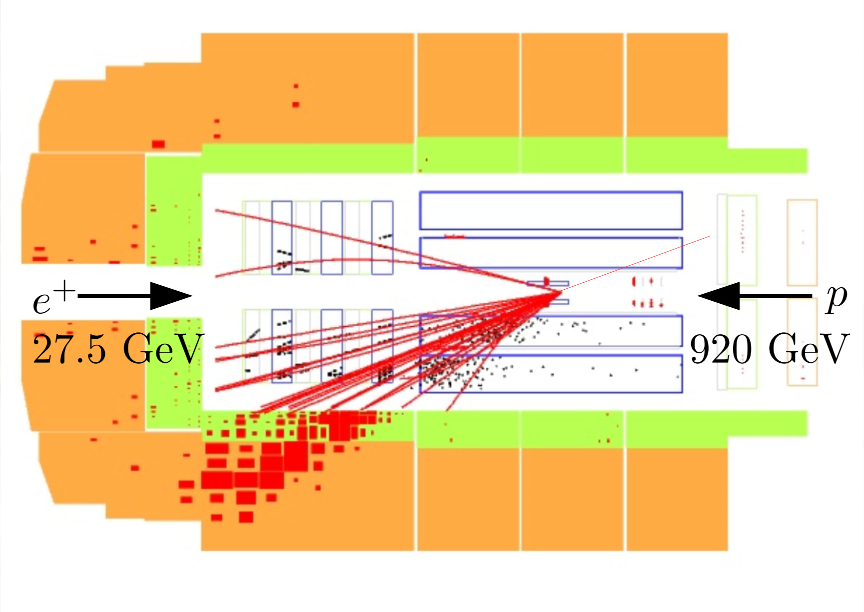


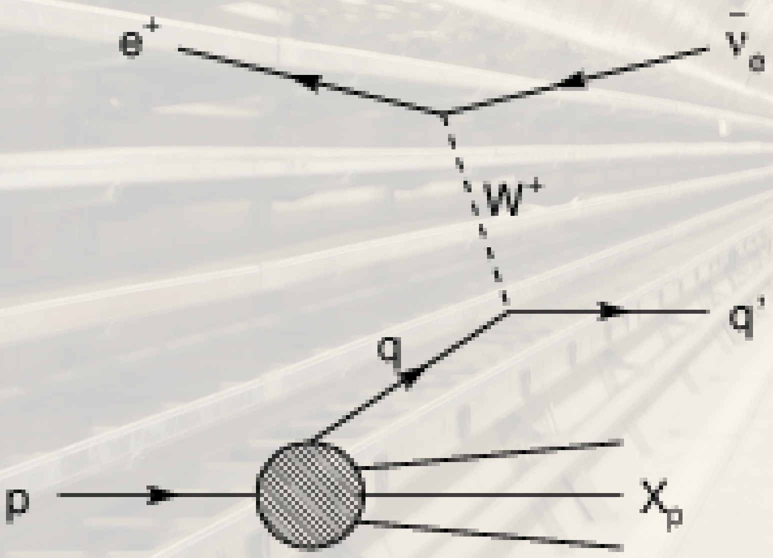
Phenomenology of Weak Interaction

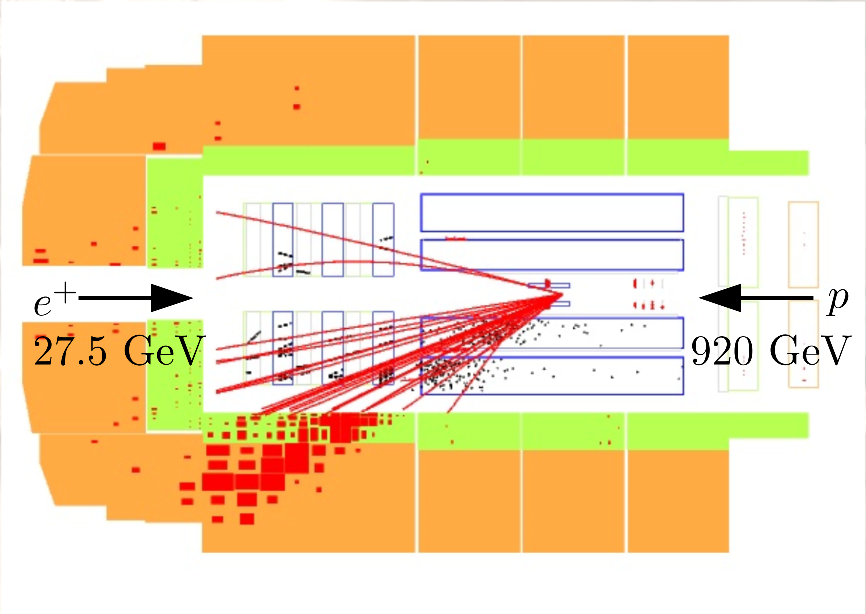
- From the view of a high energy physics scattering experiment:

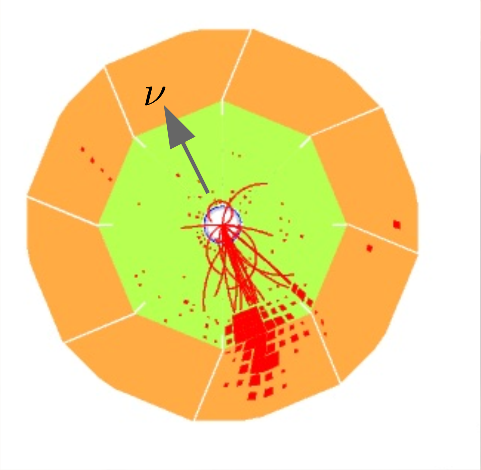




Change of Flavor & Charge

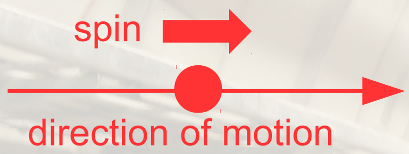


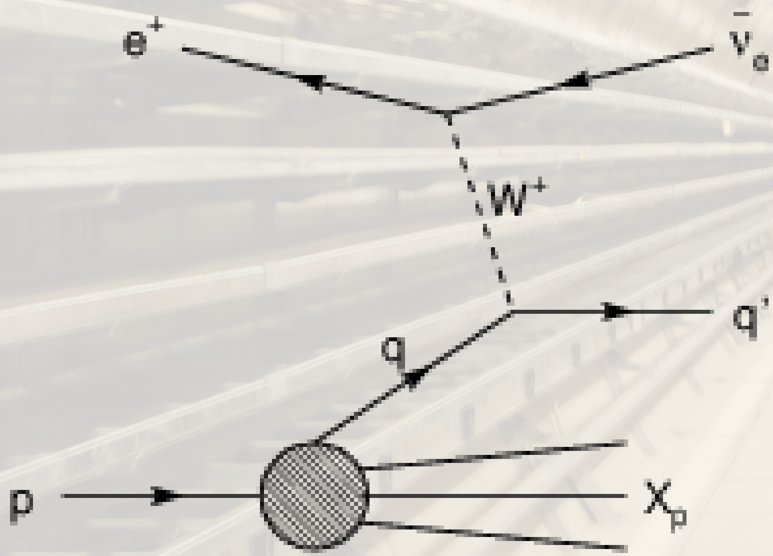


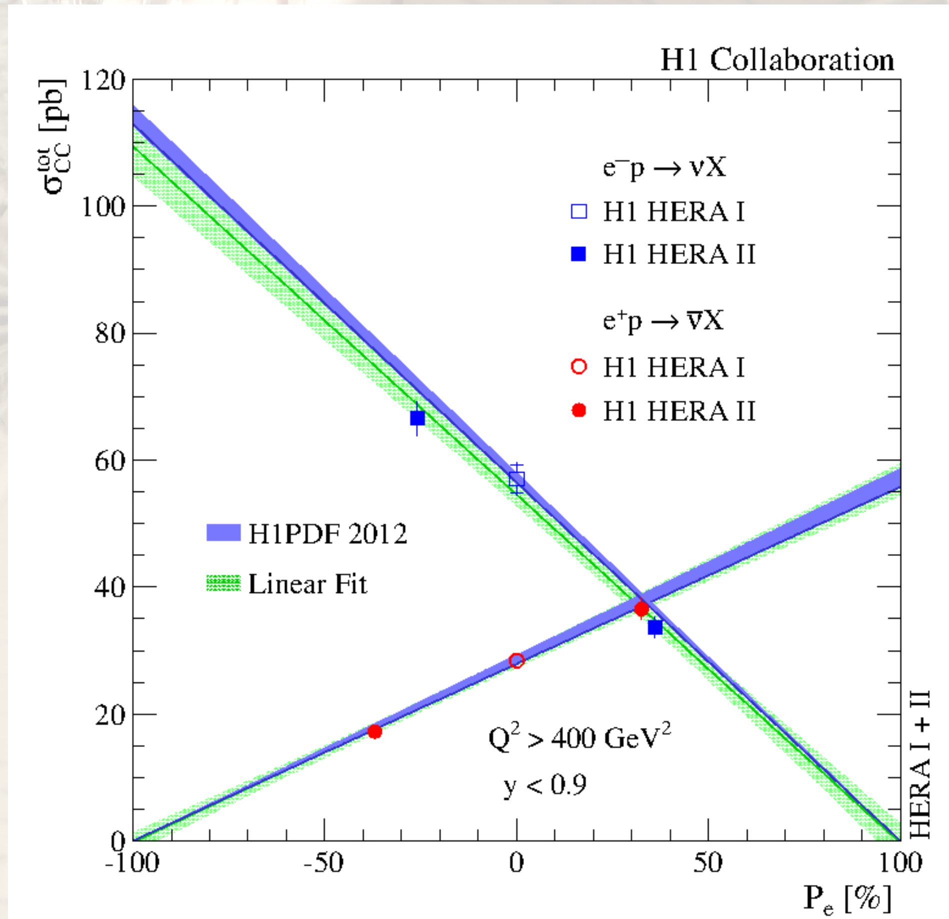


Parity Violation

- Coupling only to left-handed particles (right-handed anti-particles):



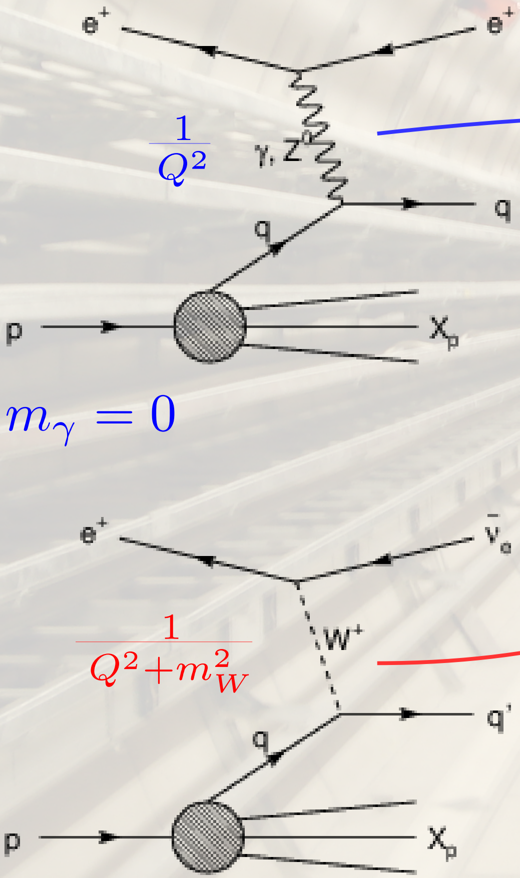




- Maximally parity violating!
- Intrinsically violating CP as well!

Heavy Mediators

- Mediation by heavy gauge bosons:

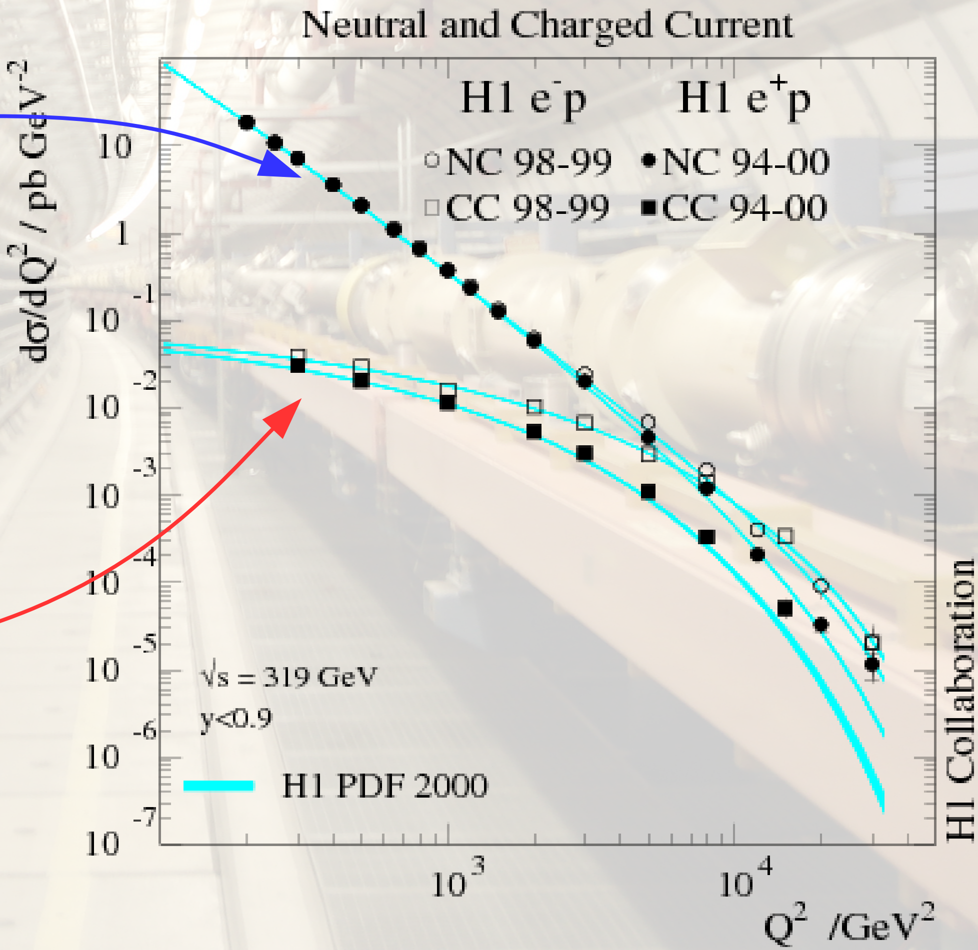


$\frac{1}{Q^2}$

$m_\gamma = 0$

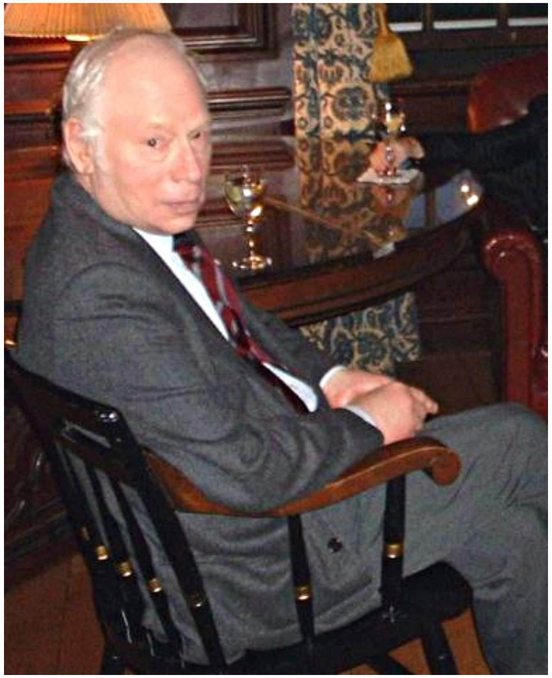
$\frac{1}{Q^2+m_W^2}$

$m_W = 85.385 \pm 0.015 \text{ GeV}$



The Model of Weak Interactions

Sheldon Glashow
(*5. December 1932)



Steven Weinberg
(*3. Mai 1933)

$SU(2)$ Space of Weak Isospin

- Example:

Transforms like a spin ½ object in space of weak isospin.

$\psi_L = \begin{pmatrix} \nu \\ e \end{pmatrix}_L$ ⟶ • *left-handed* e_L & ν form *isospin doublet*.

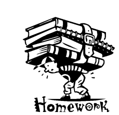
e_R ⟶ • *right-handed* e_R forms *isospin singlet*.

- *Left- & right-handed* components of fermions can be projected conveniently:

$$e = e_L + e_R \quad \begin{cases} e_L = \left(\frac{1-\gamma^5}{2}\right) e \\ e_R = \left(\frac{1+\gamma^5}{2}\right) e \end{cases} \qquad \overline{e}\gamma^\mu \left(\frac{1-\gamma^5}{2}\right) \nu = \overline{e}_L \gamma^\mu \nu_L$$

- Lagrangian w/o mass terms can be written in form:

$$\boxed{\mathcal{L}_0 = \overline{\psi}_L \gamma^\mu \partial_\mu \psi_L + \overline{e}_R \gamma^\mu \partial_\mu e_R} = \overline{e}_L \gamma^\mu \partial_\mu e_L + \overline{\nu} \gamma^\mu \partial_\mu \nu + \overline{e}_R \gamma^\mu \partial_\mu e_R$$



Covariant Derivative of $SU(2) \times U(1)$

Covariant derivative corresponding to $SU(2)$ acts on *isospin doublet* only.[1)]

$$\mathcal{L}_{IA}^{SU(2)\times U(1)} = \underbrace{\bar{\psi}_L \gamma^\mu \boxed{\left(\partial_\mu \qquad\qquad + ig W_\mu^{\mathrm{a}} \mathbf{t}^{\mathrm{a}}\right)} \psi_L}_{} \cdots$$

1) Note a different sign convention.

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$$\mathbf{t}^+ = \mathbf{t}_1 + i\,\mathbf{t}_2 = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$$ (ascending operator)

$$\mathbf{t}^- = \mathbf{t}_1 - i\,\mathbf{t}_2 = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$$ (descending operator)

$$W_\mu^a \mathbf{t}^a = \frac{1}{\sqrt{2}}\left(W_\mu^+ \mathbf{t}^+ + W_\mu^- \mathbf{t}^-\right) + W_\mu^3 \mathbf{t}^3$$

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Covariant derivative corresponding to $U(1)$ acts on *isospin doublet* (as a whole) and on *isospin singlet.*

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$SU(2) \times U(1)$ Hypercharges			
Particle	$Y_{R/L}$	I_3	Q
ν	-1	$+1/2$	
e_L	-1	$-1/2$	
e_R		0	-1

$Q = I_3 + \frac{Y}{2}$ (Gell-Mann Nischijama)

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$SU(2) \times U(1)$ Hypercharges			
Particle	$Y_{R/L}$	I_3	Q
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e_L	-1	$-1/2$	-1
e_R	-2	0	-1

$Q = I_3 + \frac{Y}{2}$ (Gell-Mann Nischijama)

1) Note a different sign convention.

$SU(2) \times U(1)$ Interactions

- Charged current interaction:

$$\mathcal{L}_{IA}^{CC} = -\frac{g}{\sqrt{2}}\left[\underbrace{\overline{\nu}\left(W_\mu^+\gamma^\mu\right)e_L}_{e\to\nu} + \underbrace{\overline{e}_L\left(W_\mu^-\gamma^\mu\right)\nu}_{\nu\to e}\right]$$

- Neutral current interaction:

$$\mathcal{L}_{IA}^{NC} = -\underbrace{\left(\frac{g}{2}W_\mu^3 - \frac{g'}{2}B_\mu\right)}_{\propto Z_\mu}(\overline{\nu}\gamma^\mu\nu) + \left(\frac{g}{2}W_\mu^3 + \frac{g'}{2}B_\mu\right)(\overline{e}_L\gamma^\mu e_L) + \frac{g'}{2}B_\mu\left(\overline{e}_R\gamma^\mu e_R\right)$$

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$$\begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta_W & -\sin\theta_W \\ \sin\theta_W & \cos\theta_W \end{pmatrix}\begin{pmatrix} W_\mu^3 \\ B_\mu \end{pmatrix}$$

$$\sin\theta_W = \frac{g'}{\sqrt{g^2+g'^2}} \quad \cos\theta_W = \frac{g}{\sqrt{g^2+g'^2}}$$

(Weinberg Rotation)

$SU(2) \times U(1)$ Interactions

- Charged current interaction:

$$\mathcal{L}_{IA}^{CC} = -\frac{g}{\sqrt{2}} \Big[\underbrace{\overline{\nu} \left(W_\mu^+ \gamma^\mu \right) e_L}_{e \to \nu} + \underbrace{\overline{e}_L \left(W_\mu^- \gamma^\mu \right) \nu}_{\nu \to e} \Big]$$

- Neutral current interaction:

$$\begin{aligned} \mathcal{L}_{IA}^{NC} = & -\frac{\sqrt{g^2 + g'^2}}{2} Z_\mu \left(\overline{\nu} \gamma_\mu \nu \right) \\ & + \frac{\sqrt{g^2 + g'^2}}{2} \left[\left(\cos^2\theta_W - \sin^2\theta_W \right) Z_\mu + 2 \sin\theta_W \cos\theta_W A_\mu \right] \left(\overline{e}_L \gamma_\mu e_L \right) \\ & + \frac{\sqrt{g^2 + g'^2}}{2} \left[-2 \sin^2\theta_W \, Z_\mu + 2 \sin\theta_W \cos\theta_W A_\mu \right] \left(\overline{e}_R \gamma_\mu e_R \right) \end{aligned}$$

What is the expression for e ?

$SU(2) \times U(1)$ Interactions

- Charged current interaction:

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What is the expression for e ? $\longrightarrow$ $e = \sqrt{g^2+g'^2}\sin\theta_W\cos\theta_W$

Skewness of the $SU(2) \times U(1)$

- Gauge boson *eigenstates of the symmetry* do not correspond to the *eigenstates* of the IA:

$$\begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta_W & -\sin\theta_W \\ \sin\theta_W & \cos\theta_W \end{pmatrix} \begin{pmatrix} W_\mu^3 \\ B_\mu \end{pmatrix}$$

- Fermion *eigenstates* of the $SU(2)$ do not correspond to the fermion *eigenstates* of the $SU(3)$:

$$\mathcal{M}_{\mathrm{CKM}} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

$$= \begin{pmatrix} c_1 & s_1 c_3 & s_1 s_3 \\ -s_1 c_2 & c_1 c_2 c_3 - s_2 s_3 e^{i\delta} & c_1 c_2 s_3 + s_2 c_3 e^{i\delta} \\ -s_1 s_2 & c_1 s_2 c_3 + c_2 s_3 e^{i\delta} & c_1 s_2 s_3 - c_2 c_3 e^{i\delta} \end{pmatrix}$$

$$c_i = \cos\vartheta_i \;;\; s_i = \sin\vartheta_i \; (i = 1 \ldots 3)$$

Self-Interactions

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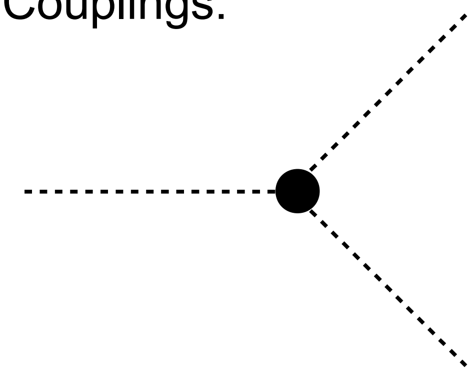
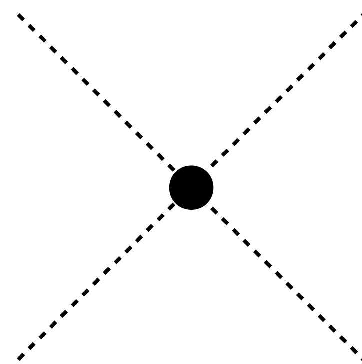
$$\mathcal{L}^{\text{gauge}} = -\frac{1}{2}\mathrm{Tr}\left(W^a_{\mu\nu}W^{a\mu\nu}\right) - \frac{1}{4}B_{\mu\nu}B^{\mu\nu}\Bigg|\quad \begin{matrix} B_\mu \rightarrow A_\mu \\ W^3_\mu \rightarrow Z_\mu \end{matrix}$$

$\boxed{\begin{matrix}\gamma \\ Z \\ W\end{matrix}}$

Triple Gauge Couplings:



Quartic Gauge Couplings:



- $SU(2)$ is a non-abelian gauge symmetry, which leads to dedicated self interactions with a predefined structure.

Which couplings are allowed (at tree level), which are not?

Self-Interactions

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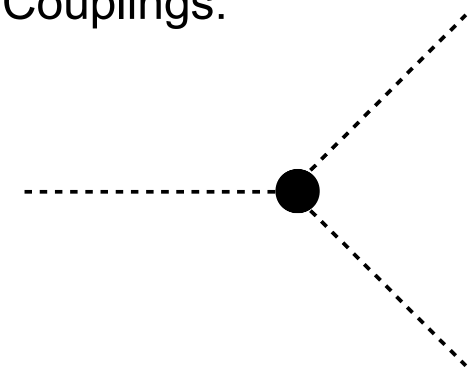

γWW $\quad ZWW$ $\quad \cancel{ZZZ}$

$\cancel{ZZ\gamma}$ $\quad \cancel{Z\gamma\gamma}$ $\quad \cancel{WWW}$

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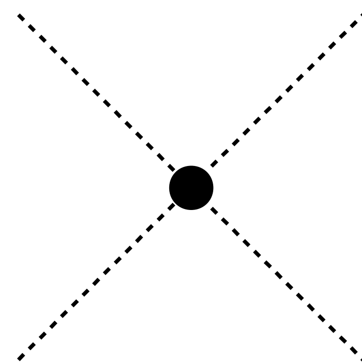

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Concluding Remarks

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- Brief review of the EWK sector of the SM (still w/o mass terms):
  - Projection to left(right)-handed states.
  - Local gauge symmetry for $SU(2) \times U(1)$ (and covariant derivatives).
  - Rotation of *eigenstates* of the symmetry into *eigenstates* of the IA.
  - Gauge symmetries imply dedicated self interactions of gauge bosons.

Sneak Preview for Next Week

- Up to now the problem of mass has been completely ignored.
- Discuss how mass terms in the Lagrangian density will compromise local gauge symmetries.
- Discuss the dynamic generation of mass via spontaneous symmetry breaking.

Backup & Homework Solutions