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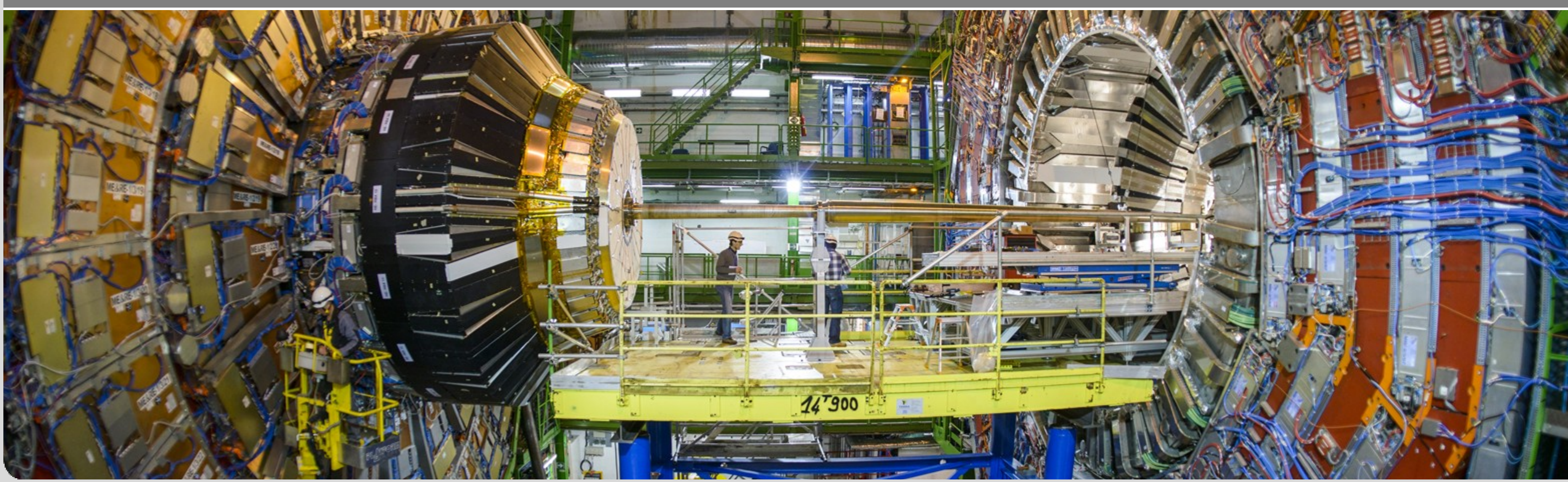
Electroweak Symmetry Breaking and the Higgs Mechanism

Roger Wolf

06. Mai 2014

INSTITUTE OF EXPERIMENTAL PARTICLE PHYSICS (IEKP) – PHYSICS FACULTY



Recap from Last Time

- Charged current interaction:

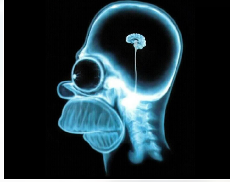
$$\mathcal{L}_{IA}^{CC} = -\frac{g}{\sqrt{2}}\left[\underbrace{\overline{\nu}\left(W_\mu^+\gamma^\mu\right)e_L}_{e\,\to\,\nu} + \underbrace{\overline{e}_L\left(W_\mu^-\gamma^\mu\right)\nu}_{\nu\,\to\,e}\right]$$

- Neutral current interaction:

$$\begin{aligned}\mathcal{L}_{IA}^{NC} = &-\frac{\sqrt{g^2+g'^2}}{2}Z_\mu\left(\overline{\nu}\gamma_\mu\nu\right)\\ &+\frac{\sqrt{g^2+g'^2}}{2}\left[\left(\cos^2\theta_W-\sin^2\theta_W\right)Z_\mu+2\sin\theta_W\cos\theta_W A_\mu\right]\left(\overline{e}_L\gamma_\mu e_L\right)\\ &+\frac{\sqrt{g^2+g'^2}}{2}\left[-2\sin^2\theta_W Z_\mu+2\sin\theta_W\cos\theta_W A_\mu\right]\left(\overline{e}_R\gamma_\mu e_R\right)\end{aligned}$$

- $SU(2)\times U(1)$ can describe electroweak IA including gauge boson self-couplings.

Quiz of the Day



- Is the mass problem the same for boson and fermion masses?
- Is the following statement true: “the Higgs boson couples proportional to the mass to all massive particles”?
- We have seen that QED does not at all have a problem with fermion masses (first lecture). Are fermion masses a problem specific to non-Abelian gauge symmetries?
- Is the Higgs boson a *Goldstone* boson? If not what is the difference between these types of particles?

Schedule for Today

3

The Higgs Mechanism in the SM

2

Spontaneous Symmetry Breaking & Higgs Mechanism

1

The Problem of Masses in the SM

The Problem of Masses in the SM

Problem 1: Massive Gauge Bosons

- Example: Abelian gauge field theories (→ first lecture)

- Transformation: $A_\mu \to A'_\mu = A_\mu + \frac{1}{e}\partial_\mu \vartheta$

- In mass term : $m_A A_\mu A^{\mu *} \to m_A A'_\mu A'^{\mu *} =$

$$m_A A_\mu A^{\mu *} + \underbrace{\tfrac{1}{e} m_A \left(A_\mu \partial^\mu \vartheta + A^{\mu *}\partial_\mu \vartheta\right) + m_A \tfrac{1}{e^2}\partial_\mu \vartheta \partial^\mu \vartheta}$$

These terms explicitly break local gauge covariance of $\mathcal{L}$.

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These terms explicitly break local gauge covariance of $\mathcal{L}$.

- Fundamental problem for all gauge field theories!

Problem 2: Massive Fermions

- No problem in $U(1)$:
  - Transformation: $\psi \to \psi' = e^{i\vartheta} \ \psi$
    $\overline{\psi} \to \overline{\psi}' = \overline{\psi} e^{-i\vartheta}$
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- So what is the problem of the $SU(2)$ in the SM?!

  It is the distinction between left-handed (ψ_L) and right-handed (ψ_R) fermions:

$$m_e \overline{e} e = m_e \overline{(e_L + e_R)}(e_L + e_R) = m_e \overline{e}_R e_L + m_e \overline{e}_L e_R$$

Dilemma of the $SU(2) \times U(1)$ Gauge Field Theory

- **Local gauge invariance...**
  - ... can motivate all interactions between elementary particles.
  - ... gives a geometrical interpretation for the presence of gauge bosons (propagate info of local phases btw space points).
  - ... predicts non trivial self-interactions between gauge bosons!

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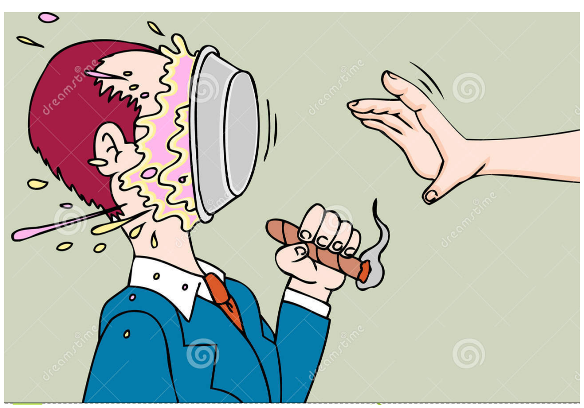
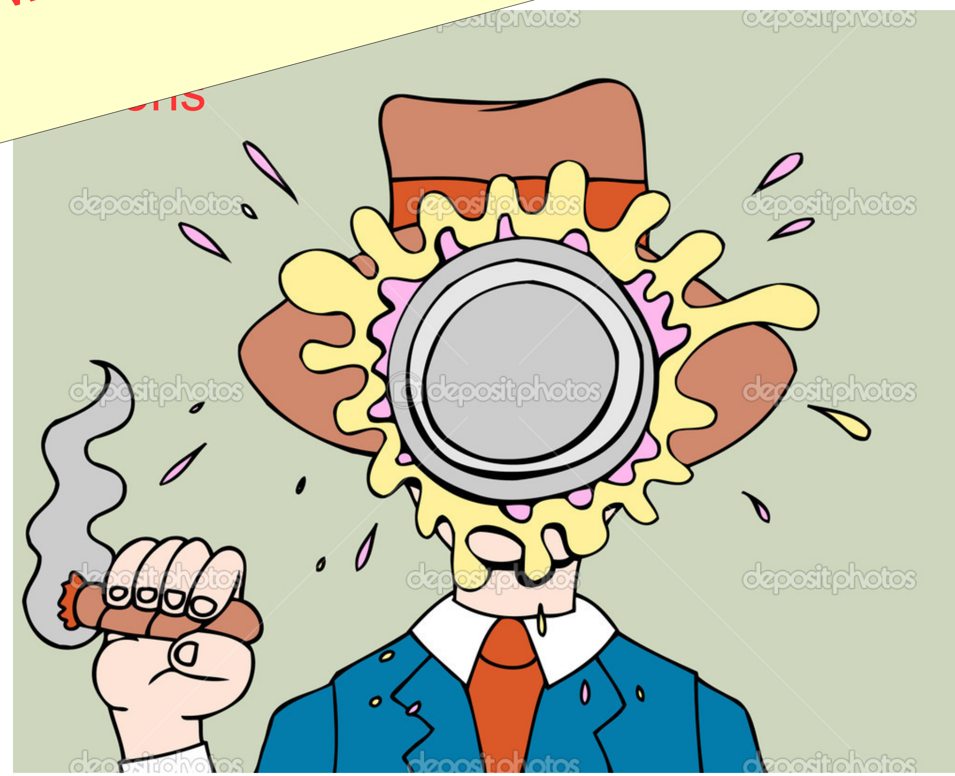
Deeply incompatible with incorporation of naive mass terms in $\mathcal{L}$!





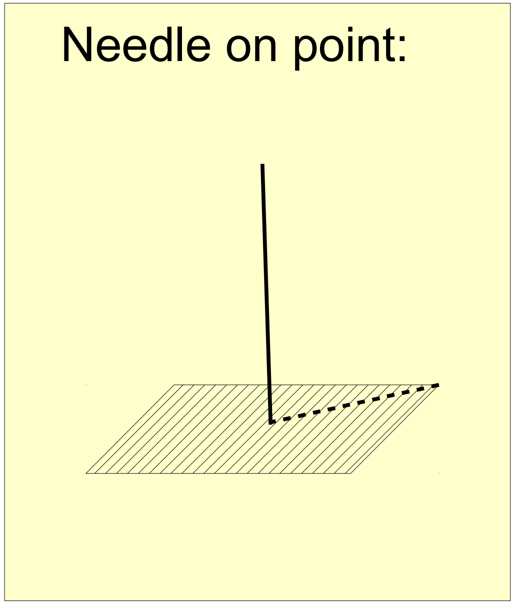
The Remedy


Bach
ORIGINAL
FLOWER REMEDIES
RESCUE
REMEDY
TO COMFORT
& REASSURE
10ml

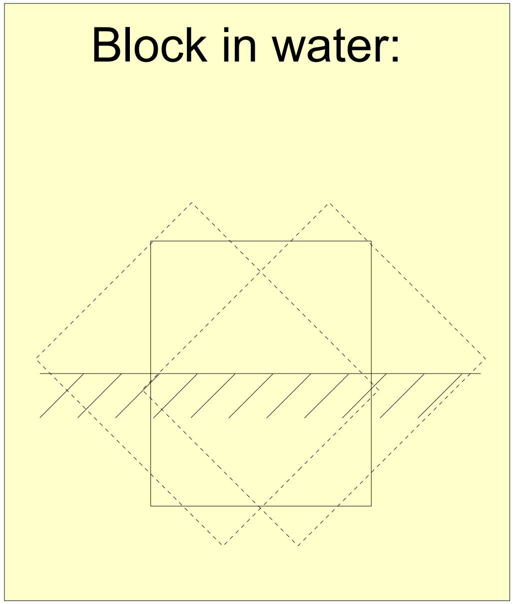

NDC# 057687-245-50
Bach
ORIGINAL
FLOWER REMEDIES
RESCUE
REMEDY
PASTILLES
NATURAL
STRESS RELIEF
ALCOHOL FREE
SUGAR FREE
50g PASTILLES
(1.7 oz)

Spontaneous Symmetry Breaking

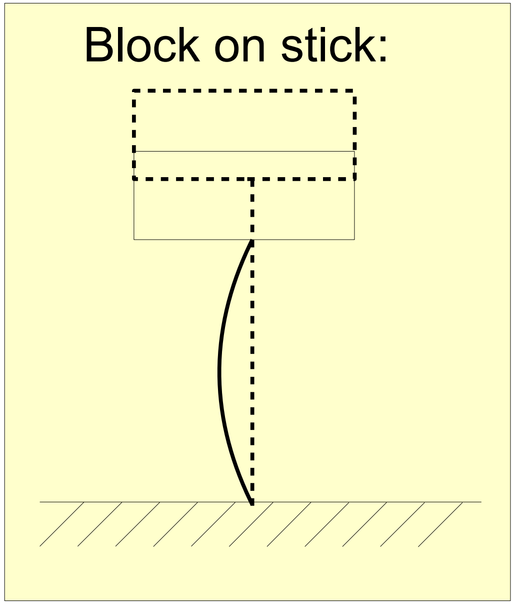
- Symmetry is present in the system (i.e. in the Lagrangian density $\mathcal{L}$).
- BUT it is broken in the ground state (i.e. in the quantum vacuum).
- Three examples (from classical mechanics):



φ symmetry



axis-symmetry

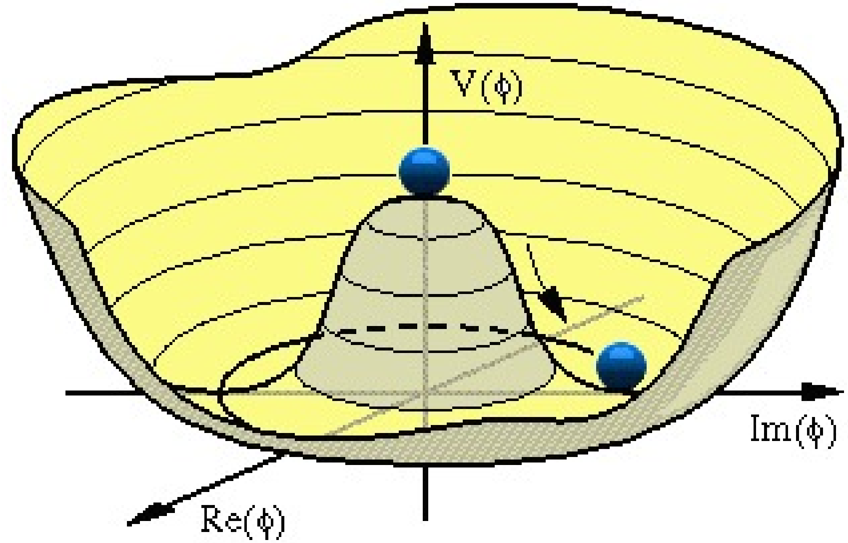


φ symmetry

Application to Particle Physics

- *Goldstone* Potential:

$$\phi = \frac{1}{\sqrt{2}}(\phi_1 + i\phi_2)$$
$$V(\phi) = -\mu^2|\phi|^2 + \lambda|\phi|^4$$
$$\mathcal{L}(\phi) = \partial_\mu\phi\partial^\mu\phi^* - V(\phi)$$



- invariant under $U(1)$ transformations (i.e. φ symmetric).
- metastable in $\phi = 0$.
- ground state breaks $U(1)$ symmetry, BUT at the same time all ground states are in-distinguishable in φ .

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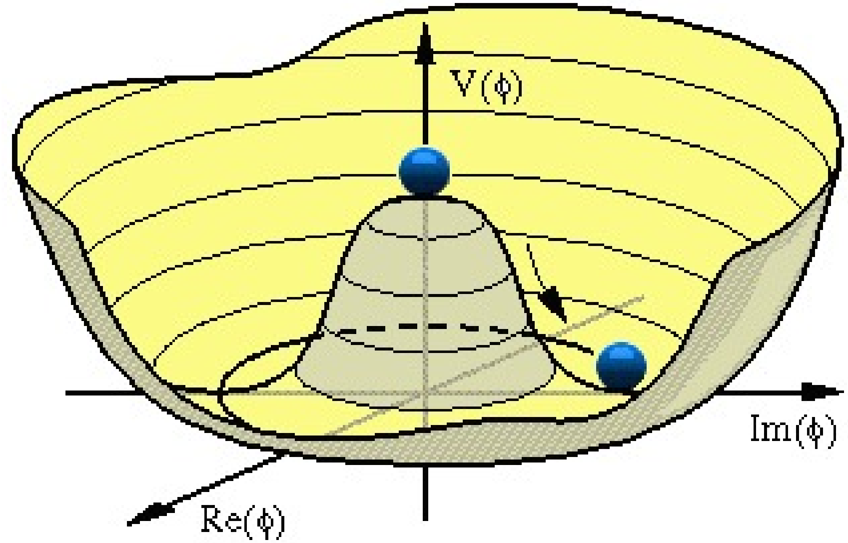

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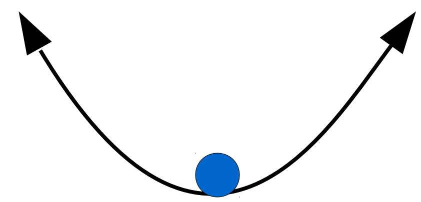
- ϕ has radial excitations in the potential $V(\phi)$.



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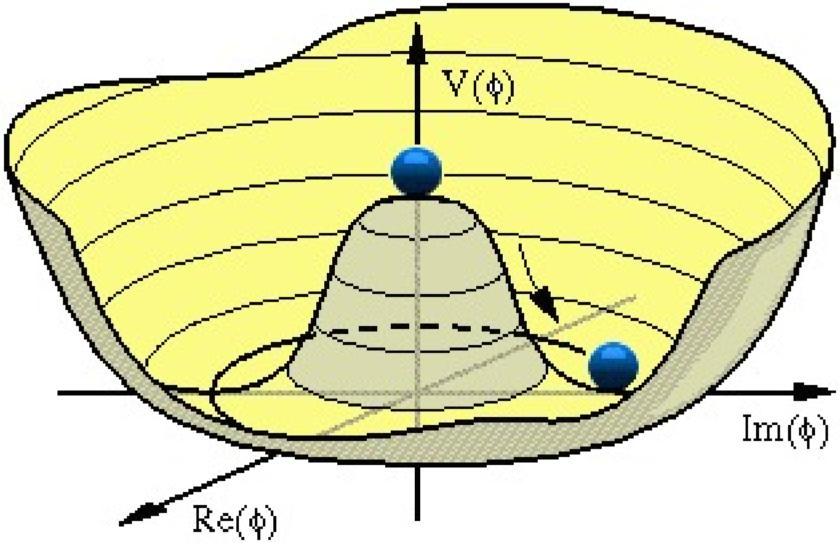

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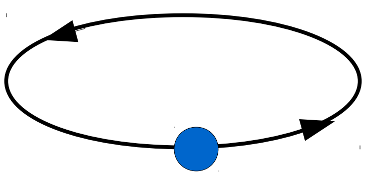
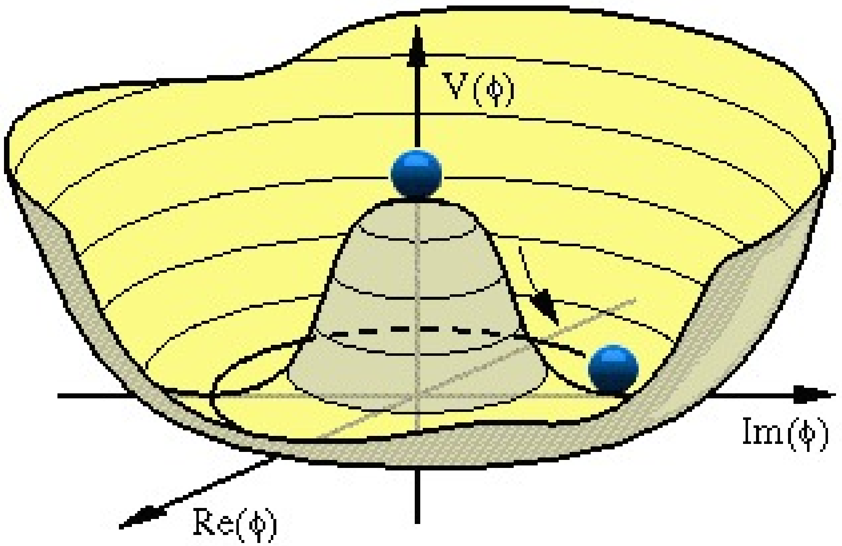
- ϕ can move freely in the circle that corresponds to the minimum of $V(\phi)$.



The *Goldstone* Theorem

- In particle physics this is formalized in the *Goldstone* Theorem:

> In a relativistic covariant quantum field theory with spontaneously broken symmetries massless particles (=*Goldstone* bosons) are created.



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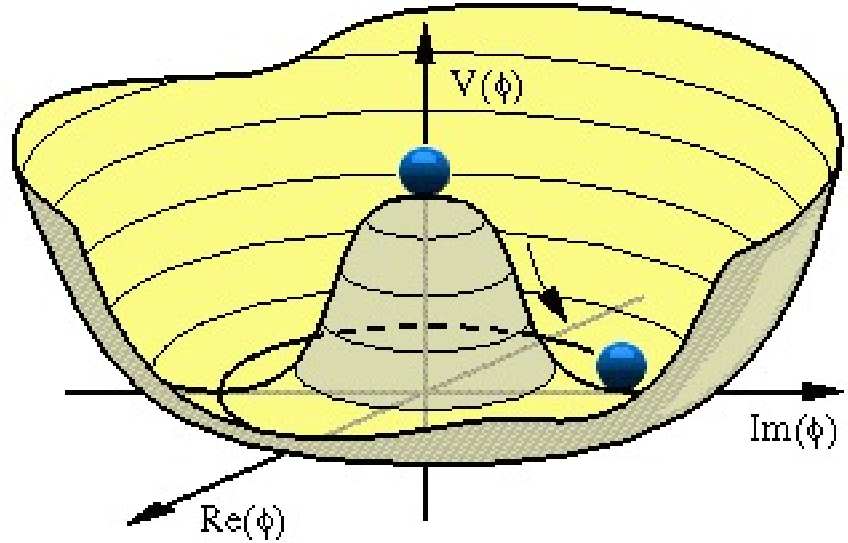

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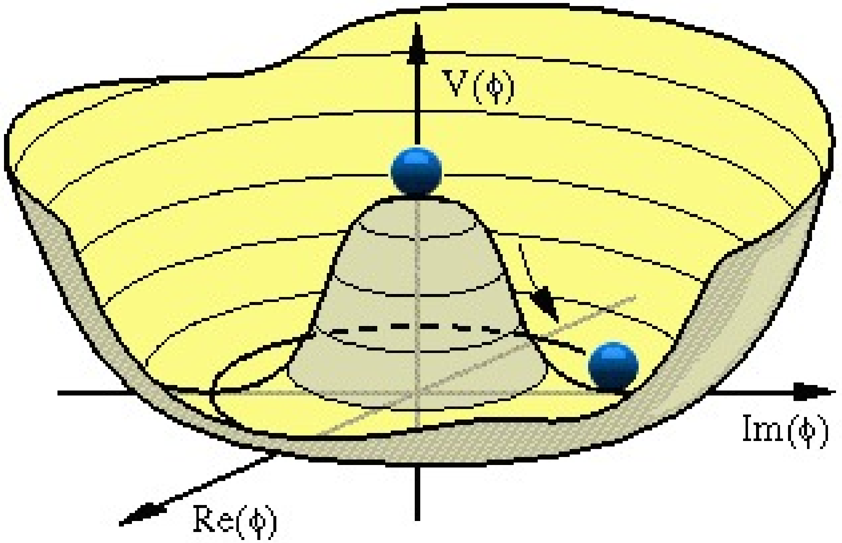

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  - Bound states, which are created by the theory (e.g. the H-atom).

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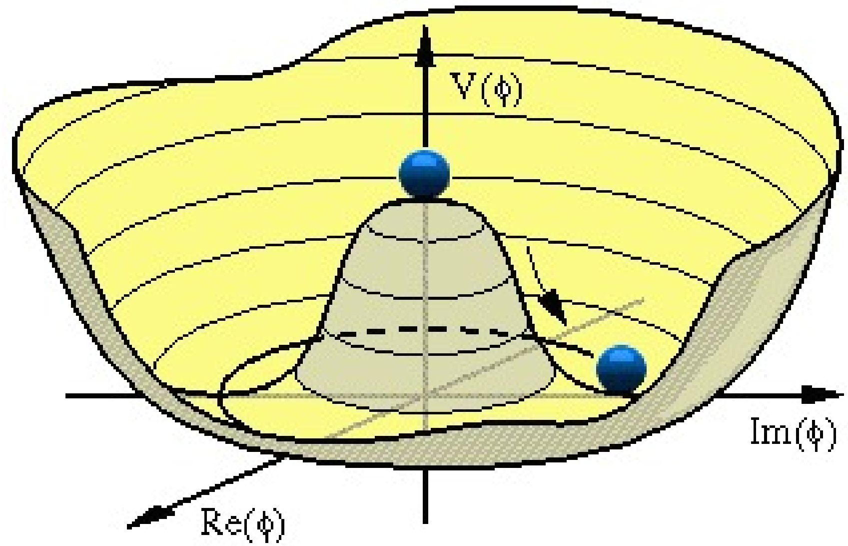


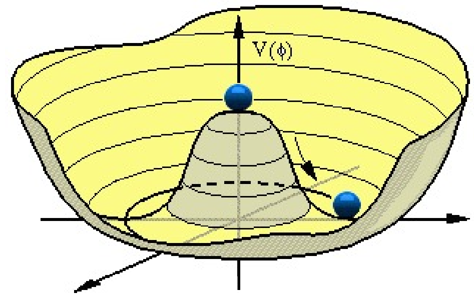
- *Goldstone* Bosons can be:
  - Elementary fields, which are already part of $\mathcal{L}$.
  - Bound states, which are created by the theory (e.g. the H-atom).
  - Unphysical or gauge degrees of freedom.

Analyzing the Energy Ground State

- The energy ground state is where the *Hameltonian*

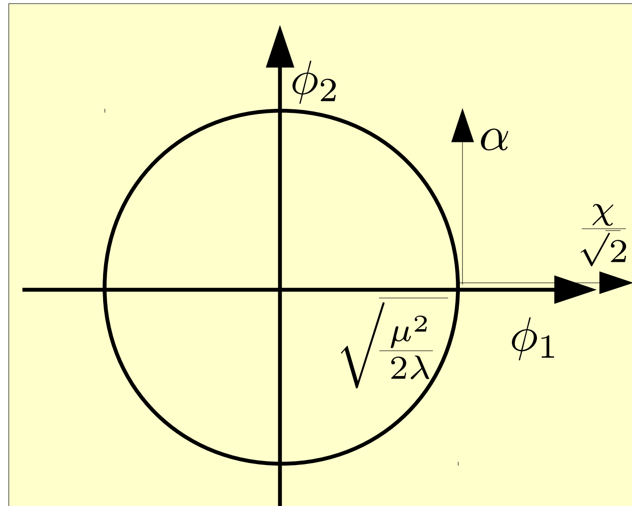
$$\mathcal{H} = \frac{\partial L}{\partial\left(\partial_\mu \phi\right)}\partial_\mu\phi + \frac{\partial L}{\partial\left(\partial^\mu \phi^*\right)}\partial^\mu\phi^* - \mathcal{L} = \partial_\mu\phi\partial^\mu\phi^* + V(\phi)$$

is minimal. This is the case for $|\phi| = \sqrt{\frac{\mu^2}{2\lambda}}$.



- To analyze the system in its physical ground state we make an expansion in an arbitrary point on the $|\phi| = \sqrt{\frac{\mu^2}{2\lambda}}$ cycle:

$$\phi(\chi,\alpha) = e^{i\alpha}\left(\sqrt{\frac{\mu^2}{2\lambda}} + \frac{\chi}{\sqrt{2}}\right)$$



Up view on *Goldstone* potential

Goldstone Bosons & Dynamic Massive Terms

- An expansion in the ground state in cylindrical coordinates leads to:

$$\mathcal{L} = \Big[\partial_\mu \phi \partial^\mu \phi^* - V(\phi)\Big]_{\phi(\chi,\alpha)} = \frac{1}{2}\partial_\mu \chi \partial^\mu \chi + \left(\sqrt{\frac{\mu^2}{2\lambda}} + \frac{\chi}{\sqrt{2}}\right)\partial_\mu \alpha \partial^\mu \alpha - V'(\chi)$$

$$\boxed{V'(\chi) = \Big[-\mu^2|\phi|^2 + \lambda|\phi|^4\Big]_{\phi(\chi)} = \underbrace{-\frac{\mu^4}{4\lambda}}_{\text{const.}} + \underbrace{\mu^2\chi^2}_{\text{dynamic mass term}} + \underbrace{\mu\sqrt{\lambda}\chi^3 + \frac{\lambda}{4}\chi^4}_{\text{self-couplings}}}$$

- Why is there no linear term in χ?

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const.

dynamic mass term

self-couplings

- Why is there no linear term in χ? $\longrightarrow$ We have performed a *Taylor* expansion in the minimum. By construction there cannot be any linear terms in there.

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- Remarks:
  - The mass term is acquired for the field χ along the radial excitation, which leads out of the minimum of $V(\chi)$. It is the term at lowest order in the *Taylor* expansion in the minimum, and therefore independent from the concrete form of $V(\chi)$ in the minimum.
  - The field α, which does not lead out of the minimum of $V(\chi)$ does not acquire a mass term. It corresponds to the *Goldstone* boson.

Extension to a Gauge Field Theory

- For simplicity reasons shown for an Abelian model:

Introduce covariant derivative

$$\phi(\chi, \alpha) = e^{i\alpha}\left(\sqrt{\frac{\mu^2}{2\lambda}} + \frac{\chi}{\sqrt{2}}\right)$$

$$\mathcal{L} = \left[\left(\partial_\mu + ieA_\mu\right)\phi\right]\left[\left(\partial^\mu + ieA^\mu\right)\phi\right]^* - V(\phi) - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}\Big|_{\phi(\chi,\alpha)}$$

$$= \left|\frac{1}{\sqrt{2}}\partial_\mu\chi e^{i\alpha} + ie^{i\alpha}\left(\sqrt{\frac{\mu^2}{\lambda}} + \frac{\chi}{\sqrt{2}}\right)\left(eA_\mu + \partial_\mu\alpha\right)\right|^2 - V'(\chi) - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

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$$A'_\mu = A_\mu + \partial_\mu\vartheta$$

How does this gauge look like?

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Mass term for A_μ :

$$\frac{e^2\mu^2}{2\lambda}A_\mu A^{\mu *}$$

Quartic and trilinear couplings with χ.

Higgs Mechanism

- *Goldstone* potential and expansion of $\phi \to \phi(\chi, \alpha)$ in the energy ground state has generated a mass term $\frac{e^2\mu^2}{2\lambda} A_\mu A^{\mu *}$ for the gauge field A_μ from the bare coupling $e^2 \left|\phi\right|^2 A_\mu A^{\mu *}$.

Higgs Mechanism

- ϕ was originally complex ($\rightarrow$ i.e. w/ 2 degrees of freedom) .
- χ is a real field, α has been absorbed into A_μ. It seems as if one degree of freedom had been lost. This is not the case:
  - as a massless particle A_μ has only two degrees of freedom (2 longitudinal polarizations).
  - as a massive particle it gains one additional degree of freedom (2 longitudinal +1 transverse polarization).

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This shuffle of degrees of freedom from the Goldstone boson(s) to the gauge boson(s) is called Higgs mechanism.

Notes on the *Goldstone* Potential

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  - it leads to spontaneous symmetry breaking.

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  - it does not distinguish any direction in space ($\rightarrow$ i.e. only depends on $|\phi|$).
  - it is bound from below and does not lead to infinite negative energies, which is a prerequisite for a stable theory.

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  - What is the dimension of ϕ ?
  - What is the dimension of μ ?
  - What is the dimension of λ ?

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  - What is the dimension of ϕ ? $[\phi] = \mathrm{GeV}^1$
  - What is the dimension of μ ? $[\mu] = \mathrm{GeV}^1$
  - What is the dimension of λ ? $[\lambda] = \mathrm{GeV}^0$

$$\mathcal{L}(\phi) = \partial_\mu\phi\partial^\mu\phi^* - V(\phi)$$
$$V(\phi) = -\mu^2|\phi|^2 + \lambda|\phi|^4$$

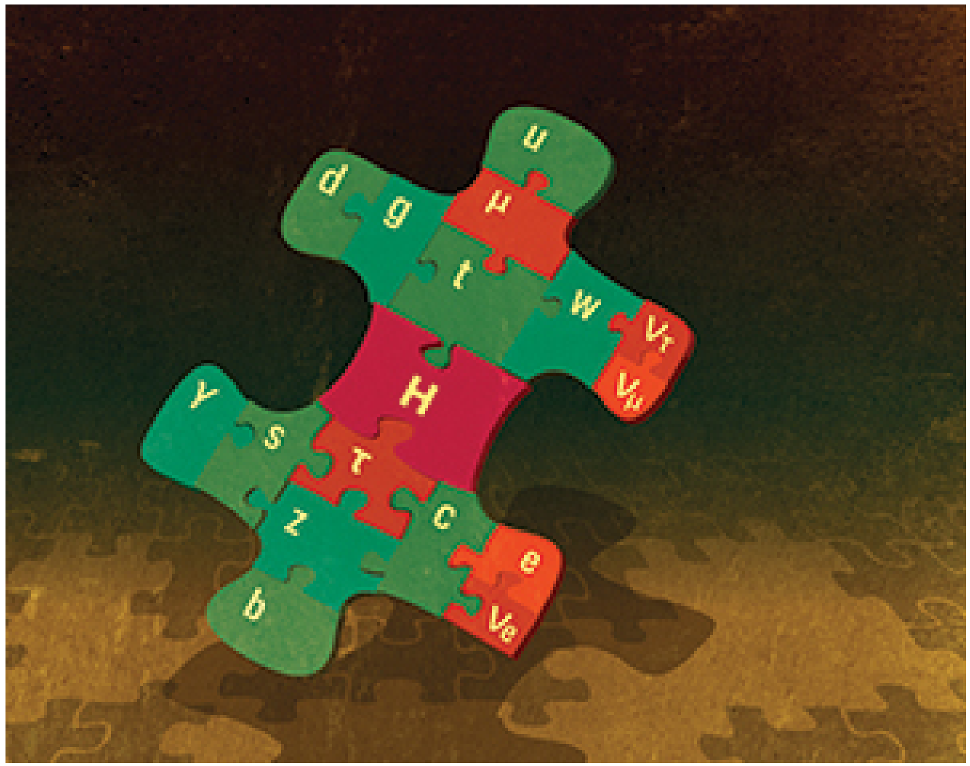
Notes on the *Goldstone* Potential

- The potential has been chosen to be cut at the order of $|\phi|^4$. This can be motivated by a dimensional analysis:
  - Due to gauge invariance ϕ has to appear in even order.
  - What is the dimension of $\mathcal{L}$? $[\mathcal{L}] = \mathrm{GeV}^4$
  - What is the dimension of ϕ ? $[\phi] = \mathrm{GeV}^1$
  - What is the dimension of μ ? $[\mu] = \mathrm{GeV}^1$
  - What is the dimension of λ ? $[\lambda] = \mathrm{GeV}^0$

$$\mathcal{L}(\phi) = \partial_\mu\phi\partial^\mu\phi^* - V(\phi)$$
$$V(\phi) = -\mu^2|\phi|^2 + \lambda|\phi|^4$$

- It would be possible to extend the potential to higher dimensions of ϕ but couplings with negative dimension will turn the theory non-renormalizable.

Final Construction of the SM



The SM without mass terms

- Compilation of the last two lectures:

Fermion kinematic

$$\mathcal{L}^{SU(2)\times U(1)} = \mathcal{L}^{\text{kin}} + \mathcal{L}^{CC} + \mathcal{L}^{NC} + \mathcal{L}^{\text{gauge}}$$

$$\mathcal{L}^{\text{kin}} = ie\gamma^\mu\partial_\mu e + i\nu\gamma^\mu\partial_\mu\nu$$

$$\mathcal{L}^{CC} = -\frac{e}{\sqrt{2}\sin\theta_W}\left[W_\mu^+\nu\gamma_\mu e_L + W_\mu^- e_L\gamma_\mu\nu\right]$$

$$\mathcal{L}^{NC} = -\frac{e}{2\sin\theta_W\cos\theta_W}Z_\mu\left[(\nu\gamma_\mu\nu) + (e_L\gamma_\mu e_L)\right] - e\left[A_\mu + \tan\theta_W Z_\mu\right](e\gamma_\mu e)$$

$$\mathcal{L}^{\text{gauge}} = -\frac{1}{2}Tr\left(W_{\mu\nu}^a W^{a\mu\nu}\right) - \frac{1}{4}B_{\mu\nu}B^{\mu\nu}\Bigg|_{\substack{B_\mu \to A_\mu \\ W_\mu^3 \to Z_\mu}}$$

The SM without mass terms

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Charged current IA

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The SM without mass terms

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Charged current IA

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$$\mathcal{L}^{SU(2)\times U(1)} = \mathcal{L}^{\text{kin}} + \mathcal{L}^{CC} + {\color{red}\mathcal{L}^{NC}} + \mathcal{L}^{\text{gauge}}$$

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The SM without mass terms

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Charged current IA

Neutral current IA

Fermion kinematic

Gauge field kinematic

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Extension by a new field ϕ

- Add ϕ as $SU(2)$ doublet field:

$$\phi = \begin{pmatrix} \phi_+ \\ \phi_0 \end{pmatrix}$$

Transformation behavior

$$\begin{cases} \phi \rightarrow \phi' = e^{i\vartheta'} G\phi \\ \phi^\dagger \rightarrow \phi'^\dagger = \phi^\dagger G^+ e^{-i\vartheta'} \\ G = e^{i\vartheta^a t^a} \in SU(2) \quad \vartheta^a, \vartheta' \in \mathbb{R} \end{cases}$$

Can you point to the *Goldstone* bosons?

$$\mathcal{L}^{SU(2)\times U(1)} = \mathcal{L}^{\text{kin}} + \mathcal{L}^{CC} + \mathcal{L}^{NC} + \mathcal{L}^{\text{gauge}} + \mathcal{L}^{\text{Higgs}}$$

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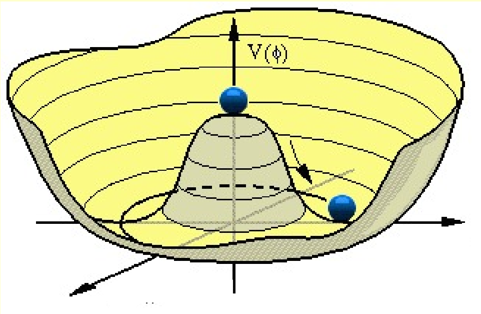
$$V(\phi) = -\mu^2 \phi^+ \phi + \lambda \left(\phi^+ \phi\right)^2$$



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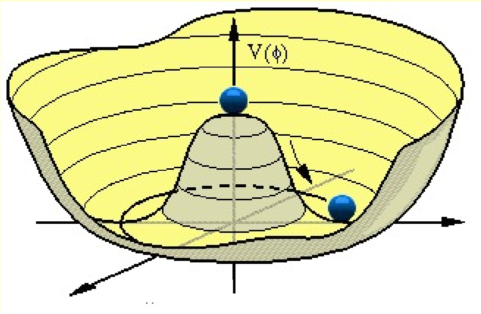

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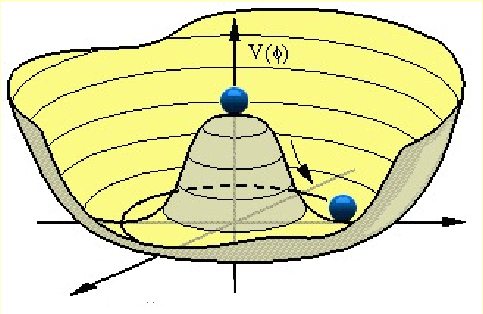

- $\mathcal{L}^{\text{Higgs}}$ is covariant under global $SU(2)$ transformations.

Extension by a new field ϕ

- Add ϕ as $SU(2)$ doublet field:

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Transformation behavior

$$\begin{cases} \phi \to \phi' = e^{i\vartheta'} G\phi \\ \phi^\dagger \to \phi'^\dagger = \phi^\dagger G^+ e^{-i\vartheta'} \\ G = e^{i\vartheta^a t^a} \in SU(2) \quad \vartheta^a, \vartheta' \in \mathbb{R} \end{cases}$$

Can you point to the *Goldstone* bosons?

- Introduce covariant derivative D_μ to enforce local gauge invariance:

$$\partial_\mu \to D_\mu = \partial_\mu + ig'\frac{Y_\phi}{2}B_\mu + igt^a W_\mu^a$$

(analogue to fermion fields)

$SU(2)\times U(1)$ Hypercharges			
Particle	Y_ϕ	I_3	Q
ϕ_+	$+1$	$+1/2$	$+1$
ϕ_0		$-1/2$	0

$Q = I_3 + \frac{Y}{2}$ (Gell-Mann Nischijama)

Expansion in the Energy Ground State of ϕ

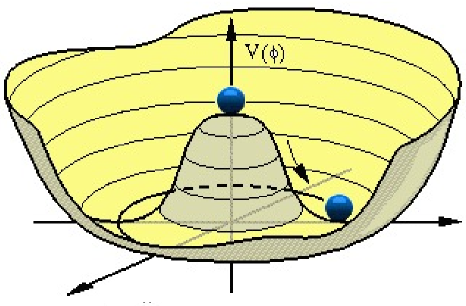
- Develop ϕ in its energy ground state at $|\phi| = \sqrt{\frac{\mu^2}{2\lambda}}$:

$$\phi = \begin{pmatrix} \phi_+ \\ \phi_0 \end{pmatrix} \xrightarrow{|\phi| = \sqrt{\frac{\mu^2}{2\lambda}}} \phi = \begin{pmatrix} 0 \\ \sqrt{\frac{\mu^2}{2\lambda}} + \frac{H}{\sqrt{2}} \end{pmatrix}$$

w/o loss of generality this can be done in the lower component of ϕ.

Non-zero vacuum expectation value.

Radial excitation field. → This is the Higgs boson!



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couples gauge fields to ϕ .

$$D_\mu \phi^\dagger D^\mu \phi$$

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$$D_\mu \phi^\dagger D^\mu \phi = \left| \left[\frac{1}{\sqrt{2}} \partial_\mu H + \left(\sqrt{\frac{\mu^2}{2\lambda}} + \frac{H}{\sqrt{2}} \right) \left(ig' \frac{Y_\phi}{2} B_\mu + i g t^a W^a_\mu \right) \right] \begin{pmatrix} 0 \\ 1 \end{pmatrix} \right|^2$$

$$\boxed{D_\mu = \partial_\mu + ig' \frac{Y_\phi}{2} B_\mu + i g t^a W^a_\mu}$$

(covariant derivative)

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Expansion in the Energy Ground State of ϕ

- Resolve products of *Pauli* matrices ($t^a \equiv \frac{1}{2}\sigma_a$):

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$$D_\mu \phi^\dagger D^\mu \phi = \left| \left[\frac{1}{\sqrt{2}} \partial_\mu H - \frac{i}{2} \left(\sqrt{\frac{\mu^2}{2\lambda}} + \frac{H}{\sqrt{2}} \right) \left(g W^3_\mu - g' B_\mu \right) \right] \begin{pmatrix} 0 \\ 1 \end{pmatrix} \right|^2 + \left| \left[\frac{i}{2} \left(\sqrt{\frac{\mu^2}{2\lambda}} + \frac{H}{\sqrt{2}} \right) g W^+_\mu \right] \begin{pmatrix} 1 \\ 0 \end{pmatrix} \right|^2$$

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  - Ascending operator t^+ (W_μ^+) shifts unit vector of the down component up.

$$D_\mu \phi^\dagger D^\mu \phi = \left| \left[\frac{1}{\sqrt{2}} \partial_\mu H - \frac{i}{2} \left(\sqrt{\frac{\mu^2}{2\lambda}} + \frac{H}{\sqrt{2}} \right) (g W_\mu^3 - g' B_\mu) \right] \begin{pmatrix} 0 \\ 1 \end{pmatrix} \right|^2 + \left| \left[\frac{i}{2} \left(\sqrt{\frac{\mu^2}{2\lambda}} + \frac{H}{\sqrt{2}} \right) g W_\mu^+ \right] \begin{pmatrix} 1 \\ 0 \end{pmatrix} \right|^2$$

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  - Descending operator t^- (W_μ^-) “destroys” unit vector of the down component.

$$D_\mu\phi^\dagger D^\mu\phi = \left|\left[\frac{1}{\sqrt{2}}\partial_\mu H - \frac{i}{2}\left(\sqrt{\frac{\mu^2}{2\lambda}} + \frac{H}{\sqrt{2}}\right)\left(gW_\mu^3 - g'B_\mu\right)\right]\left(\begin{array}{c}0\\1\end{array}\right)\right|^2$$
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Expansion in the Energy Ground State of ϕ

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  - Ascending operator t^+ (W_μ^+) shifts unit vector of the down component up.
  - Descending operator t^- (W_μ^-) “destroys” unit vector of the down component.
  - Operator t^3 switches sign for unit vector of down component.

$$D_\mu\phi^\dagger D^\mu\phi = \left|\left[\frac{1}{\sqrt{2}}\partial_\mu H - \frac{i}{2}\left(\sqrt{\frac{\mu^2}{2\lambda}} + \frac{H}{\sqrt{2}}\right)\left(gW_\mu^3 - g'B_\mu\right)\right]\left(\begin{array}{c} 0 \\ 1 \end{array}\right)\right|^2$$
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Expansion in the Energy Ground State of ϕ

- Evaluate components of absolute value squared:

$$D_\mu\phi^\dagger D^\mu\phi = \left|\left[\frac{1}{\sqrt{2}}\partial_\mu H - \frac{i}{2}\left(\sqrt{\frac{\mu^2}{2\lambda}} + \frac{H}{\sqrt{2}}\right)(gW_\mu^3 - g'B_\mu)\right]\begin{pmatrix} 0 \\ 1 \end{pmatrix}\right|^2$$

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Masses for Gauge Bosons

- By introducing ϕ as a $SU(2)$ doublet with a non-zero energy ground state we have obtained:

$$D_\mu\phi^\dagger D^\mu\phi = \tfrac{1}{2}\partial_\mu H\partial^\mu H + \frac{g^2+g'^2}{4}\left(\sqrt{\frac{\mu^2}{2\lambda}}+\frac{H}{\sqrt{2}}\right)^2 Z_\mu Z^\mu + \frac{g^2}{4}\left(\sqrt{\frac{\mu^2}{2\lambda}}+\frac{H}{\sqrt{2}}\right)^2 W_\mu^+ W^{\mu -}$$

- Dynamic mass terms for the gauge bosons: $m_Z^2 \equiv \frac{(g^2+g'^2)\mu^2}{8\lambda}$ $\quad m_W^2 \equiv \frac{g^2\mu^2}{8\lambda}$

- Characteristic trilinear and quartic couplings of the gauge bosons to the Higgs field.

- A solid prediction of the SM on the masses of the gauge bosons: $\cos\theta_W = \frac{m_W}{m_Z} \longrightarrow m_Z > m_W$

Gauge Degrees of Freedom

- We had discussed how gauge bosons obtain mass by a gauge that absorbs the *Goldstone* bosons in the theory.

- As a complex $SU(2)$ doublet ϕ has four degrees of freedom.
- In the final formulation only the radial excitation H of ϕ did remain. The *Goldstone* bosons (ϑ^a) have been absorbed into the gauge fields $W_\mu^{+/-}$ & Z_μ, which have obtained masses from this.

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- Congratulations - you got it!!!

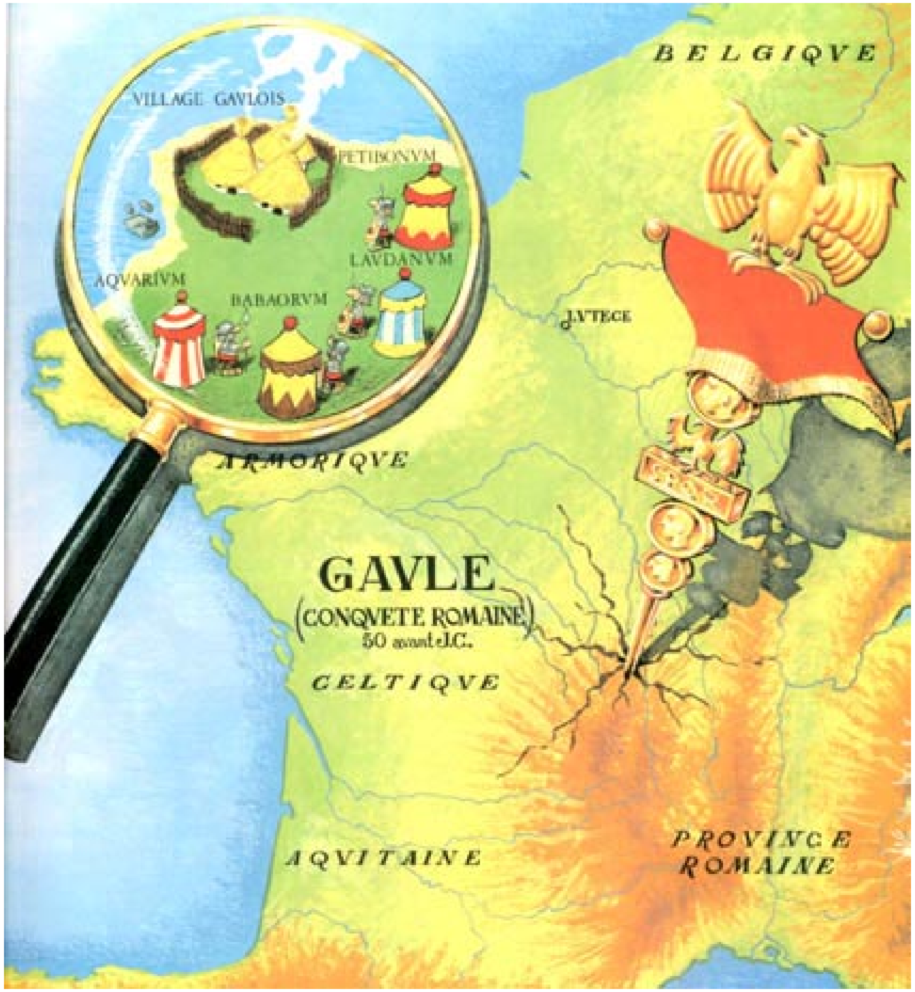
Gauge Degrees of Freedom

- We had discussed how gauge bosons obtain mass by a gauge that absorbs the *Goldstone* bosons in the theory.

- As a complex $SU(2)$ doublet ϕ has four degrees of freedom.
- In the final formulation only the radial excitation H of ϕ did remain. The *Goldstone* bosons (ϑ^a) have been absorbed into the gauge fields $W_\mu^{+/-}$ & Z_μ, which have obtained masses from this.

- Congratulations - you got it!!!
- Almost...

Solution to the Problem of Fermion Masses



Solution to the Problem of Fermion Masses

- The Higgs mechanism can also help to obtain mass terms for fermions, by coupling the fermions to ϕ .

$$\mathcal{L}^{\text{Yukawa}} = -f_e \left(\underbrace{\bar{e}_R \phi^\dagger \psi_L}_{SU(2)\ \text{singlet}}\right) + f_e^* \left(\underbrace{\bar{\psi}_L \phi e_R}_{SU(2)\ \text{singlet}}\right)$$

$$\psi_L = \begin{pmatrix} \nu \\ e_L \end{pmatrix} \qquad \phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$$

- check $SU(2)$ behavior:

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- check $U(1)$ behavior: $?$ $?$ $?$ $?$ $?$ $?$

$$Y_R = -2$$

$$Y_L = -1$$

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- check $U(1)$ behavior:

$\bar{e}_R \to +2$, $\phi^\dagger \to -1$, $\psi_L \to -1$: $U(1)$ singlet

$\bar{\psi}_L \to +1$, $\phi \to +1$, $e_R \to -2$: $U(1)$ singlet

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- check $SU(2)$ behavior:

$$\mathcal{L}^{\text{Yukawa}} = -f_e\left(\underbrace{\bar{e}_R}_{+2} \underbrace{\phi^\dagger}_{-1} \underbrace{\psi_L}_{-1}\right) + f_e^*\left(\underbrace{\bar{\psi}_L}_{+1} \underbrace{\phi}_{+1} \underbrace{e_R}_{-2}\right)$$

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$U(1)$ singlet $\qquad U(1)$ singlet

$$Y_R = -2 \qquad Y_L = -1 \qquad Y_\phi = +1$$

Manifest gauge invariant.

- NB: f can be chosen real. Residual phases can be re-defined in e_R .

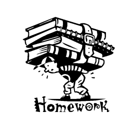
Solution to the Problem of Fermion Masses

- Expand ϕ in its energy ground state to obtain the mass terms:

$$\mathcal{L}^{\text{Yukawa}} = \underbrace{-f_e \left(\sqrt{\frac{\mu^2}{2\lambda}}\right.}_{m_e \equiv f_e\sqrt{\frac{\mu^2}{2\lambda}}} \left. + \frac{H}{\sqrt{2}}\right) \underbrace{(\bar{e}_R e_L + \bar{e}_L e_R)}_{\bar{e}e} = -m_e \left(1 + \sqrt{\frac{\lambda}{\mu^2}} H\right) \bar{e}e$$

- We obtained the desired mass term and a coupling to the Higgs boson field, which is proportional to the fermion mass.

- Check the relation: $\bar{e}e = \bar{e}_R e_L + \bar{e}_L e_R$



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  On slide 11 I told you that terms of type $\bar{e}e = \bar{e}_R e_L + \bar{e}_L e_R$ break gauge invariance.
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Concluding Remarks

- The Higgs mechanism = incorporation of spontaneous symmetry breaking into a gauge field theory. This leads to the fact that the gauge bosons eat up the *Goldstone* bosons, which exist in the system and gain mass on this.

- This mechanism leaves one or more degrees of freedom of radial excitations in the potential behind as Higgs boson(s).

- The Higgs boson obtains its mass from the *Goldstone* potential. The gauge bosons obtain their mass from their coupling to ϕ via the covariant derivative. The Fermions obtain their mass via a direct Yukawa coupling to ϕ .

- Gauge bosons couple to Higgs like $\propto m^2_{\text{gauge}}H$, fermion fields couple to Higgs like $\propto m_f H$.

Sneak Preview for Next Week

- Wrap up what we have learned during the last three lectures.
- Discuss the way from Lagrangian to measurable quantities ($\rightarrow$ Feynman rules).
- Discuss loop corrections and higher orders to tree level calculations (pictorially).
- Constraints on m_H within the theory itself.

Backup & Homework Solutions