Exercises to Lecture 3: Electroweak Symmetry Breaking and the Higgs Mechanism

Exercise 7 (Projectors):

In the lecture you have seen the relation:

$$\overline{e}e = \overline{e}_R e_L + \overline{e}_L e_R$$

Proof that this relation is correct. Hint: for this start with $\overline{e}e = \overline{(e_L + e_R)}(e_L + e_R)$ and show that $\overline{e}_L e_L = \overline{e}_R e_R \equiv 0$. Make use of the properties of the projectors to left- an right-handed states.

Exercise 8 (*Goldstone* potential):

In the lecture you have been introduced to the *Goldstone* potential:

$$V(\phi) = -\mu^2|\phi|^2 + \lambda|\phi|^4$$

a)
Proof that this potential indeed has its minimum in $|\phi| = \sqrt{\frac{\mu^2}{2\lambda}}$.

b)
In the lecture you have seen an expansion of the field in cylindrical coordinates. Try yourself in an expansion in Cartesian coordinated in the point $\phi(u,v) = \sqrt{\frac{\mu^2}{2\lambda}} + \frac{1}{\sqrt{2}}(u+iv)$. You may also try $\phi(u,v) = i\sqrt{\frac{\mu^2}{2\lambda}} + \frac{1}{\sqrt{2}}(u+iv)$ and check the difference.

Exercise 9 (*Higgs mechanism in QED with a massive photon*):

Consider a hypothetical QED model with a massive photon. This model shall be described by the Lagrangian density:

$$\mathcal{L} = \overline{\psi}\left(i\gamma^\mu D_\mu - m_e\right)\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}m_A^2 A_\mu A^\mu$$

a)
Show that the mass term of the photon $\frac{1}{2}m_A^2 A_\mu A^\mu$ violates $U(1)$ gauge symmetry, while the massless Lagrangian density does not.

b)
Introduce such a mass term via the Higgs mechanism: introduce a scalar complex field, which transforms under the $U(1)$ gauge symmetry like

$$\phi \to \phi' = e^{ie\vartheta(x)}\phi$$

with a Lagrangian density and a spontaneous symmetry breaking potential of form

$$\mathcal{L} = (D^\mu \phi^*)(D_\mu \phi) - V(\phi^* \phi)$$

$$V(\phi^* \phi) = \lambda \left(\phi^* \phi - \frac{v^2}{2} \right)^2$$

and expand the field as $\phi = \frac{1}{\sqrt{2}}(v + h(x))$.

c)
Show that a *Yukawa* interaction term of type

$$\mathcal{L}_{\text{Yukawa}} = f_e |\phi| \overline{\psi} \psi$$

modifies the electron mass in the model and express the electron mass in terms of the "bare" electron mass m_e, the *Yukawa* coupling f_e and the vacuum expectation value of the Higgs field v.