Exercises to Lecture 4: From Lagrange Density to Observable

Exercise 10 (Time evolution of *Green's* function):

In the lecture we have discussed that there are two *Green's* functions as solutions to the *Dirac* equation depending on the evolution behavior, backwards or forwards in time. Assume a *spinor*

$\phi(x') = u(k)e^{-ik_0t' + i\vec{k}\vec{x}'}$

and proof that for the *Green's* function for $t > t'$ you obtain

$\phi(x) = i \int \mathrm{d}^3x' K(x - x')\gamma^0 \phi(t', \vec{x}')$

Note: make use of the fact that $u(k)$ fulfills the *Dirac* equation. Show that using the *Green's* function for $t < t'$ leads to 0. This proofs that particles with positive energy evolve forward in time. You could show the corresponding for particles with negative energy that evolve backwards in time.

Exercise 11 (Triviality bound on Higgs boson mass):

In the lecture you have discussed the solution of the renormalization group equation (at one loop level) for the Higgs self-coupling λ for $Q^2 \ll m_H$:

$\lambda(Q^2) = \frac{\lambda(\mathrm{v}^2)}{1 - \frac{3}{4\pi^2}\lambda(\mathrm{v}^2)\log(Q^2/\mathrm{v}^2)}$

Note that $\mathrm{v} = \sqrt{\frac{\mu^2}{\lambda}} = 246\,\mathrm{GeV}$. Calculate the *Landau* pole, where the denominator turns to zero. Obtain a formula for m_H from this equation from which you can derive upper bounds on m_H. Calculate the upper bounds for the assumption that the SM is a valid and perturbative theory up to scales of $Q = 10^3\,\mathrm{GeV}$ ($Q = 10^{16}\,\mathrm{GeV}$). Note: you can use the fact that $m_H^2 = 2\mu^2$.

Exercise 12 (Stability bound on Higgs boson mass):

In the lecture you have also seen the solution of the renormalization group equation for λ for the case that $m_H \ll m_t$:

$\lambda(Q^2) = \lambda(\mathrm{v}^2) - \frac{3}{16\pi^2}\frac{m_t^4}{\mathrm{v}^4}\log\left(Q^2/\mathrm{v}^2\right)$

Derive the formula this time to set lower limits on m_H. Calculate the upper bounds for the assumption that the SM is a valid and perturbative theory up to scales of $Q = 10^3\,\mathrm{GeV}$ ($Q = 10^{16}\,\mathrm{GeV}$).