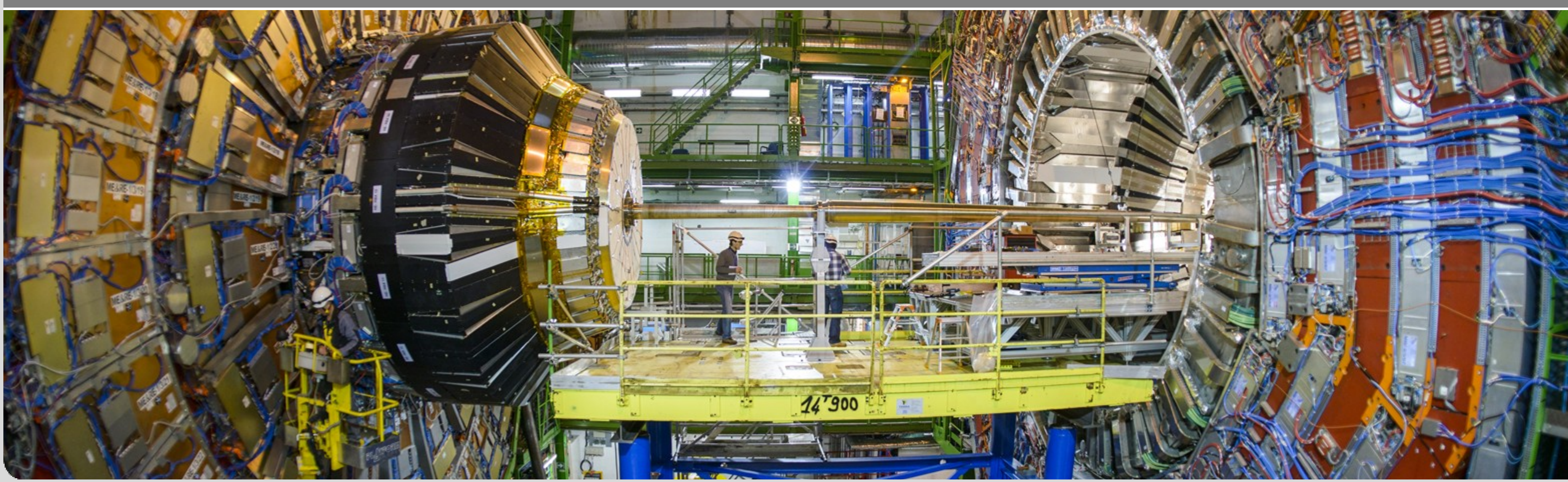
Karlsruhe Institute of Technology

Electroweak Sector of the SM

Roger Wolf

23. April 2015

INSTITUTE OF EXPERIMENTAL PARTICLE PHYSICS (IEKP) – PHYSICS FACULTY



Schedule for Today

1

Review of Lie-Groups:

- $U(1)$ & $SU(2)$
- (Non-) Abelian Gauge theories

2

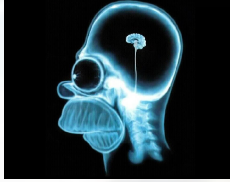
Phenomenology of Weak Interaction

3

Sketch of the Electroweak Sector of the SM:

- Left (Right)-handed States
- Local $SU(2) \times U(1)$ Symmetry
- Weinberg Rotation

Quiz of the Day



- Are normal normal rotation in $\mathbb{R}^3$ Abelian or non-Abelian?

- The W boson only couples to left-handed particles! Does the Z boson also couple only to left-handed particles?

- Are the following gauge boson self-couplings allowed: ZWW, $WWWW$?

Recap from Last Time

Gauge Field Theories:

$$\psi(\vec{x},t) \to \psi'(\vec{x},t) = e^{i\vartheta}\psi(\vec{x},t)$$
$$\overline{\psi}(\vec{x},t) \to \overline{\psi}'(\vec{x},t) = \overline{\psi}(\vec{x},t)e^{-i\vartheta}$$

→ *(Local Gauge Invariance)*

$$\partial_\mu \to D_\mu = \partial_\mu - ieA_\mu$$
$$D_\mu \to D'_\mu = D_\mu - i\partial_\mu\vartheta$$
$$A_\mu \to A'_\mu = A_\mu + \frac{1}{e}\partial_\mu\vartheta$$

→ *(Covariant Derivative)*

$$F_{\mu\nu} \equiv [D_\mu, D_\nu] = \partial_\mu A_\nu - \partial_\nu A_\mu$$
$$F_{\mu\nu} \to F'_{\mu\nu} = F_{\mu\nu}$$

→ *(Field Strength Tensor)*

$$\mathcal{L} = \overline{\psi}\left(i\gamma^\mu D_\mu - m\right)\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

→ *(Lagrange Density)*

Review of Lie-Groups

Marius Sophus Lie

(*17. December 1842, † 18. February 1899)

Unitary Transformations

$U(1)$ phase transformation

$$\psi(\vec{x},t) \rightarrow \psi'(\vec{x},t) = e^{i\vartheta}\psi(\vec{x},t)$$

- $U(1)$ is a group of unitary transformations in $\mathbb{R}^n$ with the following properties:

$$\mathbf{G} \in U(n) \qquad \mathbf{G}^\dagger\mathbf{G} = \mathbb{I}_n \qquad \det\mathbf{G} = \pm 1$$

- Splitting an additional phase from $\mathbf{G}$ one can reach that $\det\mathbf{G} = 1$:

$$U(n) = U(1) \times SU(n)$$

$\det\mathbf{G} = \pm 1$ $\det\mathbf{G} = +1$

(*Unitary Transformations*) (*Special Unitary Transformations*)

Infinitesimal → Finite Transformations

- The $SU(n)$ can be composed from infinitesimal transformations with a continuous parameter $\vartheta \in \mathbb{R}$:

$$\mathbf{G}|_{\text{finite}} = \mathbb{I}_n + i\vartheta_{\text{finite}}\mathbf{t} \; (\vartheta_{\text{finite}} \in \mathbb{R}\,,\; \mathbf{t} \in \mathcal{M}(\mathrm{n}\times\mathrm{n}))$$

$$\mathbf{G}|_{\text{finite}} = \left(\mathbb{I}_n + i\,\frac{\vartheta_{\text{finite}}}{m}\mathbf{t}\right)^m \xrightarrow{m\to\infty} e^{i\vartheta_{\text{finite}}\cdot\mathbf{t}}$$

→ t generators of $\mathbf{G}$.

→ t define structure of $\mathbf{G}$.

- The set of $\mathbf{G}$ forms a *Lie-Group*.
- The set of t forms the tangential-space or *Lie-Algebra*.

Properties of $\mathbf{t}$

- Hermitian:

$$\mathbf{G}^\dagger\mathbf{G} = \mathbb{I}_n$$

$$= \left(\mathbb{I}_n - i\vartheta\mathbf{t}^\dagger\right)\left(\mathbb{I}_n + i\vartheta\mathbf{t}\right) = \mathbb{I}_n + i\vartheta\underbrace{\left(\mathbf{t} - \mathbf{t}^\dagger\right)}_{} + O(\vartheta^2)$$

$$\boxed{\mathbf{t} = \mathbf{t}^\dagger}$$

- Traceless (example $SU(n)$):

$$\det\mathbf{G} = \det\left(\mathbb{I}_n + i\vartheta\mathbf{t}\right)$$

$$= 1 + i\vartheta\mathrm{Tr}(\mathbf{t}) + O(\vartheta^2) \overset{!}{=} 1$$

$$\boxed{Tr(\mathbf{t}) = 0}$$

- Dimension of tangential space:

$$\begin{pmatrix} * & * & * & * & * \\ * & * & * & * & \\ * & * & * & & \\ * & * & & & \\ * & & & & \end{pmatrix}$$

- n real entries in diagonal.
- $1/2 \cdot n(n-1)$ complex entries in off-diagonal.
- -1 for $SU(n)$ for det req.

> - $U(n)$ has n^2 generators.
> - $SU(n)$ has (n^2-1) generators.

Examples that appear in the SM ($U(1)$)

- **$U(1)$ Transformations (equivalent to $O(2)$):**
  - Number of generators: $1^2 \quad = 1$ **NB:** what is the Generator?

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- $U(1)$ Transformations (equivalent to $O(2)$):
  - Number of generators: $1^2 \quad = 1$ **NB:** what is the Generator? ⟶ The generator is 1.

Examples that appear in the SM ($SU(2)$)

- $SU(2)$ Transformations (equivalent to $O(3)$):
  - Number of generators: $(2^2-1)=3$ i.e. there are 3 matrices $\{\mathbf{t}_j\}$, which form a basis of traceless hermitian matrices, for which the following relation holds:

$$\mathbf{G} = e^{i\sum_{j=1}^{3}\vartheta_j \mathbf{t}_j}$$

  - Explicit representation:

$$\boxed{\mathbf{t}_j = \frac{1}{2}\sigma_j \qquad (j=1\dots3)}$$

(*3 Pauli Matrices*)

$$\boxed{[\mathbf{t}_i,\mathbf{t}_j] = i\epsilon_{ijk}\mathbf{t}_k}$$

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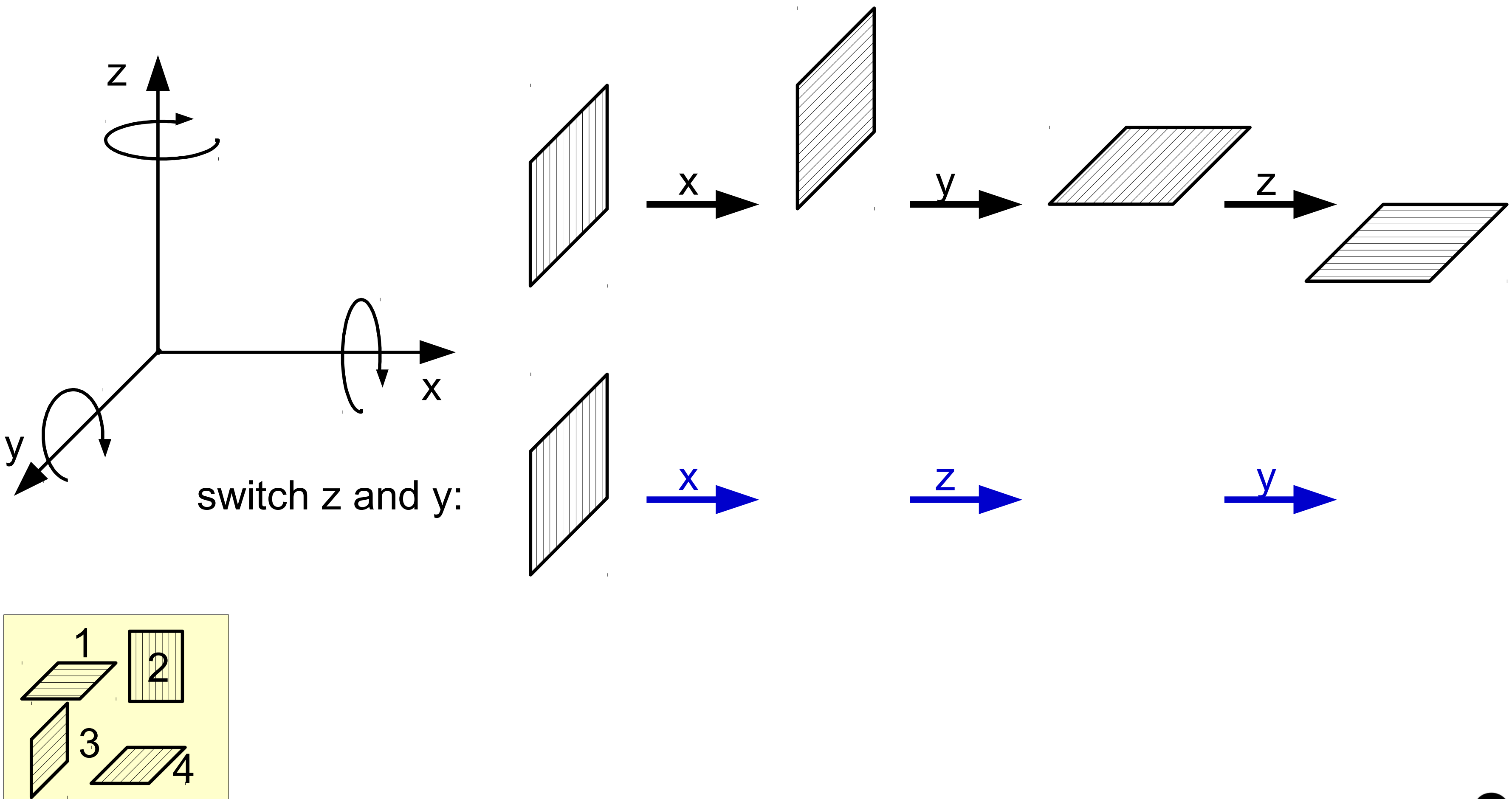
- algebra closes.
- structure constants of $SU(2)$.

Non-Abelian Symmetry Transformations

- Example $O(3)$ (90° rotations in $\mathbb{R}^3$):



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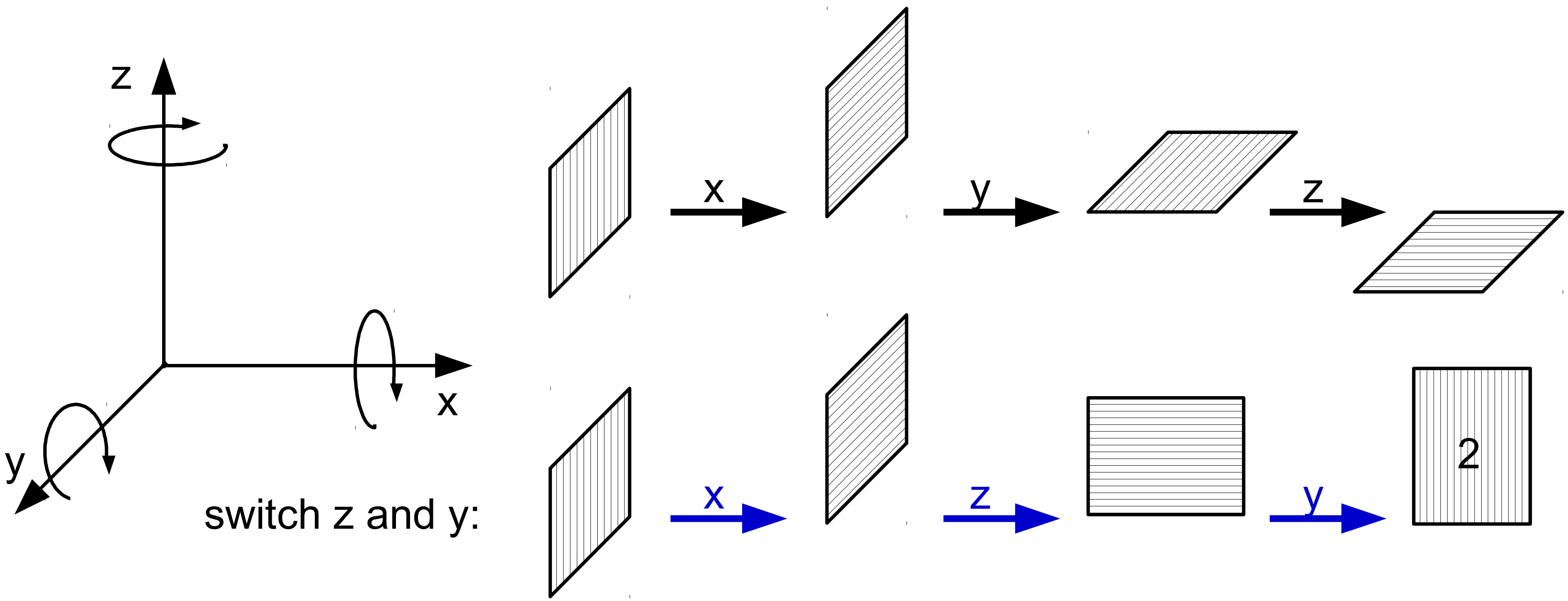

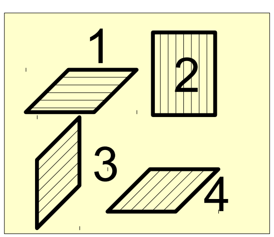

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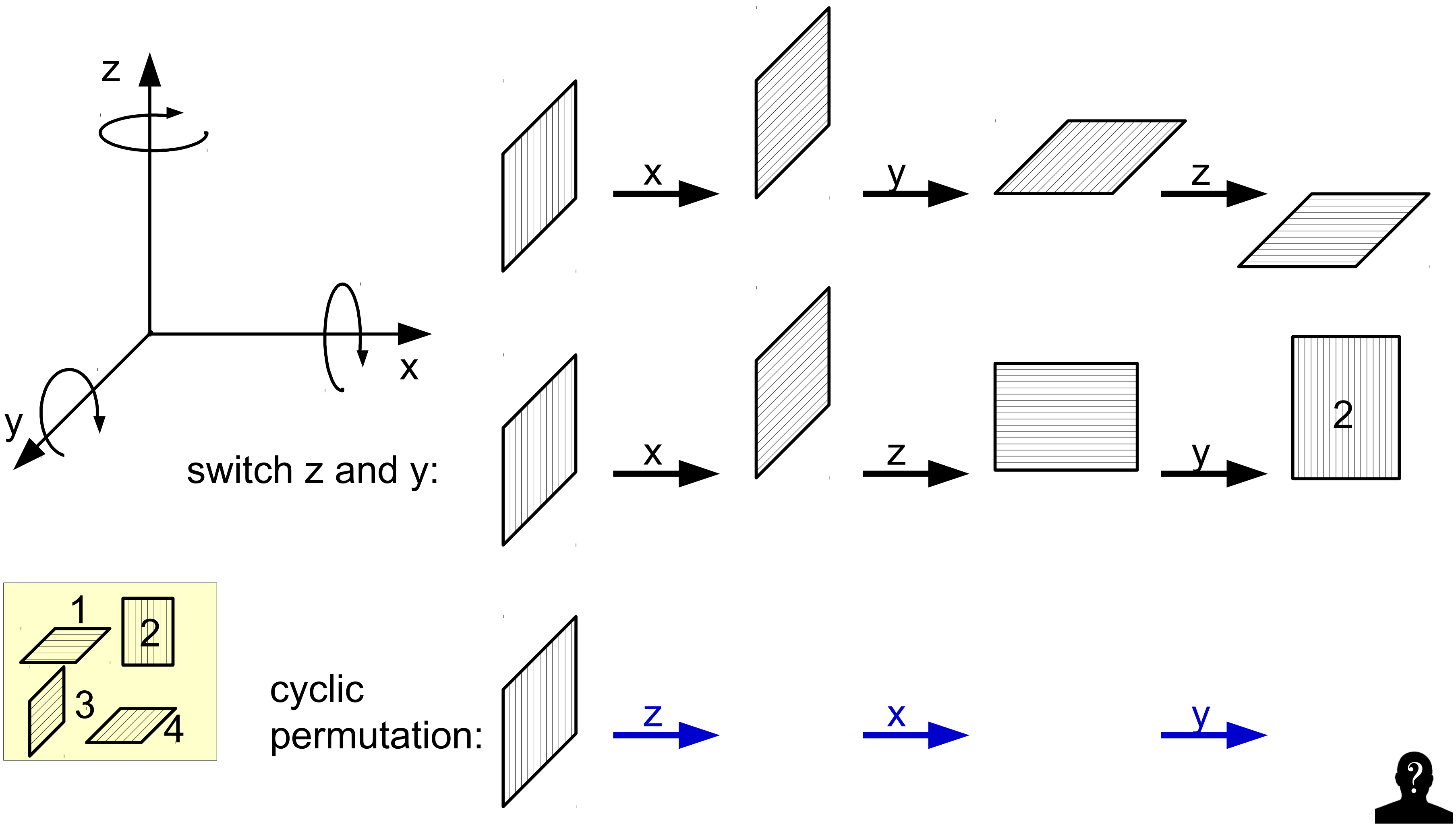

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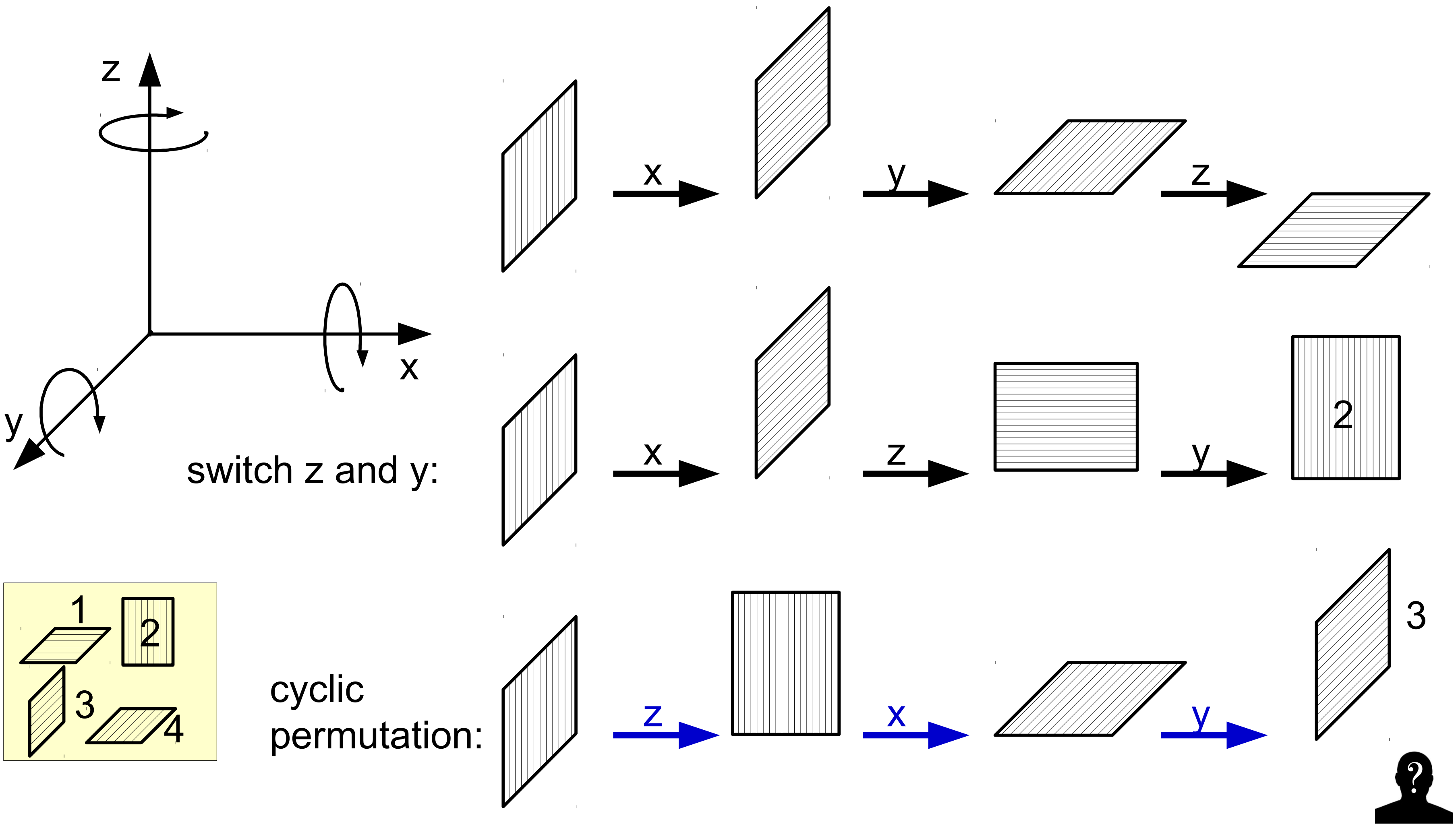

Examples that appear in the SM ($SU(3)$)

- $SU(3)$Transformations (equivalent to $O(4)$):
  - Number of generators: $(3^2 - 1) = 8$ ($\rightarrow$ *8 Gell-Mann Matrices*)

Abelian vs. Non-Abelian Gauge Theories

Abelian:

$$\psi(\vec{x},t) \to \psi'(\vec{x},t) = e^{i\vartheta}\psi(\vec{x},t)$$
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$$\partial_\mu \;\to D_\mu = \partial_\mu \; -ieA_\mu$$
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$$F_{\mu\nu} \equiv [D_\mu, D_\nu] = \partial_\mu A_\nu - \partial_\nu A_\mu$$

$$F_{\mu\nu} \to F'_{\mu\nu} = F_{\mu\nu}$$

$$\mathcal{L} = \overline{\psi}\left(i\gamma^\mu D_\mu - m\right)\psi - \tfrac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

Non-Abelian:

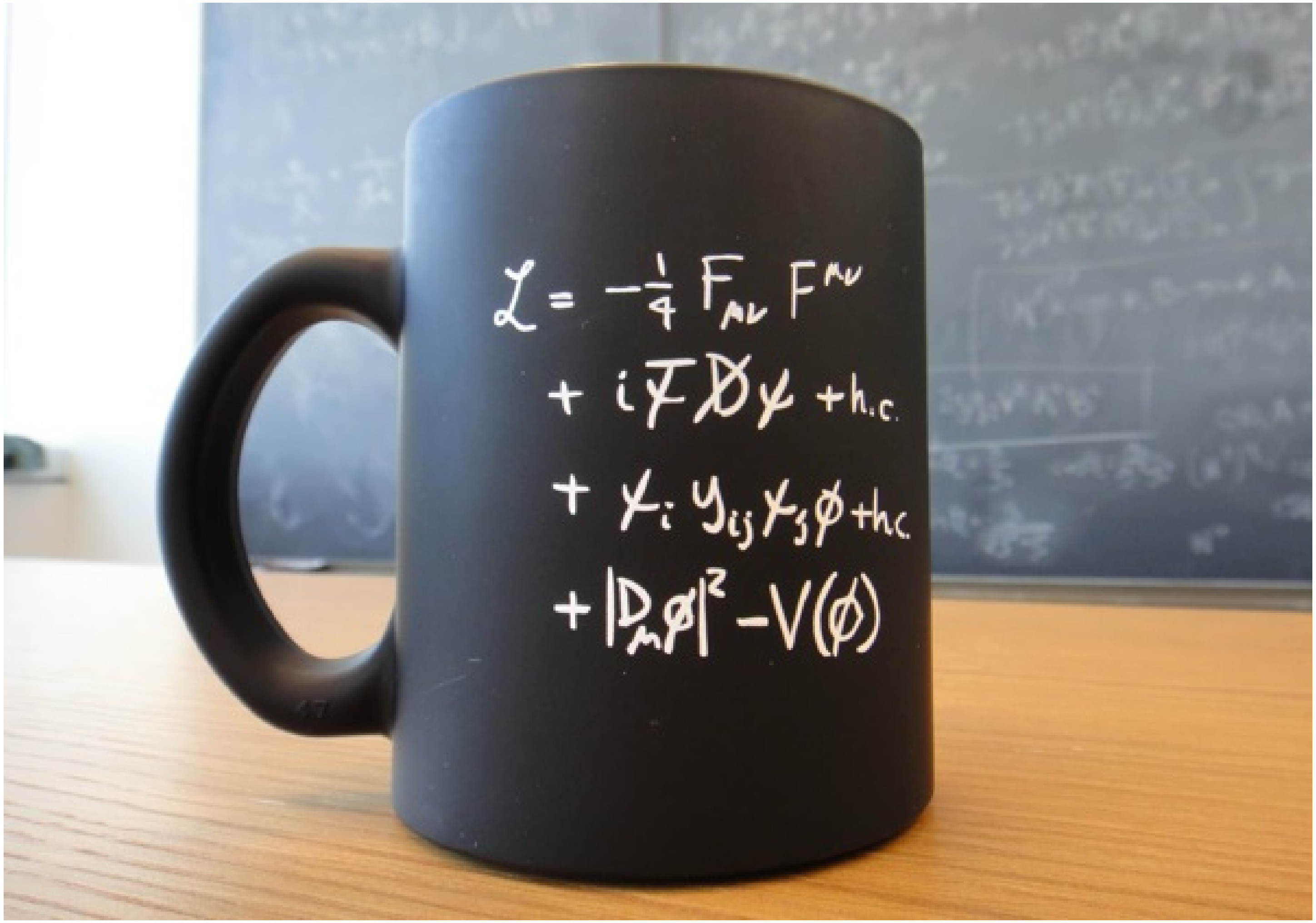
$$\psi(\vec{x},t) \to \psi'(\vec{x},t) = e^{i\vartheta_\mathrm{a}\mathbf{t}_\mathrm{a}}\psi(\vec{x},t)$$
$$\overline{\psi}(\vec{x},t) \to \overline{\psi}'(\vec{x},t) = \overline{\psi}(\vec{x},t)e^{-i\vartheta_\mathrm{a}\mathbf{t}_\mathrm{a}}$$

$$\partial_\mu \;\to D_\mu = \partial_\mu \; - igW_{\mu,\mathrm{a}}\mathbf{t}_\mathrm{a}$$
$$D_\mu \to D'_\mu = D_\mu - i\left[\vartheta_\mathrm{a}\mathbf{t}_\mathrm{a}, D_\mu\right]$$
$$W_\mu \to W'_\mu = W_\mu + i\left[\vartheta_\mathrm{a}\mathbf{t}_\mathrm{a}, W_{\mu,\mathrm{a}}\mathbf{t}_\mathrm{a}\right] + \tfrac{1}{g}\partial_\mu\left(\vartheta_\mathrm{a}\mathbf{t}_\mathrm{a}\right)$$
$$W_{\mu\nu} \equiv [D_\mu, D_\nu] = \partial_\mu W_\nu - \partial_\nu W_\mu - ig\left[W_\mu, W_\nu\right]$$

$$W_{\mu\nu} \to W'_{\mu\nu} = W_{\mu\nu} - i\left[\vartheta_\mathrm{a}\mathbf{t}_\mathrm{a}, W_{\mu\nu}\right]$$

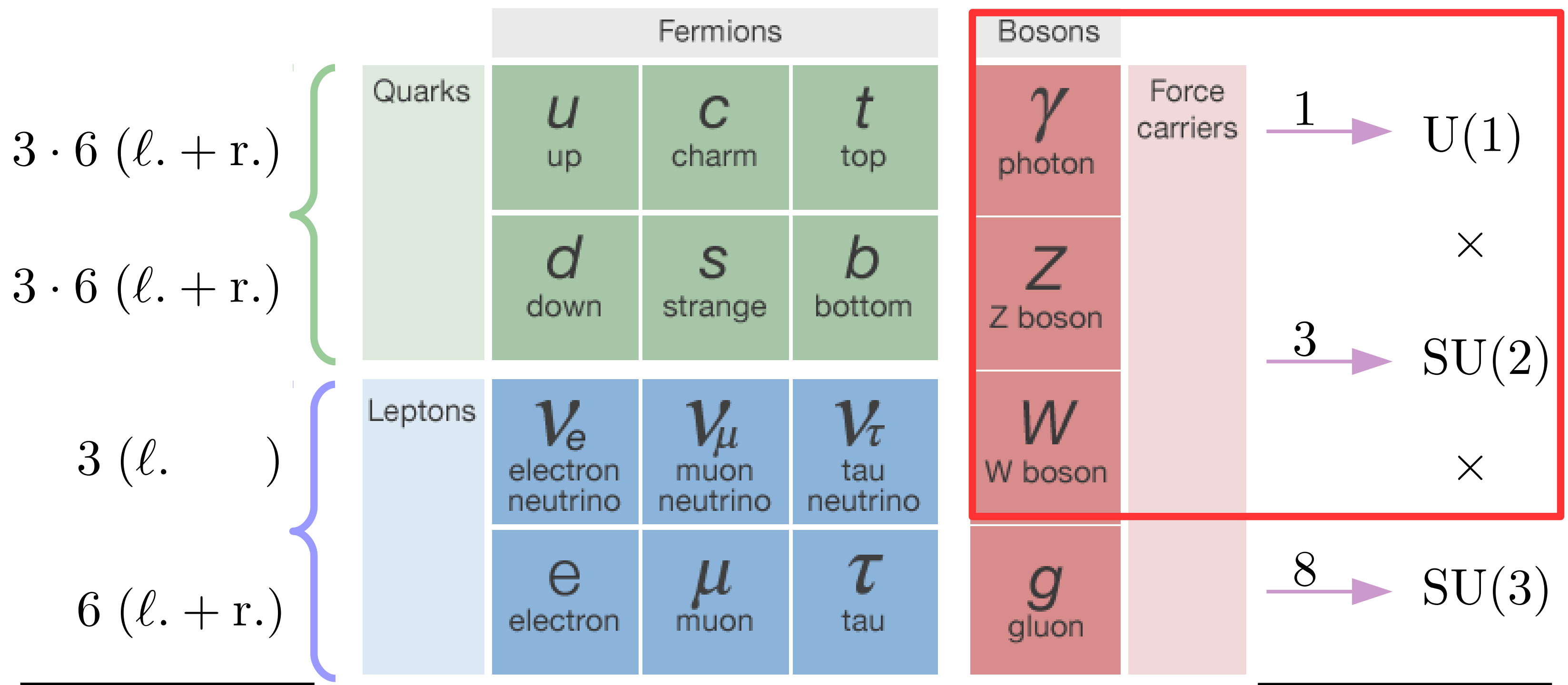
$$\mathcal{L} = \overline{\psi}\left(i\gamma^\mu D_\mu - m\right)\psi - \tfrac{1}{4}W_{\mathrm{a}\mu\nu}W^{\mathrm{a}\mu\nu}$$

The SM of Particle Physics



Constituents and Interactions of the SM

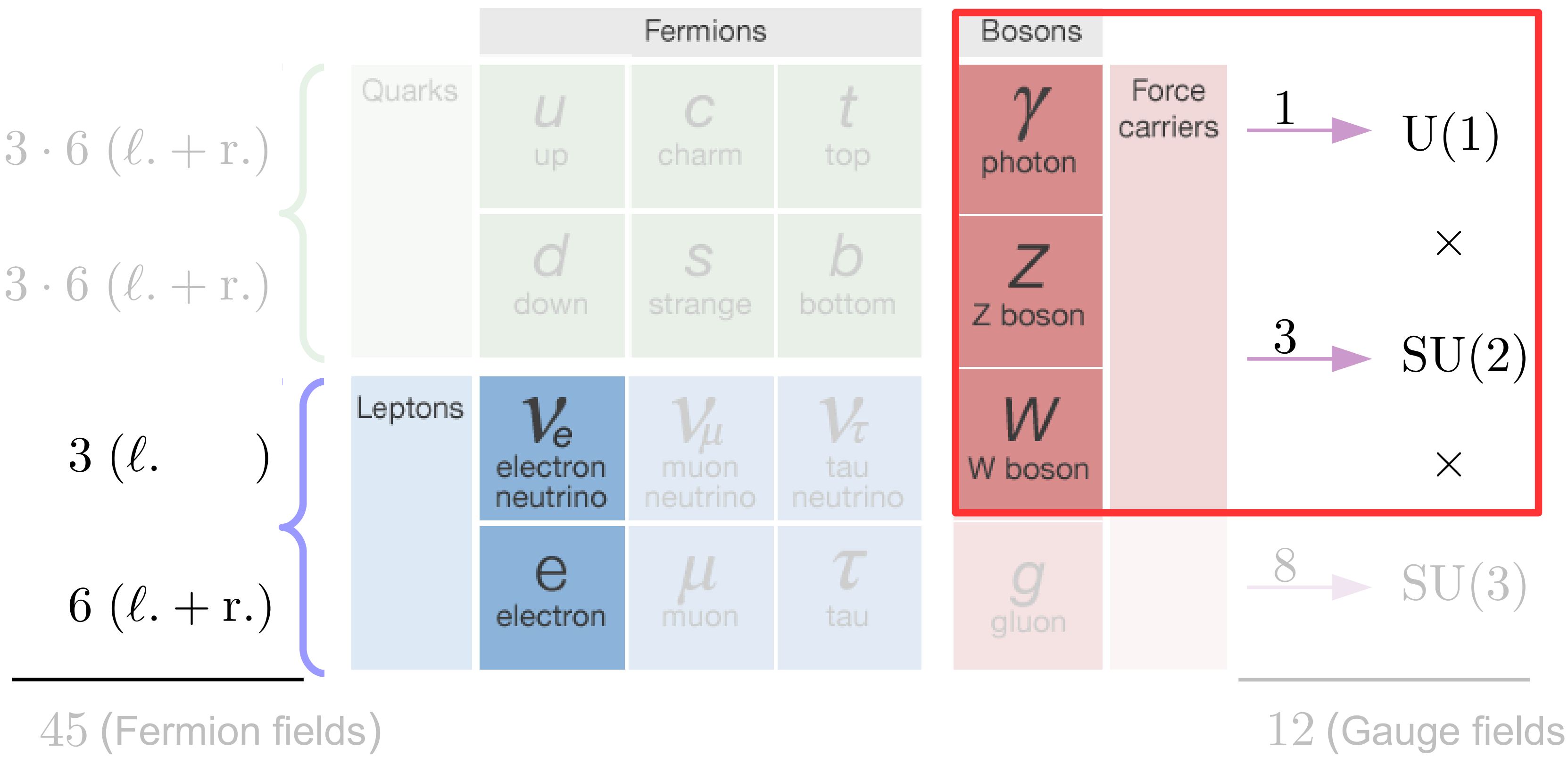
18 free parameters



45 (Fermion fields)

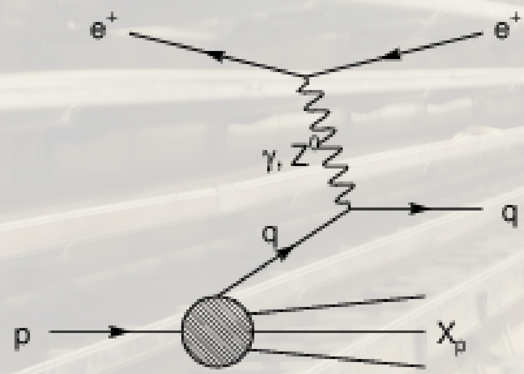
12 (Gauge fields)

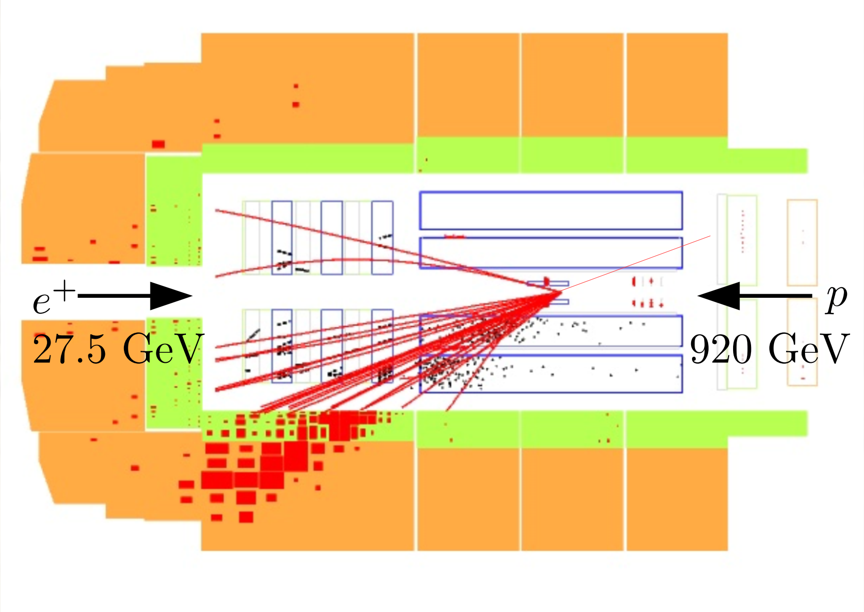
Constituents and Interactions of the SM



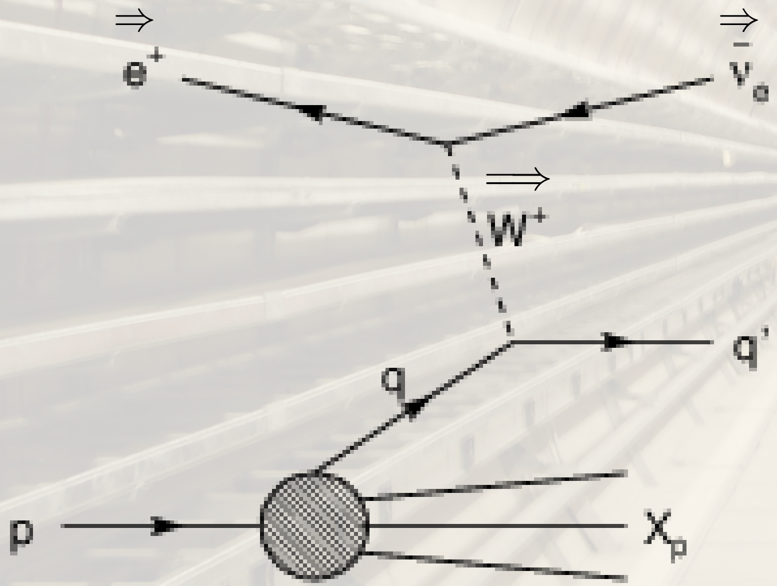
Phenomenology of Weak Interaction

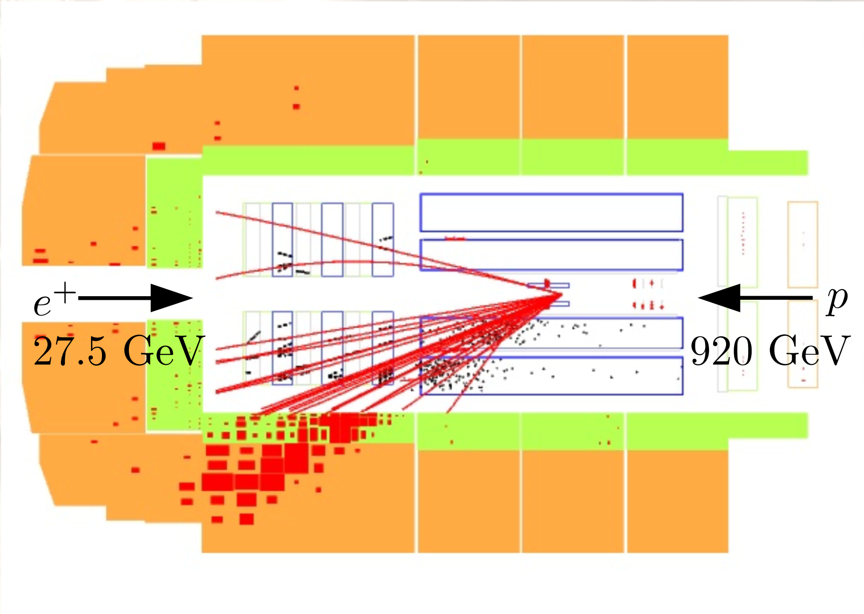
- From the view of a high energy physics scattering experiment:

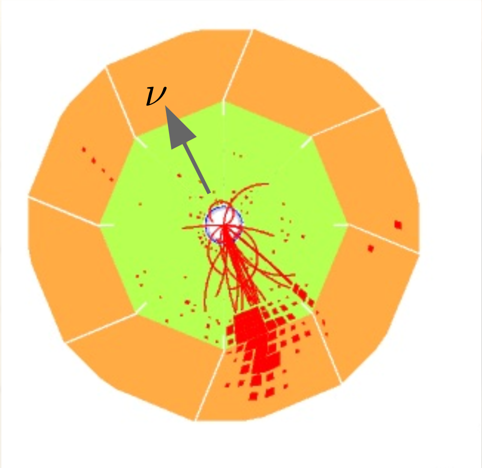




Change of Flavor & Charge

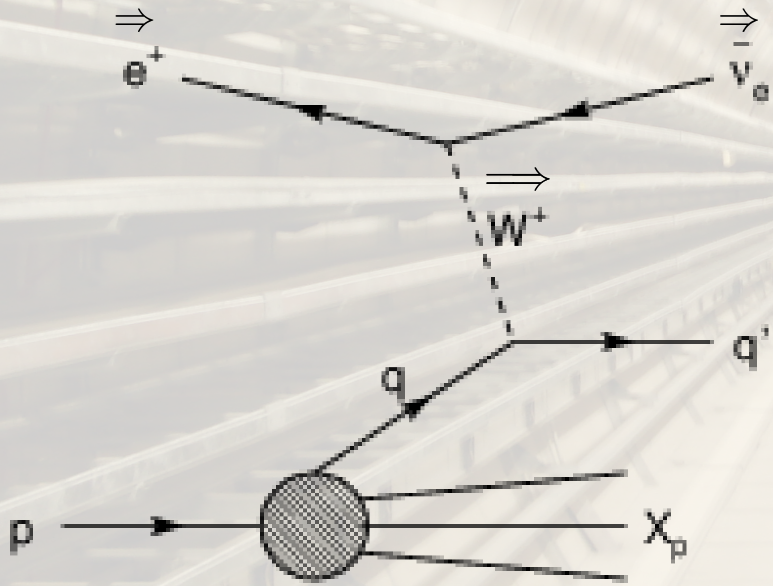


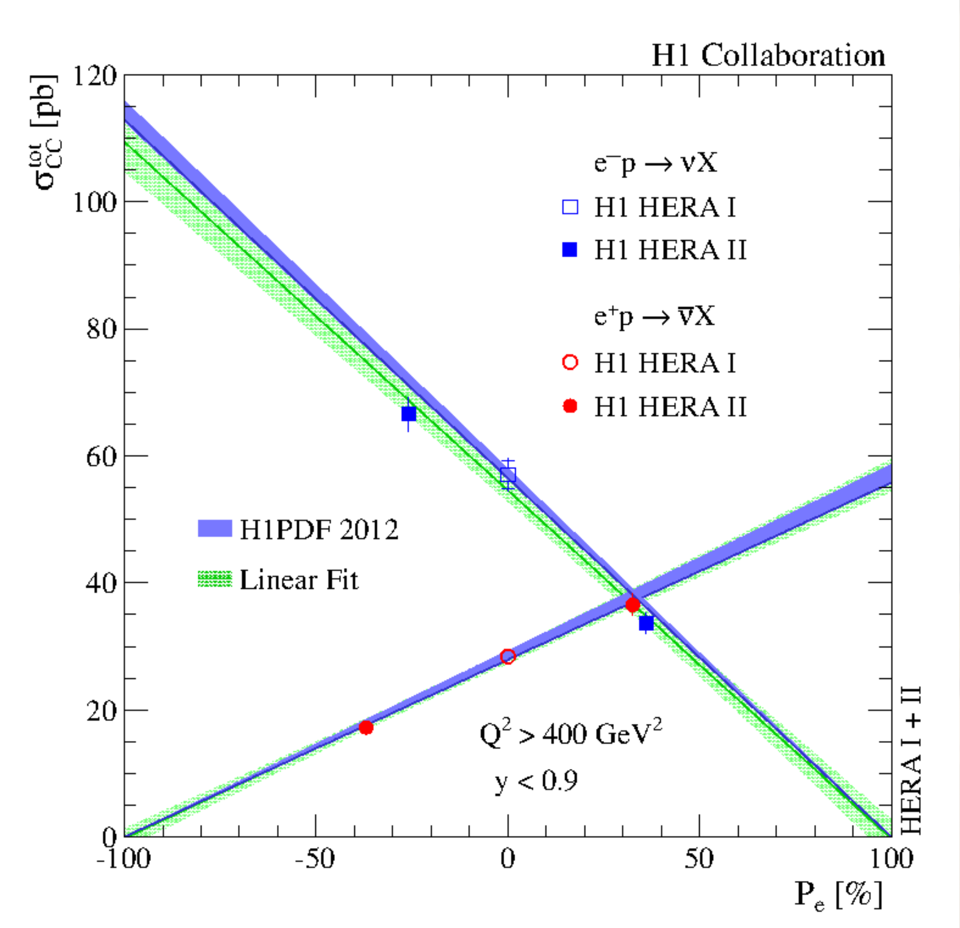




Parity Violation

- W bosons couple only to left-handed particles (right-handed anti-particles):

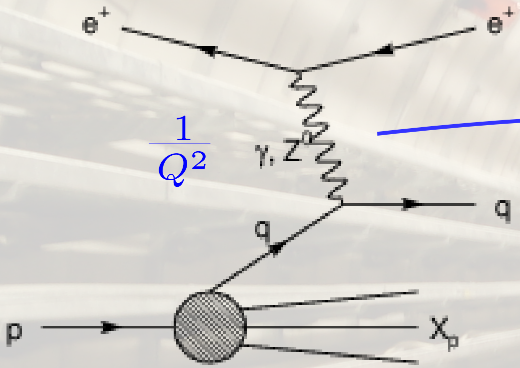




- Maximally parity violating!
- Intrinsically violating CP as well!

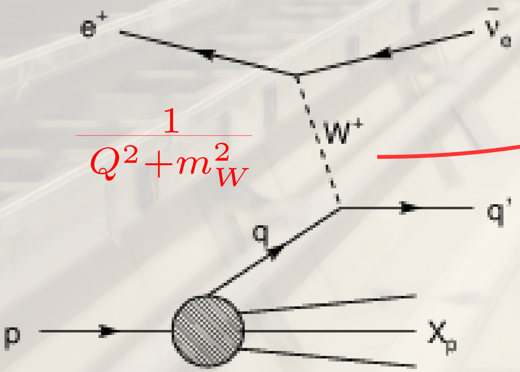
Heavy Mediators

- Mediation by heavy gauge bosons:



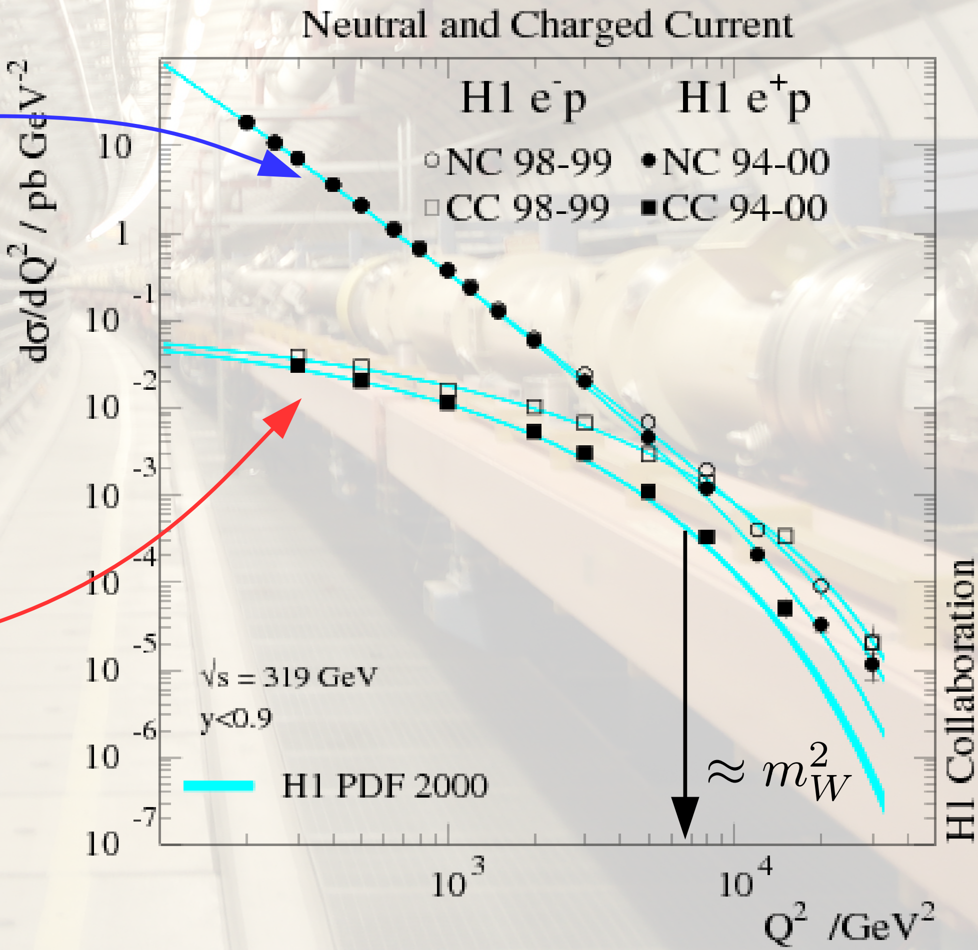
$\frac{1}{Q^2}$

$m_\gamma = 0$



$\frac{1}{Q^2+m_W^2}$

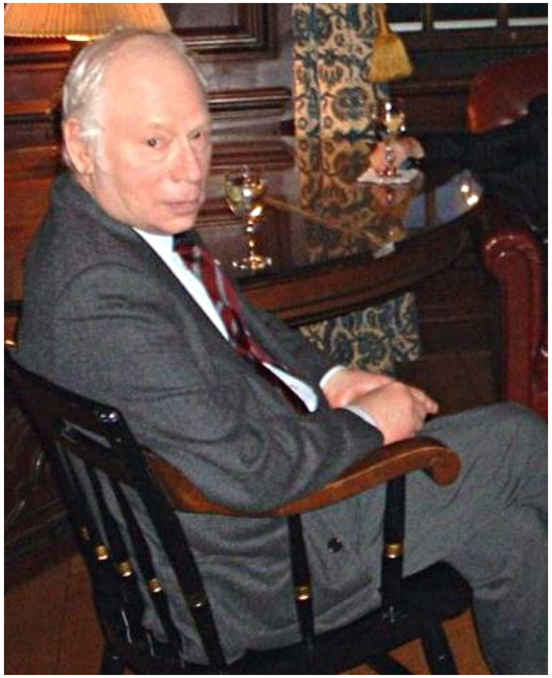
$m_W = 85.385 \pm 0.015$ GeV



$\approx m_W^2$

The Model of Weak Interactions

Sheldon Glashow
(*5. December 1932)



Steven Weinberg
(*3. Mai 1933)

$SU(2)$ Space of Weak Isospin

- Example:

Transforms like a spin ½ object in space of weak isospin.

$\psi_L = \begin{pmatrix} \nu \\ e \end{pmatrix}_L$ $\longrightarrow$ • *left-handed* e_L & ν form *isospin doublet*.

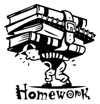
e_R $\longrightarrow$ • *right-handed* e_R forms *isospin singlet*.

- *Left- & right-handed* components of fermions can be projected conveniently:

$$e = e_L + e_R \quad \begin{cases} e_L = \left(\frac{1-\gamma^5}{2}\right) e \\ e_R = \left(\frac{1+\gamma^5}{2}\right) e \end{cases} \qquad \overline{e}\gamma^\mu \left(\frac{1-\gamma^5}{2}\right) \nu = \overline{e}_L \gamma^\mu \nu_L$$

- Lagrangian w/o mass terms can be written in form:

$$\boxed{\mathcal{L}_0 = \overline{\psi}_L \gamma^\mu \partial_\mu \psi_L + \overline{e}_R \gamma^\mu \partial_\mu e_R} = \overline{e}_L \gamma^\mu \partial_\mu e_L + \overline{\nu} \gamma^\mu \partial_\mu \nu + \overline{e}_R \gamma^\mu \partial_\mu e_R$$



Covariant Derivative of $SU(2) \times U(1)$

Covariant derivative corresponding to $SU(2)$ acts on *isospin doublet* only.[1)]

$$\mathcal{L}_{IA}^{SU(2)\times U(1)} = \underbrace{\bar{\psi}_L \gamma^\mu \boxed{\left(\partial_\mu \qquad\qquad + ig W_\mu^{\mathrm{a}} \mathbf{t}^{\mathrm{a}}\right)} \psi_L}\cdots$$

1) Note a different sign convention.

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$$\mathbf{t}^+ = \mathbf{t}_1 + i\,\mathbf{t}_2 = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$$ (ascending operator)

$$\mathbf{t}^- = \mathbf{t}_1 - i\,\mathbf{t}_2 = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$$ (descending operator)

$$\mathbf{t}^3 = {}^1\!/_2 \cdot \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$W_\mu^a \mathbf{t}^a = \frac{1}{\sqrt{2}}\left(W_\mu^+ \mathbf{t}^+ + W_\mu^- \mathbf{t}^-\right) + W_\mu^3 \mathbf{t}^3$$

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Covariant Derivative of $SU(2) \times U(1)$

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Covariant derivative corresponding to $U(1)$ acts on *isospin doublet* (as a whole) and on *isospin singlet.*

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$SU(2) \times U(1)$ Hypercharges			
Particle	$Y_{R/L}$	I_3	Q
ν	-1	$+1/2$	
e_L	-1	$-1/2$	
e_R		0	-1

$Q = I_3 + \frac{Y}{2}$ (Gell-Mann Nischijama)

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$SU(2) \times U(1)$ Hypercharges			
Particle	$Y_{R/L}$	I_3	Q
ν	-1	$+1/2$	0
e_L	-1	$-1/2$	-1
e_R	-2	0	-1

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1) Note a different sign convention.

$SU(2) \times U(1)$ Interactions

Karlsruhe Institute of Technology

- Charged current interaction:

$$\mathcal{L}_{IA}^{CC} = -\frac{g}{\sqrt{2}}\left[\underbrace{\overline{\nu}\left(W_\mu^+\gamma^\mu\right)e_L}_{e \to \nu} + \underbrace{\overline{e}_L\left(W_\mu^-\gamma^\mu\right)\nu}_{\nu \to e}\right]$$

from t^+

from t^-

- Neutral current interaction:

$$\mathcal{L}_{IA}^{NC} = -\underbrace{\left(\frac{g}{2}W_\mu^3 - \frac{g'}{2}B_\mu\right)}_{\propto Z_\mu}(\overline{\nu}\gamma^\mu\nu) + \left(\frac{g}{2}W_\mu^3 + \frac{g'}{2}B_\mu\right)(\overline{e}_L\gamma^\mu e_L) + \frac{g'}{2}B_\mu(\overline{e}_R\gamma^\mu e_R)$$

from t^3

$SU(2) \times U(1)$ Interactions

- Charged current interaction:

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from t^+

from t^-

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from t^3

$$\begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta_W & -\sin\theta_W \\ \sin\theta_W & \cos\theta_W \end{pmatrix}\begin{pmatrix} W_\mu^3 \\ B_\mu \end{pmatrix}$$

$$\sin\theta_W = \frac{g'}{\sqrt{g^2+g'^2}} \quad \cos\theta_W = \frac{g}{\sqrt{g^2+g'^2}}$$

(Weinberg Rotation)

$SU(2) \times U(1)$ Interactions

- Charged current interaction:

$$\mathcal{L}_{IA}^{CC} = -\frac{g}{\sqrt{2}}\left[\underbrace{\overline{\nu}\left(W_\mu^+\gamma^\mu\right)e_L}_{e\,\to\,\nu} + \underbrace{\overline{e}_L\left(W_\mu^-\gamma^\mu\right)\nu}_{\nu\,\to\,e}\right]$$

Desired behavior: A_μ couples to left- and right handed component of e in the same way!

- Neutral current interaction:

$$\begin{aligned}\mathcal{L}_{IA}^{NC} = &-\frac{\sqrt{g^2+g'^2}}{2}Z_\mu\left(\overline{\nu}\gamma_\mu\nu\right)\\ &+\frac{\sqrt{g^2+g'^2}}{2}\left[\left(\cos^2\theta_W-\sin^2\theta_W\right)Z_\mu+2\sin\theta_W\cos\theta_W A_\mu\right]\left(\overline{e}_L\gamma_\mu e_L\right)\\ &+\frac{\sqrt{g^2+g'^2}}{2}\left[-2\sin^2\theta_W Z_\mu+2\sin\theta_W\cos\theta_W A_\mu\right]\left(\overline{e}_R\gamma_\mu e_R\right)\end{aligned}$$

What is the expression for e ?

$SU(2) \times U(1)$ Interactions

- Charged current interaction:

$$\mathcal{L}_{IA}^{CC} = -\frac{g}{\sqrt{2}}\left[\underbrace{\overline{\nu}\left(W_\mu^+\gamma^\mu\right)e_L}_{e\,\to\,\nu} + \underbrace{\overline{e}_L\left(W_\mu^-\gamma^\mu\right)\nu}_{\nu\,\to\,e}\right]$$

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What is the expression for e ? $\longrightarrow$ $e = \sqrt{g^2+g'^2}\sin\theta_W\cos\theta_W$

NB: Skewness of the $SU(2) \times U(1)$

- Gauge boson *eigenstates of the symmetry* do not correspond to the *eigenstates* of the IA:

$$\begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta_W & -\sin\theta_W \\ \sin\theta_W & \cos\theta_W \end{pmatrix} \begin{pmatrix} W_\mu^3 \\ B_\mu \end{pmatrix}$$

- Quark *eigenstates* of the $SU(2)$ do not correspond to the quark *eigen-states* of the $SU(3)$ (NB: which are the mass *eigenstates*):

$$\mathcal{M}_{\mathrm{CKM}} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

$$= \begin{pmatrix} c_1 & s_1 c_3 & s_1 s_3 \\ -s_1 c_2 & c_1 c_2 c_3 - s_2 s_3 e^{i\delta} & c_1 c_2 s_3 + s_2 c_3 e^{i\delta} \\ -s_1 s_2 & c_1 s_2 c_3 + c_2 s_3 e^{i\delta} & c_1 s_2 s_3 - c_2 c_3 e^{i\delta} \end{pmatrix}$$

$$c_i = \cos\vartheta_i \; ; \; s_i = \sin\vartheta_i \; (i = 1...3)$$

Non-Abelian Gauge Structure of $SU(2)$

$$\mathcal{L}^{\text{gauge}} = -\frac{1}{2}\mathrm{Tr}\left(W^a_{\mu\nu}W^{a\mu\nu}\right) - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} \Bigg| \quad \begin{array}{l} B_\mu \to A_\mu \\ W^3_\mu \to Z_\mu \end{array}$$

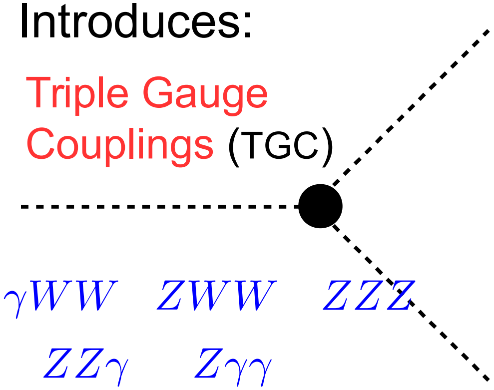
$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$$

$$W_{\mu\nu} = \partial_\mu W^a_\nu - \partial_\nu W^a_\mu + ig\left[W^a_\mu, W^a_\nu\right]$$

- Implies lepton universality of weak interaction.
  (→extensively tested @ LEP)

- Introduces:

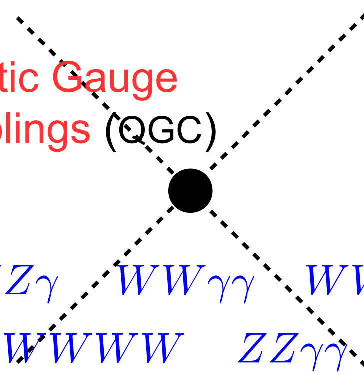
Triple Gauge Couplings (TGC)



γWW $\quad ZWW$ $\quad ZZZ$

$ZZ\gamma$ $\quad Z\gamma\gamma$

Quartic Gauge Couplings (QGC)



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Which couplings are allowed (at tree level), which are not?

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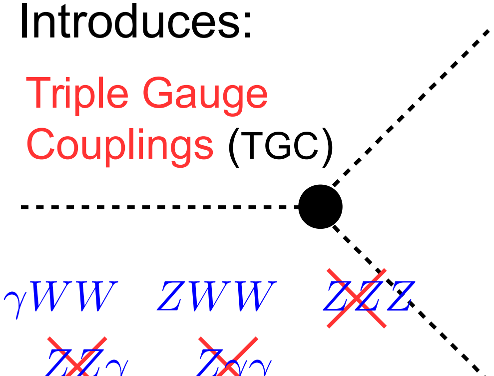

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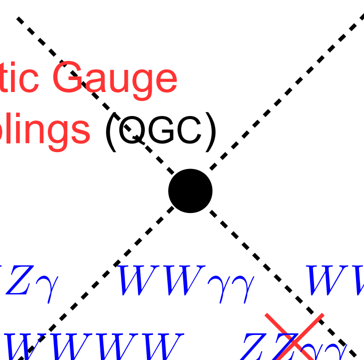

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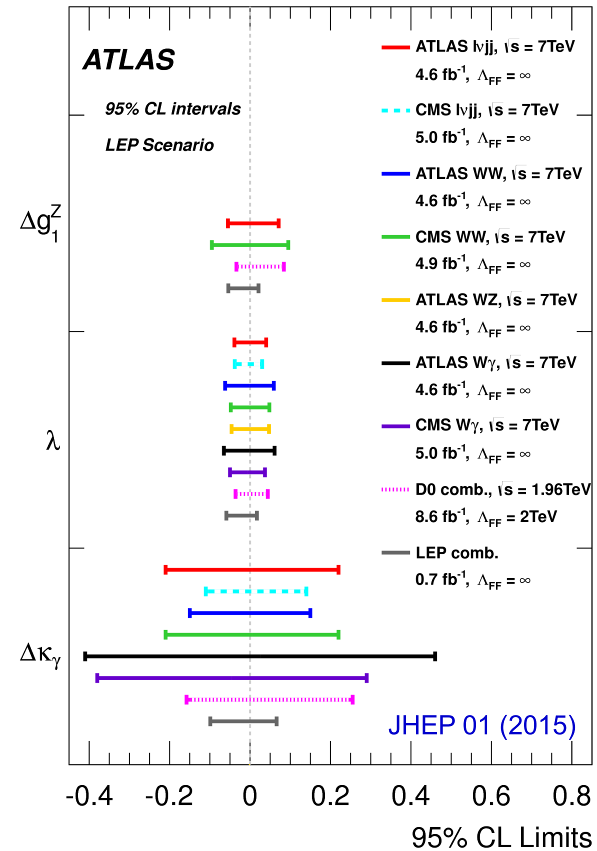

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Concluding Remarks

Karlsruhe Institute of Technology

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  - No "*EWK symmetry* conservation"!
  - ...

Sneak Preview for Next Week

- Up to now the problem of mass has been completely ignored.
- Discuss how mass terms in the Lagrangian density will compromise local gauge symmetries.
- Discuss the dynamic generation of mass via spontaneous symmetry breaking.

Backup & Homework Solutions