# Electroweak Symmetry Breaking and the Higgs Mechanism

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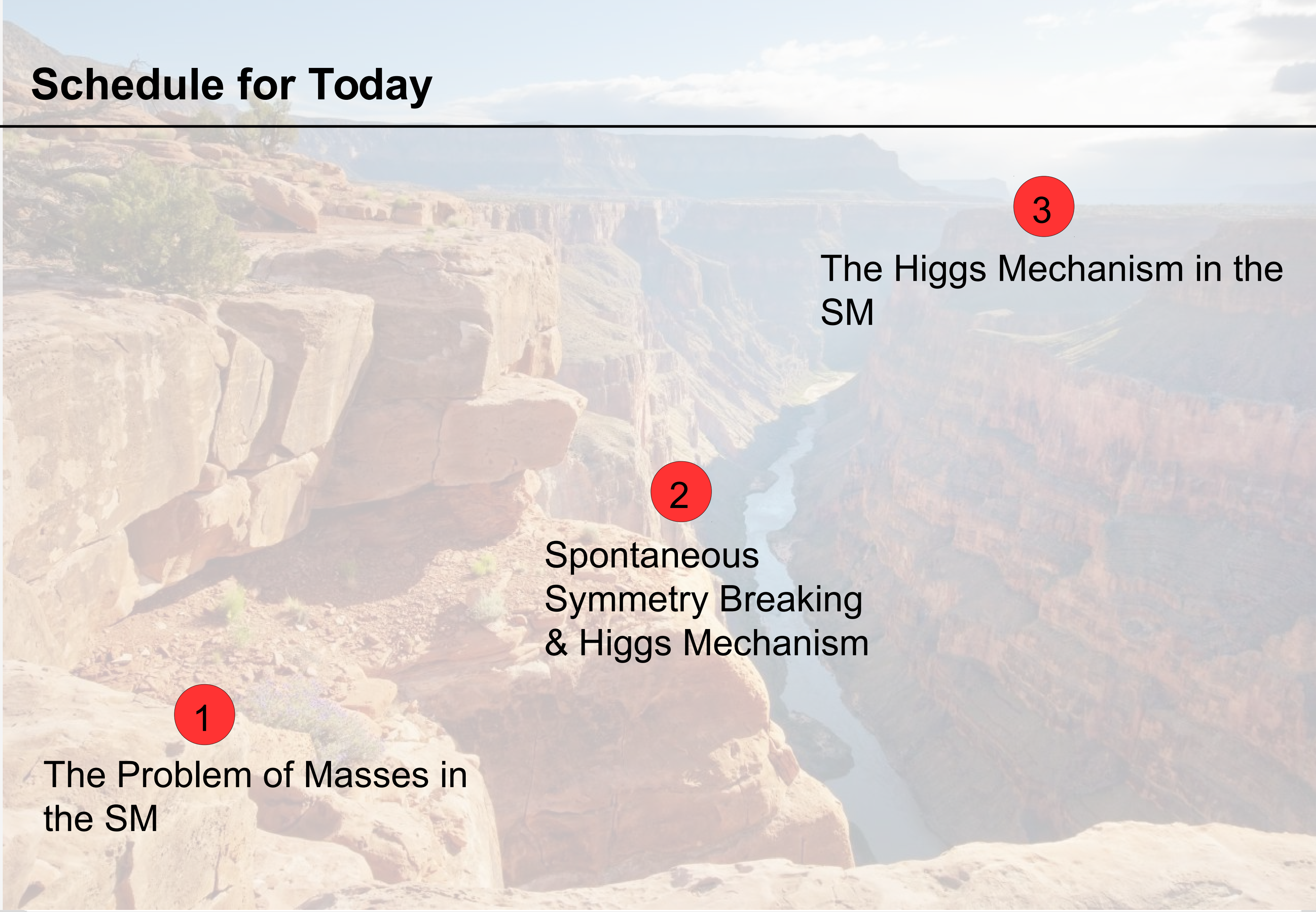
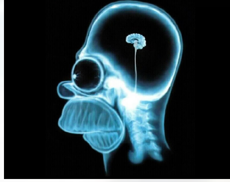
INSTITUTE OF EXPERIMENTAL PARTICLE PHYSICS (IEKP) – PHYSICS FACULTY

# Schedule for Today



1. The Problem of Masses in the SM
2. Spontaneous Symmetry Breaking & Higgs Mechanism
3. The Higgs Mechanism in the SM

# Quiz of the Day



- Is the mass problem the same for bosons and fermions?
- Is the following statement true: “the Higgs boson couples proportional to the mass to all massive particles”?
- We have seen that QED does not at all have a problem with fermion masses (c.f. first lecture). Are fermion masses a problem specific to non-Abelian gauge symmetries?
- Is the Higgs boson a *Goldstone* boson?

# Recap from Last Time

- Charged current interaction:

$$\mathcal{L}_{IA}^{CC} = -\frac{g}{\sqrt{2}}\left[\underbrace{\overline{\nu}\left(W_\mu^+\gamma^\mu\right)e_L}_{e\to\nu} + \underbrace{\overline{e}_L\left(W_\mu^-\gamma^\mu\right)\nu}_{\nu\to e}\right]$$

- Neutral current interaction:

$$\begin{aligned}\mathcal{L}_{IA}^{NC} = &-\frac{\sqrt{g^2+g'^2}}{2}Z_\mu\left(\overline{\nu}\gamma_\mu\nu\right)\\ &+\frac{\sqrt{g^2+g'^2}}{2}\left[\left(\cos^2\theta_W-\sin^2\theta_W\right)Z_\mu+2\sin\theta_W\cos\theta_W A_\mu\right]\left(\overline{e}_L\gamma_\mu e_L\right)\\ &+\frac{\sqrt{g^2+g'^2}}{2}\left[\qquad\qquad -2\sin^2\theta_W\,Z_\mu+2\sin\theta_W\cos\theta_W A_\mu\right]\left(\overline{e}_R\gamma_\mu e_R\right)\end{aligned}$$

- $SU(2)\times U(1)$ can describe electroweak IA including gauge boson self-couplings.

# The Problem of Masses in the SM

# Problem 1: Massive Gauge Bosons

- Example: Abelian gauge field theories (→ first lecture)

- Transformation: $A_\mu \to A'_\mu = A_\mu + \frac{1}{e}\partial_\mu \vartheta$

- In mass term : $m_A A_\mu A^{\mu *} \to m_A A'_\mu A'^{\mu *} =$

$$m_A A_\mu A^{\mu *} + \underbrace{\tfrac{1}{e} m_A \left(A_\mu \partial^\mu \vartheta + A^{\mu *} \partial_\mu \vartheta\right) + m_A \tfrac{1}{e^2} \partial_\mu \vartheta \partial^\mu \vartheta}$$

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These terms explicitly break local gauge covariance of $\mathcal{L}$.

- Fundamental problem for all gauge field theories!

# Problem 2: Massive Fermions

- No problem in $U(1)$ :

- Transformation: $\psi \rightarrow \psi' = e^{i\vartheta} \; \psi$

  $\overline{\psi} \rightarrow \overline{\psi}' = \overline{\psi} e^{-i\vartheta}$

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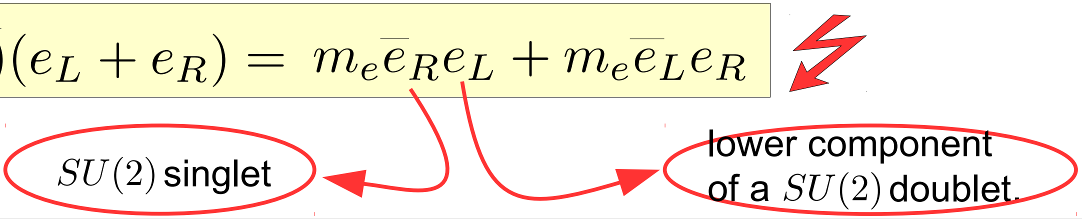
- So what is the problem of the $SU(2)$ in the SM?!

  It is the distinction between left-handed ( $\psi_L$ ) and right-handed ( $\psi_R$ ) fermions:

$$m_e \bar{e} e = m_e \overline{(e_L + e_R)}(e_L + e_R) = m_e \bar{e}_R e_L + m_e \bar{e}_L e_R$$



$SU(2)$ singlet

lower component of a $SU(2)$ doublet.

# Dilemma of the $SU(2) \times U(1)$ Gauge Field Theory

- **Local gauge invariance...**
  - ... can motivate all interactions between elementary particles.
  - ... gives a geometrical interpretation for the presence of gauge bosons (propagate info of local phases btw space points).
  - ... predicts non trivial self-interactions between gauge bosons!

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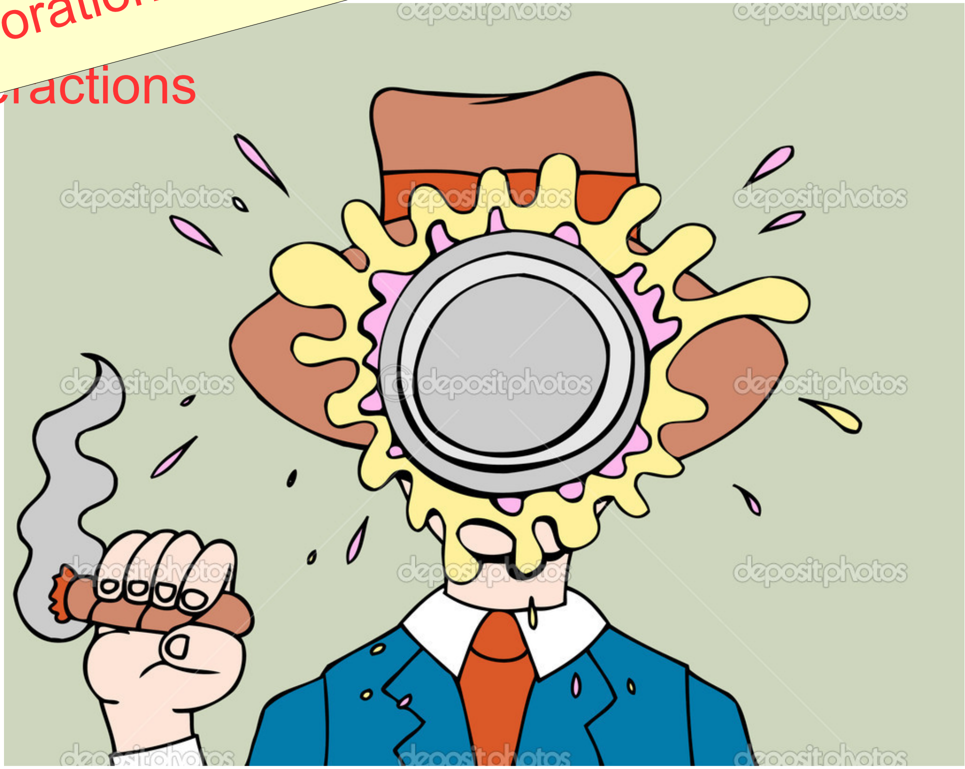
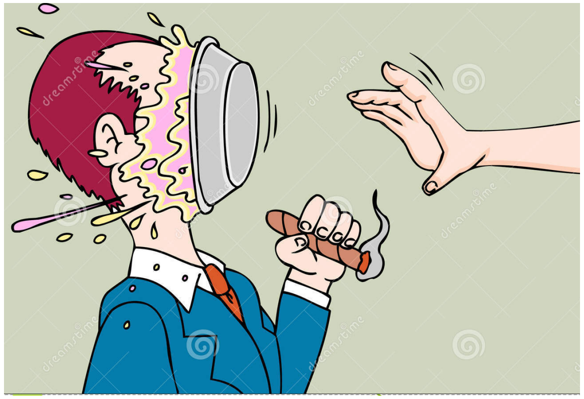
Incompatible with incorporation of naive mass terms in $\mathcal{L}$!





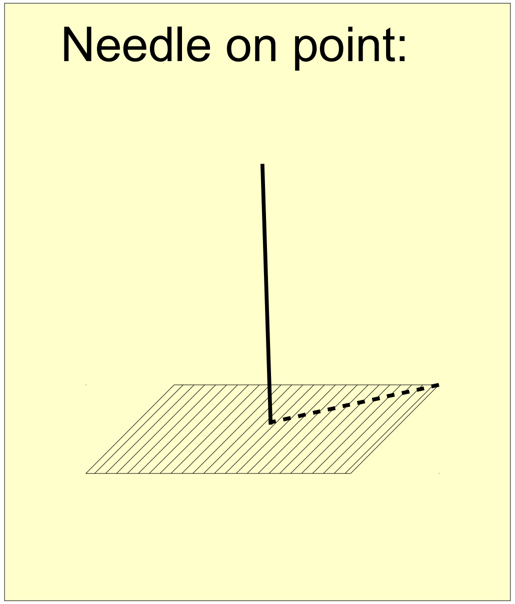
# The Remedy


Bach
ORIGINAL
FLOWER REMEDIES
RESCUE
REMEDY
TO COMFORT
& REASSURE
10ml
RESCUE

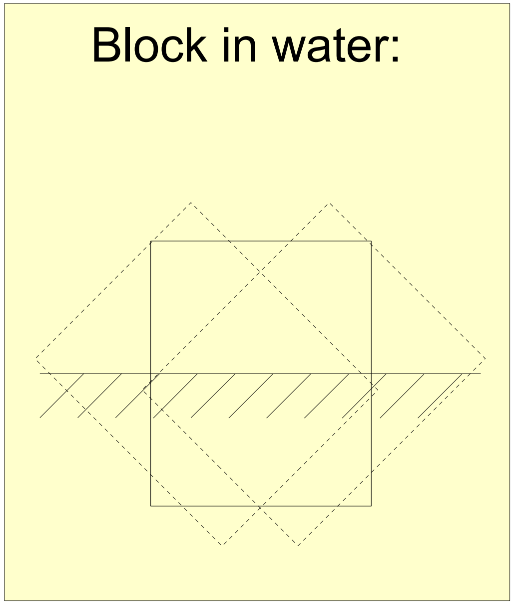

NDC# 057687-245-50
Bach
ORIGINAL
FLOWER REMEDIES
RESCUE
REMEDY
PASTILLES
NATURAL
STRESS RELIEF
ALCOHOL FREE
SUGAR FREE
50g PASTILLES
(1.7 oz)
A Homeopathic Remedy
RESCUE

# Spontaneous Symmetry Breaking

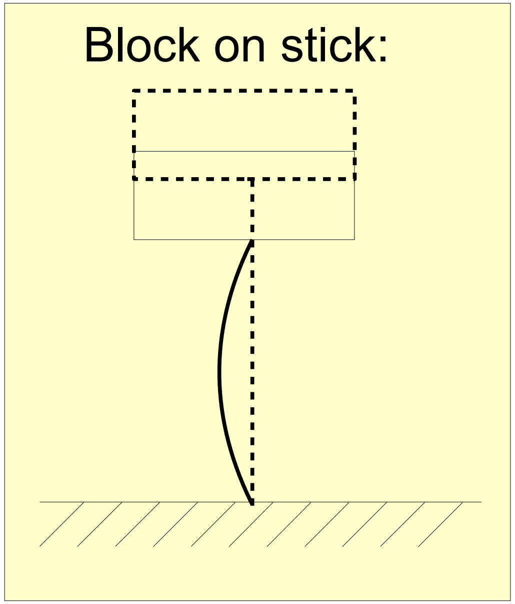
- Symmetry is present in the system (i.e. in the Lagrangian density $\mathcal{L}$ ).
- BUT it is broken in the ground state (i.e. in the quantum vacuum).
- Three examples (from classical mechanics):


Needle on point:

$\varphi$ symmetry


Block in water:

axis-symmetry

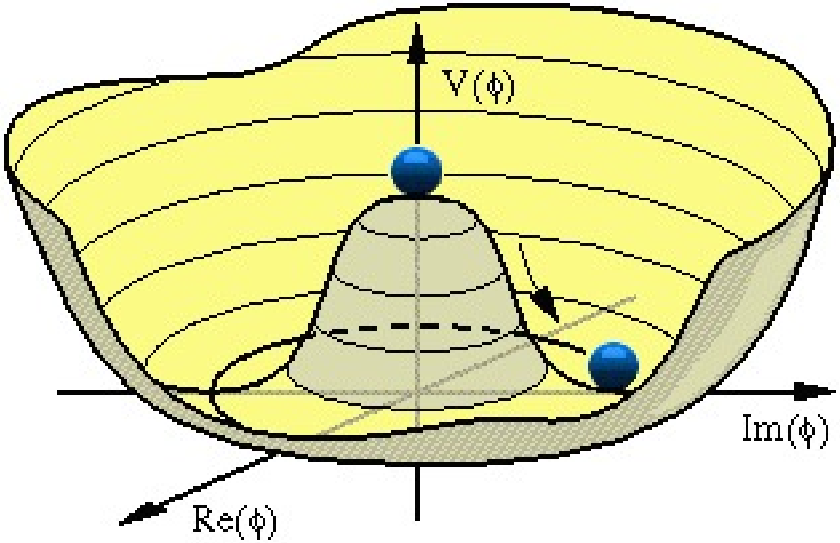

Block on stick:

$\varphi$ symmetry

# Application to Particle Physics

- *Goldstone* Potential:

$$\begin{aligned} \phi &= \frac{1}{\sqrt{2}}(\phi_1 + i\phi_2) \\ V(\phi) &= -\mu^2|\phi|^2 + \lambda|\phi|^4 \\ \mathcal{L}(\phi) &= \partial_\mu\phi\partial^\mu\phi^* - V(\phi) \end{aligned}$$



- invariant under $U(1)$ transformations (i.e. $\varphi$ symmetric).
- metastable in $\phi = 0$.
- ground state breaks $U(1)$ symmetry, BUT at the same time all ground states are in-distinguishable in $\varphi$.

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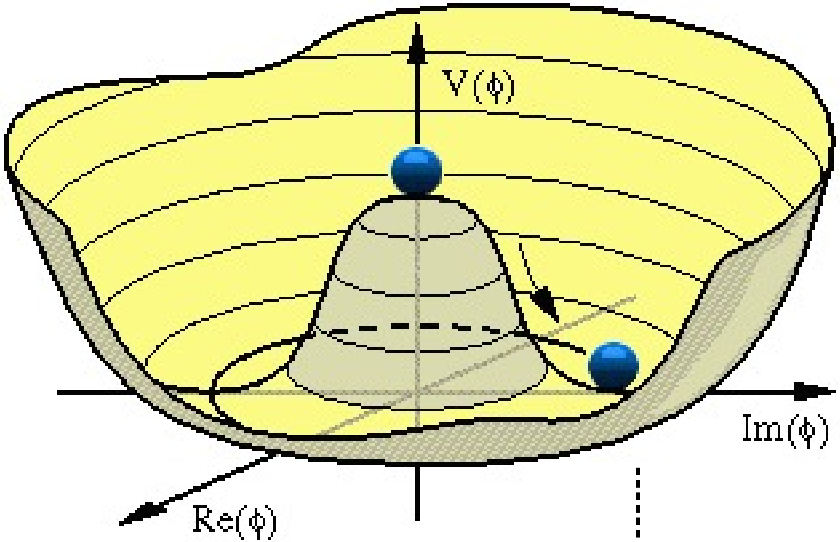

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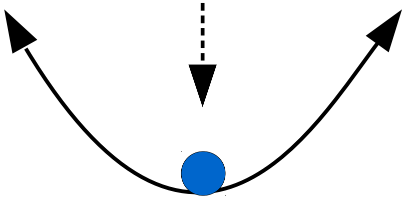
- $\phi$ has radial excitations in the potential $V(\phi)$.



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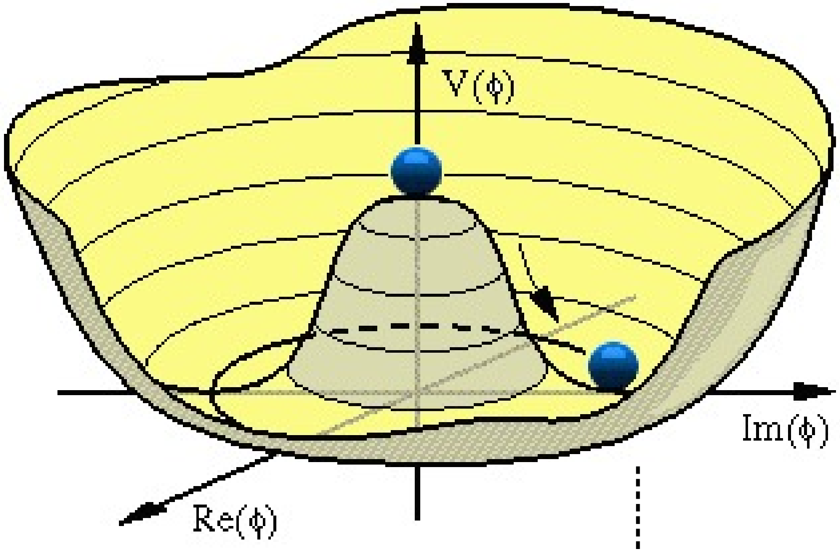

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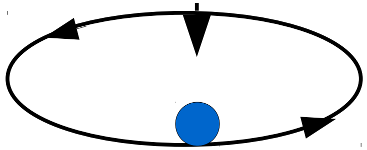
  - ground state breaks $U(1)$ symmetry, BUT at the same time all ground states are in-distinguishable in $\varphi$.

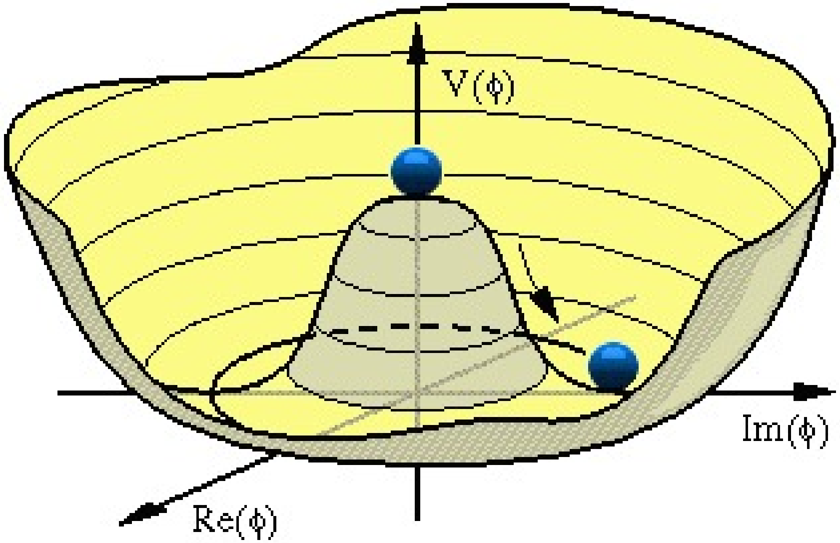
- $\phi$ can "move freely" in the circle that corresponds to the minimum of $V(\phi)$.



# The *Goldstone* Theorem

- In particle physics this is formalized in the *Goldstone* Theorem:

In a relativistic covariant quantum field theory with spontaneously broken symmetries massless particles (=*Goldstone* bosons) are created.



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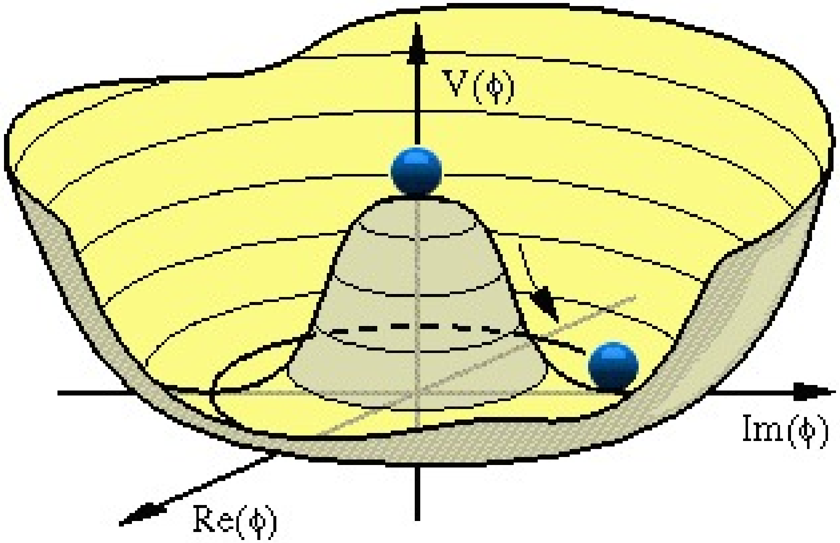

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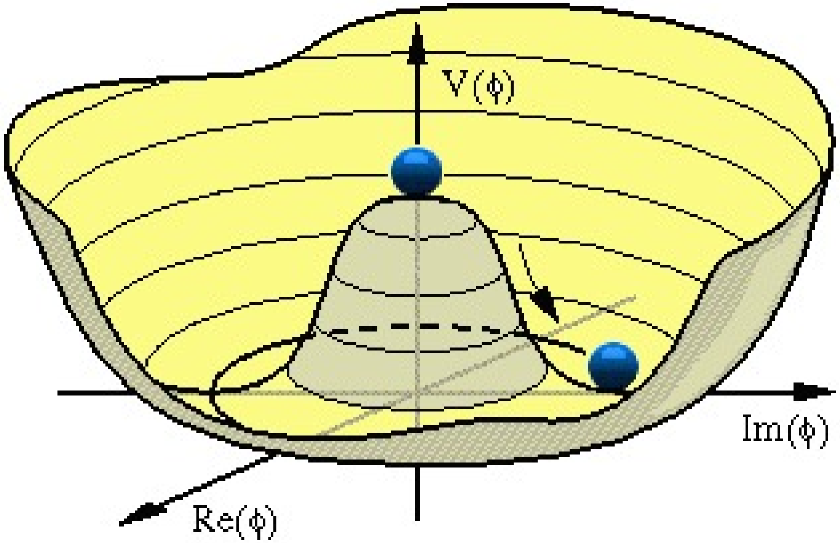

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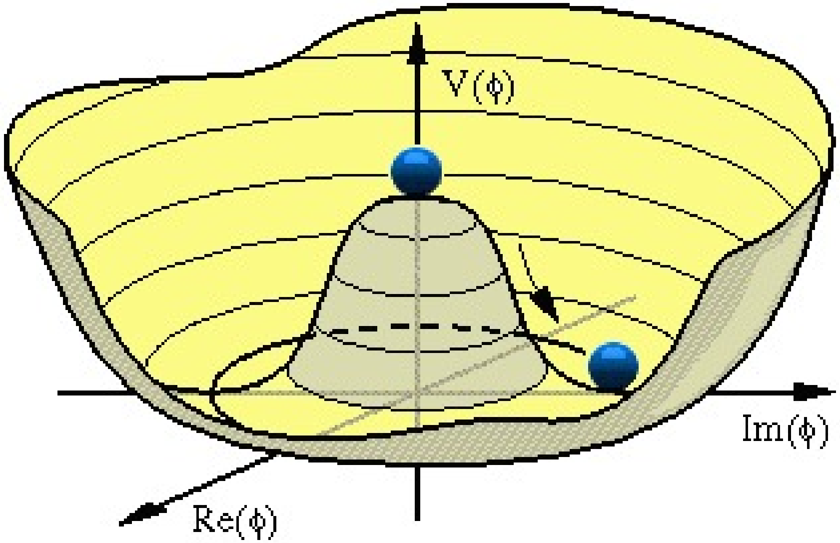


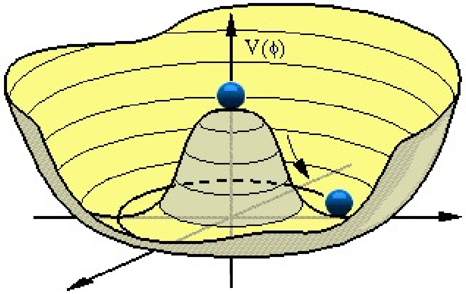
- *Goldstone* Bosons can be:
  - Elementary fields, which are already part of $\mathcal{L}$ .
  - Bound states, which are created by the theory (e.g. the H-atom, Cooper-pairs, ...).
  - Unphysical or gauge degrees of freedom.

# Analyzing the Energy Ground State

- The energy ground state is where the *Hameltonian*

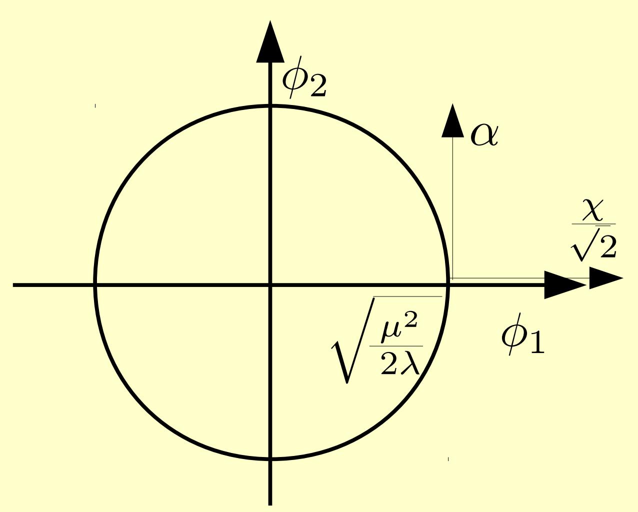
$$\mathcal{H} = \frac{\partial L}{\partial\left(\partial_\mu \phi\right)}\partial_\mu\phi + \frac{\partial L}{\partial\left(\partial^\mu \phi^*\right)}\partial^\mu\phi^* - \mathcal{L} = \partial_\mu\phi\partial^\mu\phi^* + V(\phi)$$

is minimal. This is the case for $|\phi| = \sqrt{\frac{\mu^2}{2\lambda}}$ .



- To analyze the system in its physical ground state we make an expansion in an arbitrary point on the $|\phi| = \sqrt{\frac{\mu^2}{2\lambda}}$ cycle:

$$\phi(\chi,\alpha) = e^{i\alpha}\left(\sqrt{\frac{\mu^2}{2\lambda}} + \frac{\chi}{\sqrt{2}}\right)$$



Up view on *Goldstone* potential

# *Goldstone* Bosons & Dynamic Massive Terms

- An expansion in the ground state in cylindrical coordinates leads to:

$$\mathcal{L} = \Big[\partial_\mu\phi\partial^\mu\phi^* - V(\phi)\Big]_{\phi(\chi,\alpha)} = \frac{1}{2}\partial_\mu\chi\partial^\mu\chi + \left(\sqrt{\frac{\mu^2}{2\lambda}} + \frac{\chi}{\sqrt{2}}\right)^2 \partial_\mu\alpha\partial^\mu\alpha - V'(\chi)$$

$$V'(\chi) = \Big[-\mu^2|\phi|^2 + \lambda|\phi|^4\Big]_{\phi(\chi)} = -\frac{\mu^4}{4\lambda} + \mu^2\chi^2 + \underbrace{\mu\sqrt{\lambda}\chi^3 + \frac{\lambda}{4}\chi^4}$$

*const.*

*dynamic mass term*

*self-couplings*

- Why is there no linear term in $\chi$?

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- Why is there no linear term in $\chi$? → We have performed a *Taylor* expansion in the minimum. By construction there cannot be any linear terms in there.

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- **Remarks:**
  - The mass term is acquired for the field $\chi$ along the radial excitation, which leads out of the minimum of $V(\chi)$. It is the term at lowest order in the *Taylor* expansion in the minimum, and therefore independent from the concrete form of $V(\chi)$ in the minimum.
  - The field $\alpha$, which does not lead out of the minimum of $V(\chi)$ does not acquire a mass term. It corresponds to the *Goldstone* boson.

# Extension to a Gauge Field Theory

- For simplicity reasons shown for an Abelian model:

Introduce covariant derivative

$$\phi(\chi,\alpha) = e^{i\alpha}\left(\sqrt{\frac{\mu^2}{2\lambda}} + \frac{\chi}{\sqrt{2}}\right)$$

$$\mathcal{L} = [(\partial_\mu + ieA_\mu)\,\phi]\,[(\partial^\mu + ieA^\mu)\,\phi]^* - V(\phi) - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}|_{\phi(\chi,\alpha)}$$

$$= \left|\frac{1}{\sqrt{2}}\partial_\mu\chi e^{i\alpha} + ie^{i\alpha}\left(\sqrt{\frac{\mu^2}{\lambda}} + \frac{\chi}{\sqrt{2}}\right)(eA_\mu + \partial_\mu\alpha)\right|^2 - V'(\chi) - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

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Remove by proper gauge:

$$A'_\mu = A_\mu + \partial_\mu\vartheta$$

How does this gauge look like?

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How does this gauge look like? $\vartheta = -\frac{1}{e}\alpha$

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&= \frac{1}{2}\partial_\mu\chi\partial^\mu\chi + \left(\left(\sqrt{\frac{\mu^2}{2\lambda}} + \frac{\chi}{\sqrt{2}}\right) eA'_\mu \qquad\right)^2 - V'(\chi) - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}
\end{aligned}$$

Mass term for $A_\mu$ :

$$\frac{e^2\mu^2}{2\lambda}A_\mu A^{\mu *}$$

Quartic and tri-linear couplings with $\chi$.

# Higgs Mechanism

- *Goldstone* potential and expansion of $\phi \to \phi(\chi, \alpha)$ in the energy ground state has generated a mass term $\frac{e^2\mu^2}{2\lambda} A_\mu A^{\mu *}$ for the gauge field $A_\mu$ from the bare coupling $e^2 \left|\phi\right|^2 A_\mu A^{\mu *}$.

# Higgs Mechanism

- $\phi$ was originally complex (→ i.e. w/ 2 degrees of freedom) .
- $\chi$ is a real field, $\alpha$ has been absorbed into $A_\mu$. It seems as if one degree of freedom had been lost. This is not the case:
  - as a massless particle $A_\mu$ has only two degrees of freedom (±1 helicity states).
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This shuffle of degrees of freedom from the Goldstone boson(s) to the gauge boson(s) is called Higgs mechanism.

# Notes on the *Goldstone* Potential

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  - it does not distinguish any direction in space (→ i.e. only depends on $|\phi|$ ).
  - it is bound from below and does not lead to infinite negative energies, which is a prerequisite for a stable theory.
  - it is the simplest potential with these features.

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  - What is the dimension of $\mathcal{L}$ ?

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- The potential has been chosen to be cut at the order of $|\phi|^4$. This can be motivated by a dimensional analysis:
  - Due to gauge invariance $\phi$ has to appear in even order (c.f. transformation behavior of objects in first lecture).
  - What is the dimension of $\mathcal{L}$ ? $[\mathcal{L}] = \mathrm{GeV}^4$
  - What is the dimension of $\phi$ ?
  - What is the dimension of $\mu$ ?
  - What is the dimension of $\lambda$ ?

$$\mathcal{L}(\phi) = \partial_\mu\phi\partial^\mu\phi^* - V(\phi)$$

$$V(\phi) = -\mu^2|\phi|^2 + \lambda|\phi|^4$$

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$$\begin{aligned} \mathcal{L}(\phi) &= \partial_\mu \phi \partial^\mu \phi^* - V(\phi) \\ V(\phi) &= -\mu^2 |\phi|^2 + \lambda |\phi|^4 \end{aligned}$$

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$$\mathcal{L}(\phi) = \partial_\mu\phi\partial^\mu\phi^* - V(\phi)$$

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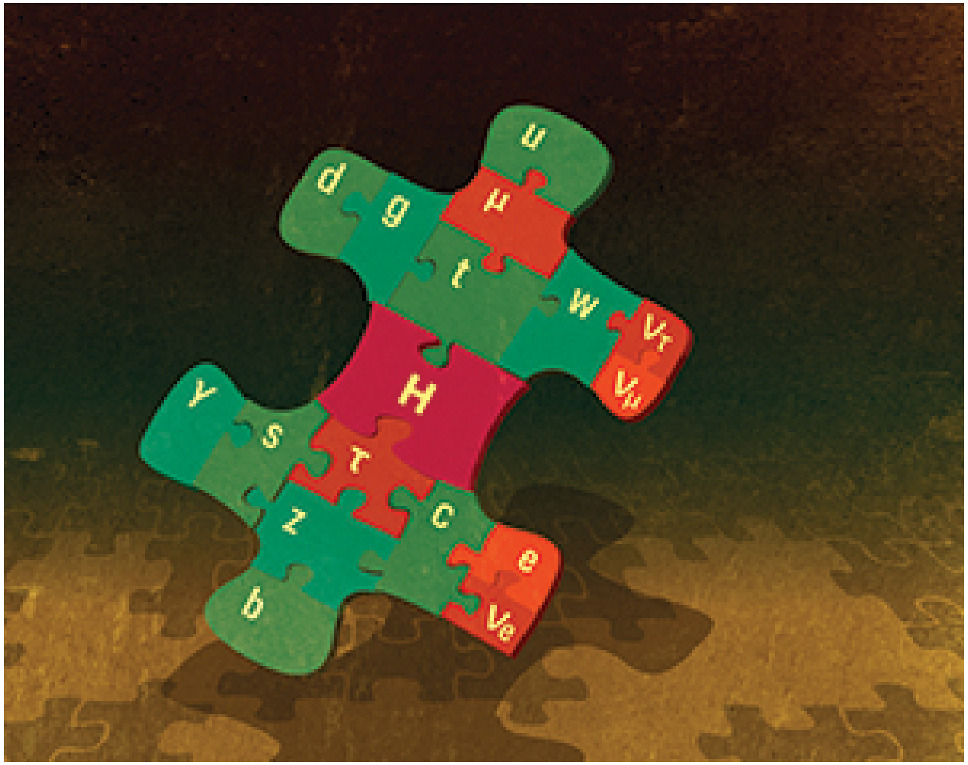
- **NB:** It would be possible to extend the potential to higher dimensions of $\phi$ but couplings with negative dimension will turn the theory non-renormalizable.

# Final Construction of the SM



# The SM without mass terms

- Compilation of the last two lectures:

Fermion kinematics

$$\mathcal{L}^{SU(2)\times U(1)} = \mathcal{L}^{\text{kin}} + \mathcal{L}^{CC} + \mathcal{L}^{NC} + \mathcal{L}^{\text{gauge}}$$

$$\mathcal{L}^{\text{kin}} = i\bar{e}\gamma^{\mu}\partial_{\mu}e + i\bar{\nu}\gamma^{\mu}\partial_{\mu}\nu$$

$$\mathcal{L}^{CC} = -\frac{e}{\sqrt{2}\sin\theta_W}\left[W_{\mu}^{+}\bar{\nu}\gamma_{\mu}e_L + W_{\mu}^{-}\bar{e}_L\gamma_{\mu}\nu\right]$$

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$$\mathcal{L}^{\text{gauge}} = -\frac{1}{2}Tr\left(W_{\mu\nu}^{a}W^{a\mu\nu}\right) - \frac{1}{4}B_{\mu\nu}B^{\mu\nu}\bigg|$$

$$B_{\mu} \to A_{\mu}$$
$$W_{\mu}^{3} \to Z_{\mu}$$

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- Compilation of the last two lectures:

Charged current IA

Fermion kinematics

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Neutral current IA

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Charged current IA

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Gauge field kinematic

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# Extension by a new field $\phi$

- Add $\phi$ as $SU(2)$ doublet field:

$$\phi = \begin{pmatrix} \phi_+ \\ \phi_0 \end{pmatrix}$$

Transformation behavior

$$\left\{\begin{array}{l} \phi \rightarrow \phi' = e^{i\vartheta'} G\phi \\ \phi^\dagger \rightarrow \phi'^\dagger = \phi^\dagger G^\dagger e^{-i\vartheta'} \\ G = e^{i\vartheta^a t^a} \in SU(2) \quad \vartheta^a, \vartheta' \in \mathbb{R} \end{array}\right.$$

Can you point to the *Goldstone* bosons?

$$\mathcal{L}^{SU(2)\times U(1)} = \mathcal{L}^{\text{kin}} + \mathcal{L}^{CC} + \mathcal{L}^{NC} + \mathcal{L}^{\text{gauge}} + \mathcal{L}^{\text{Higgs}}$$

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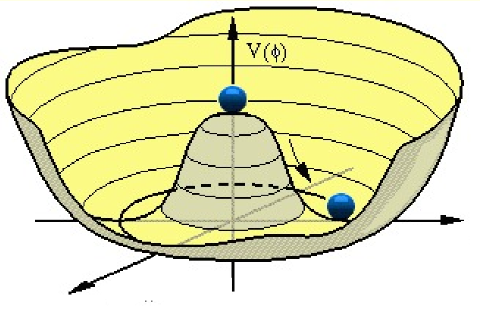
$$V(\phi) = -\mu^2 \phi^\dagger \phi + \lambda \left(\phi^\dagger \phi\right)^2$$



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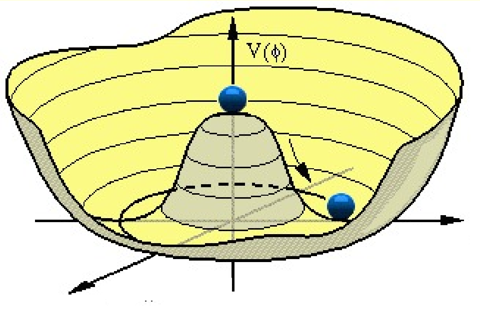

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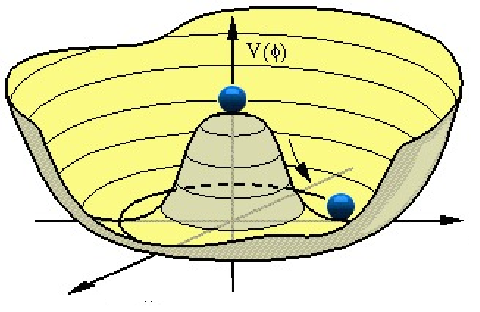

- $\mathcal{L}^{\text{Higgs}}$ is covariant under global $SU(2)$ transformations.

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Can you point to the *Goldstone* bosons?

- Introduce covariant derivative $D_\mu$ to enforce local gauge invariance:

$$\boxed{\partial_\mu \rightarrow D_\mu = \partial_\mu + ig'\frac{Y_\phi}{2}B_\mu + igW_\mu^a \mathbf{t}^a}$$

(analogue to fermion fields)

| $SU(2) \times U(1)$ Hypercharges | | | |
|---|---|---|---|
| Particle | $Y_\phi$ | $I_3$ | $Q$ |
| $\phi_+$ | $+1$ | $+{}^1/_2$ | $+1$ |
| $\phi_0$ | | $-{}^1/_2$ | $0$ |

$Q = I_3 + \frac{Y}{2}$ (Gell-Mann Nischijama)

# Expansion in the Energy Ground State of $\phi$

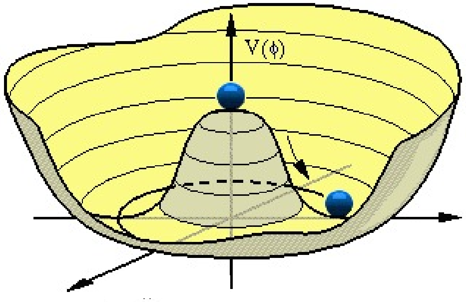
- Develop $\phi$ in its energy ground state at $|\phi| = \sqrt{\frac{\mu^2}{2\lambda}}$ :

$$\phi = \begin{pmatrix} \phi_+ \\ \phi_0 \end{pmatrix} \xrightarrow{|\phi| = \sqrt{\frac{\mu^2}{2\lambda}}} \phi = \begin{pmatrix} 0 \\ \sqrt{\frac{\mu^2}{2\lambda}} + \frac{H}{\sqrt{2}} \end{pmatrix}$$

**NB:** In principle this can be done anywhere in the minimum. For a consistent model it is done in the lower component of $\phi$.

Non-zero vacuum expectation value.

Radial excitation field. → This is the Higgs boson!



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couple gauge fields to $\phi$ :

$$D_\mu \phi^\dagger D^\mu \phi$$

$$D_\mu = \partial_\mu + ig' \frac{Y_\phi}{2} B_\mu + igW_\mu^a \mathbf{t}^a$$

(covariant derivative)

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# Expansion in the Energy Ground State of $\phi$

- Resolve products of *Pauli* matrices ( $\mathbf{t}^a \equiv \frac{1}{2}\sigma_a$ ):

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  - Ascending operator $\mathbf{t}^+$ (of $W_\mu^+$ ) shifts unit vector of the down component up.

$$D_\mu \phi^\dagger D^\mu \phi = \left| \left[ \frac{1}{\sqrt{2}} \partial_\mu H - \frac{i}{2} \left( \sqrt{\frac{\mu^2}{2\lambda}} + \frac{H}{\sqrt{2}} \right) (g W_\mu^3 - g' B_\mu) \right] \begin{pmatrix} 0 \\ 1 \end{pmatrix} \right|^2$$
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  - Ascending operator $\mathbf{t}^+$(of $W_\mu^+$ ) shifts unit vector of the down component up.
  - Descending operator $\mathbf{t}^-$(of $W_\mu^-$) “destroys” unit vector of the down component.

$$D_\mu \phi^\dagger D^\mu \phi = \left| \left[ \frac{1}{\sqrt{2}} \partial_\mu H - \frac{i}{2} \left( \sqrt{\frac{\mu^2}{2\lambda}} + \frac{H}{\sqrt{2}} \right) (g W_\mu^3 - g' B_\mu) \right] \begin{pmatrix} 0 \\ 1 \end{pmatrix} \right|^2$$
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$\mathbf{t}^{-}$

# Expansion in the Energy Ground State of $\phi$

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  - Ascending operator $\mathbf{t}^+$ (of $W^+_\mu$ ) shifts unit vector of the down component up.
  - Descending operator $\mathbf{t}^-$ (of $W^-_\mu$ ) “destroys” unit vector of the down component.
  - Operator $\mathbf{t}^3$ switches sign for unit vector of down component.

$$D_\mu\phi^\dagger D^\mu\phi = \left|\left[\frac{1}{\sqrt{2}}\partial_\mu H - \frac{i}{2}\left(\sqrt{\frac{\mu^2}{2\lambda}} + \frac{H}{\sqrt{2}}\right)\left(gW^3_\mu - g'B_\mu\right)\right]\begin{pmatrix}0\\1\end{pmatrix}\right|^2 + \left|\left[\frac{i}{2}\left(\sqrt{\frac{\mu^2}{2\lambda}} + \frac{H}{\sqrt{2}}\right)gW^+_\mu\right]\begin{pmatrix}1\\0\end{pmatrix}\right|^2$$

# Expansion in the Energy Ground State of $\phi$

- Evaluate components of absolute value squared:

$$D_\mu\phi^\dagger D^\mu\phi = \left|\left[\frac{1}{\sqrt{2}}\partial_\mu H - \frac{i}{2}\left(\sqrt{\frac{\mu^2}{2\lambda}} + \frac{H}{\sqrt{2}}\right)(gW_\mu^3 - g'B_\mu)\right]\begin{pmatrix}0\\1\end{pmatrix}\right|^2$$

$$+\left|\left[\frac{i}{2}\left(\sqrt{\frac{\mu^2}{2\lambda}} + \frac{H}{\sqrt{2}}\right)gW_\mu^+\right]\begin{pmatrix}1\\0\end{pmatrix}\right|^2$$

$$D_\mu\phi^\dagger D^\mu\phi = \tfrac{1}{2}\partial_\mu H\partial^\mu H$$
$$+\tfrac{g^2+g'^2}{4}\left(\sqrt{\tfrac{\mu^2}{2\lambda}} + \tfrac{H}{\sqrt{2}}\right)^2 Z_\mu Z^\mu + \tfrac{g^2}{4}\left(\sqrt{\tfrac{\mu^2}{2\lambda}} + \tfrac{H}{\sqrt{2}}\right)^2 W_\mu^+ W^{\mu -}$$

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- Evaluate components of absolute value squared:

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$$+\left|\left[\frac{i}{2}\left(\sqrt{\frac{\mu^2}{2\lambda}} + \frac{H}{\sqrt{2}}\right)gW_\mu^+\right]\begin{pmatrix}1\\0\end{pmatrix}\right|^2$$

$$D_\mu\phi^\dagger D^\mu\phi = \tfrac{1}{2}\partial_\mu H\partial^\mu H$$
$$+\tfrac{g^2+g'^2}{4}\left(\sqrt{\tfrac{\mu^2}{2\lambda}} + \tfrac{H}{\sqrt{2}}\right)^2 Z_\mu Z^\mu + \tfrac{g^2}{4}\left(\sqrt{\tfrac{\mu^2}{2\lambda}} + \tfrac{H}{\sqrt{2}}\right)^2 W_\mu^+ W^{\mu-}$$

$$\left(gW_\mu^3 - g'B_\mu\right) \equiv \sqrt{g^2+g'^2}\,Z_\mu$$

# Masses for Gauge Bosons

- By introducing $\phi$ as a $SU(2)$ doublet with a non-zero energy ground state we have obtained:

$$D_\mu\phi^\dagger D^\mu\phi = \tfrac{1}{2}\partial_\mu H\partial^\mu H + \underbrace{\frac{g^2+g'^2}{4}\left(\sqrt{\frac{\mu^2}{2\lambda}}+\frac{H}{\sqrt{2}}\right)^2 Z_\mu Z^\mu}_{} + \underbrace{\frac{g^2}{4}\left(\sqrt{\frac{\mu^2}{2\lambda}}+\frac{H}{\sqrt{2}}\right)^2 W_\mu^+W^{\mu-}}_{}$$

- Dynamic mass terms for the gauge bosons: $m_Z^2 \equiv \frac{(g^2+g'^2)\mu^2}{8\lambda}$ $\qquad m_W^2 \equiv \frac{g^2\mu^2}{8\lambda}$

- Characteristic tri-linear and quartic couplings of the gauge bosons to the Higgs field.

- A solid prediction of the SM on the masses of the gauge bosons: $\cos\theta_W = \frac{m_W}{m_Z} \longrightarrow m_Z > m_W$

# Gauge Degrees of Freedom

- We had discussed how gauge bosons obtain mass by a gauge that absorbs the *Goldstone* bosons in the theory.

- As a complex $SU(2)$ doublet $\phi$ has four degrees of freedom.
- In the final formulation only the radial excitation $H$ of $\phi$ did remain. The *Goldstone* bosons ( $\vartheta^a$ ) have been absorbed into the gauge fields $W_\mu^{+/-}$ & $Z_\mu$, which have obtained masses from this.

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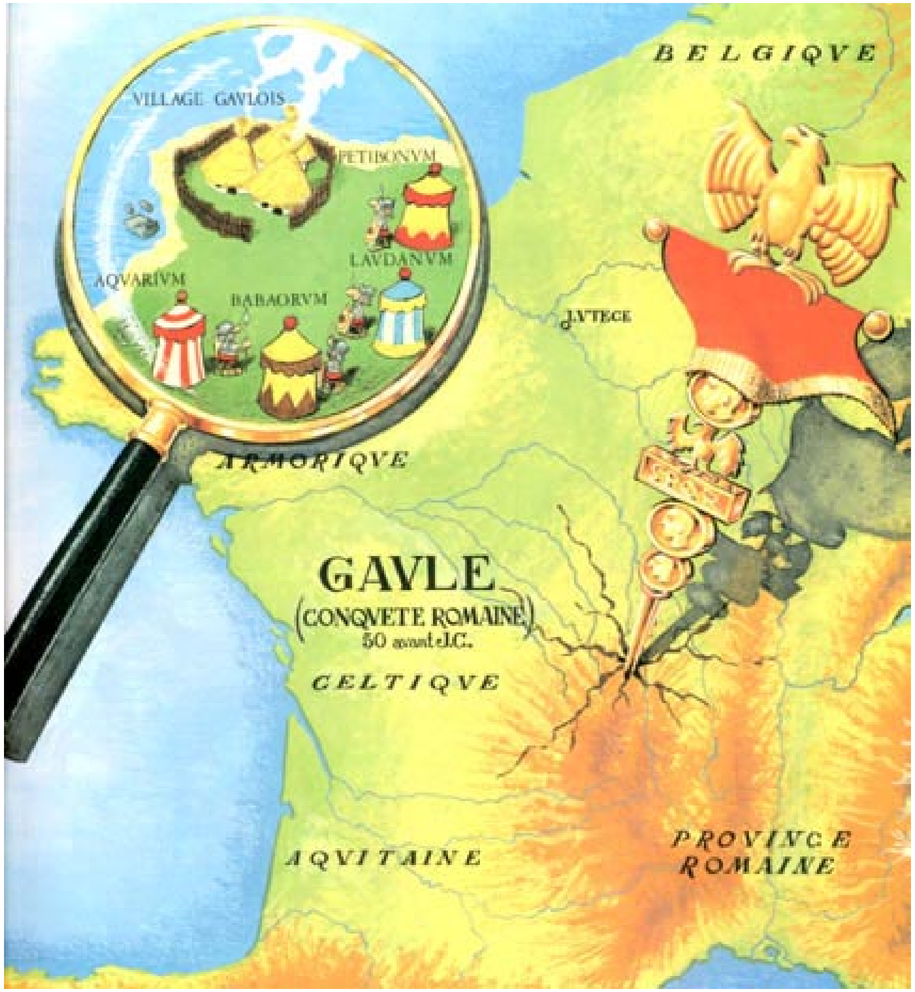
- Almost...

# Solution to the Problem of Fermion Masses



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- The Higgs mechanism can also help to obtain mass terms for fermions, by coupling the fermions to $\phi$ .

$$\phi \to \phi' = e^{i\vartheta'} G\phi$$

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$$\mathcal{L}^{\text{Yukawa}} = -f_e \left(\bar{e}_R \underbrace{\phi^\dagger \psi_L}_{SU(2)\ \text{singlet}}\right) + f_e^* \left(\underbrace{\bar{\psi}_L \phi}_{SU(2)\ \text{singlet}} e_R\right)$$

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Karlsruhe Institute of Technology

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Manifest gauge invariant.

- NB: $f$ can be chosen real. Residual phases can be re-defined in $e_R$ .

# Solution to the Problem of Fermion Masses

- Expand $\phi$ in its energy ground state to obtain the mass terms:

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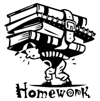

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  Did I lie to you? When yes, when?

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# Concluding Remarks

- Higgs mechanism = incorporation of spontaneous symmetry breaking into a gauge field theory. This leads to the fact that the gauge bosons eat up the *Goldstone* bosons, which exist in the system and gain mass on them.

- This mechanism can leave one or more degrees of freedom e.g. of radial excitations in the potential behind as Higgs boson(s).

- The Higgs boson obtains its mass from the *Goldstone* potential. The gauge bosons obtain their mass from their coupling to $\phi$ via the covariant derivative. The Fermions obtain their mass via a direct Yukawa coupling to $\phi$.

- Gauge bosons couple to the Higgs like $\propto m^2_{\mathrm{gauge}}H$, fermion fields couple to the Higgs like $\propto m_f H$.

# Sneak Preview for Next Week

- Wrap up what we have learned during the last three lectures.

- Discuss the way from Lagrangian to measurable quantities (→ Feynman rules).

- Discuss loop corrections and higher orders to tree level calculations (pictorially).

- Constraints on $m_H$ within the theory itself.