Teilchenphysik II (Higgs-Physik) (SS 2016)

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Exercises Sheet 04

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Exercise 13: Massive Fermions in the SM (presence)

General $SU(2)$ rotations can be described by the following rotation matrix:

$$\begin{aligned}\mathcal{U} &= \cos(\vartheta/2) + i\sin(\vartheta/2)\cdot\sum\lambda_j\sigma_j \\ &= \begin{pmatrix} \cos(\vartheta/2) - i\lambda_3\sin(\vartheta/2) & (-\lambda_2 - i\lambda_1)\sin(\vartheta/2) \\ (\lambda_2 - i\lambda_1)\sin(\vartheta/2) & \cos(\vartheta/2) + i\lambda_3\sin(\vartheta/2) \end{pmatrix}\end{aligned}$$

$$\mathcal{U}^\dagger = \begin{pmatrix} \cos(\vartheta/2) + i\lambda_3\sin(\vartheta/2) & (\lambda_2 + i\lambda_1)\sin(\vartheta/2) \\ (-\lambda_2 + i\lambda_1)\sin(\vartheta/2) & \cos(\vartheta/2) - i\lambda_3\sin(\vartheta/2) \end{pmatrix}$$

where $\{\sigma_j\}$ are the Pauli matrices and $\sum\lambda_j^2 = 1$. You have seen this formula already in the first lecture in the context of the transformation of spinors. With the help of this information proof that:

$$\begin{aligned}m_e\bar{e}e \to m_e\bar{e}'e' = m_e\Big(&\cos(\vartheta/2)(\bar{e}_Re_L + \bar{e}_Re_L) + \lambda_2\sin(\vartheta/2)(\bar{e}_R\nu + \bar{\nu}e_L)\Big) \\ &+ im_e\sin(\vartheta/2)\Big(\lambda_3(\bar{e}_Re_L + \bar{e}_Re_L) + \lambda_1(\bar{e}_R\nu + \bar{\nu}e_L)\Big)\end{aligned}$$

which demonstrates explicitly that fermionic mass terms break local gauge invariance. Note that

$$\bar{e}e = \bar{e}_Le_R + \bar{e}_Re_L$$

You have shown this identity explicitly in Exercise 9 e).

Exercise 14: Goldstone Potential (homework)

In the lecture we have introduced the *Goldstone* potential

$$V(\phi) = -\mu^2|\phi|^2 + \lambda|\phi|^4$$

a) Proof that this potential has indeed a minimum at

$$|\phi| = \sqrt{\frac{\mu^2}{2\lambda}}$$

b) In the lecture you have seen an expansion of the field in cylindrical coordinates. Try yourself an expansion in *Cartesian* coordinates in the point

$$\phi(u,v) = \sqrt{\frac{\mu^2}{2\lambda}} + \frac{1}{\sqrt{2}}(u + iv)$$

What changes if you do an expansion in the point

$$\phi(u,v) = i\sqrt{\frac{\mu^2}{2\lambda}} + \frac{1}{\sqrt{2}}(u+iv)$$

what is the role of the fields u and v?

Exercise 15: Massive Photons in QED (homework)

Consider a hypothetical QED model with a massive photon. This model shall be described by the Lagrangian density

$$\mathcal{L} = \overline{\psi}\left(i\gamma^\mu D_\mu - m_e\right)\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}m_A^2 A_\mu A^\mu$$

a) Show that the mass term of the photon

$$\frac{1}{2}m_A^2 A_\mu A^\mu$$

violates the $U(1)$ gauge symmetry. In the following we will introduce such a mass term via the Higgs mechanism.

b) Introduce a scalar complex field, which transforms under the $U(1)$ gauge symmetry like

$$\phi \to \phi' = e^{ie\vartheta(x)}\phi$$

with a spontaneous symmetry breaking potential and a Lagrangian density of form

$$\mathcal{L} = (D^\mu\phi)^*(D_\mu\phi) - V(\phi^*\phi)$$
$$V(\phi^*\phi) = \lambda\left(\phi^*\phi - \frac{v^2}{2}\right)^2$$

and expand the field as $\phi = \frac{1}{\sqrt{2}}(v + h(x))$.

c) Show that a *Yukawa* interaction term of type

$$\mathcal{L}_{\text{Yukawa}} = y|\phi|\overline{\psi}\psi$$

modifies the electron mass in the model and express the electron mass in terms of the "bare" electron mass m_e, the *Yukawa* coupling y and the vacuum expectation value of the Higgs field v.

Exercise 16: Dimensional Analysis (presence)

For the discussion of the choice of the Higgs potential, complete the dimension of the following objects:

- $[\mathcal{L}] =?$
- $[\phi] =?$
- $[\mu] =?$
- $[\lambda] =?$

for the Higgs term of the Lagrangian density:

$$\begin{aligned}\mathcal{L}(\phi) &= \partial_\mu \phi \partial^\mu \phi^* - V(\phi) \\ V(\phi) &= -\mu^2 |\phi|^2 + \lambda |\phi|^4\end{aligned}$$