Teilchenphysik II (Higgs-Physik) (SS 2016)

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Exercises Sheet 05

http://www-ekp.physik.uni-karlsruhe.de/~quast/ Release: Thu, 13.05.2016

Exercise 17: Coupling of Gauge Bosons to the Higgs Boson (presence)

In the lecture you have seen how the mass terms for the gauge bosons $W^{\pm}_{\mu}$ and Z_{μ} emerge dynamically from their coupling to the Higgs field ϕ via the covariant derivative. The corresponding terms are:

$$\frac{g^2 + g'^2}{4}\left(\sqrt{\frac{\mu^2}{2\lambda}} + \frac{H}{\sqrt{2}}\right)^2 Z_{\mu}Z^{\mu}$$

and

$$\frac{g^2}{4}\left(\sqrt{\frac{\mu^2}{2\lambda}} + \frac{H}{\sqrt{2}}\right)^2 W^{+}_{\mu}W^{\mu-}$$

Express these terms by the corresponding mass of the gauge boson and the vacuum expectation value $v = \sqrt{\frac{\mu^2}{2\lambda}}$ and show the explicit proportionality of the coupling of the gauge bosons to the Higgs boson field H.

Exercise 18: $SU(2)$ behavior of *Yukawa* coupling terms (presence)

To obtain mass terms for fermions, in the lecture we have introduced a special *Yukawa* coupling term of form

$$\mathcal{L}^{\text{Yukawa}} = -f_e\left(\overline{e}_R\phi^{\dagger}\psi_L\right) - f_e^{*}\left(\overline{\psi}_L\phi e_R\right)$$

into the Lagrangian density. We remind you of the $SU(2)$ properties of ψ and ϕ:

$$\psi_L = \begin{pmatrix} \nu \\ e_L \end{pmatrix} \qquad \psi_L \to \psi'_L = e^{iY_L\vartheta'}G\psi_L \qquad \phi = \begin{pmatrix} \phi^{+} \\ \phi^{0} \end{pmatrix} \qquad \phi \to \phi' = e^{iY_{\phi}\vartheta'}G\phi$$

$$\overline{\psi}_L = \begin{pmatrix} \overline{\nu} & \overline{e}_L \end{pmatrix} \qquad \overline{\psi}_L \to \overline{\psi}'_L = \overline{\psi}_L G^{\dagger}e^{-iY_L\vartheta'} \qquad \phi^{\dagger} = \begin{pmatrix} \phi^{+*} & \phi^{0*} \end{pmatrix} \qquad \phi^{\dagger} \to \phi'^{\dagger} = \phi^{\dagger}G^{\dagger}e^{-iY_{\phi}\vartheta'}$$

Check the $SU(2)$ and the $U(1)$ behavior of the terms $\overline{e}_R\phi^{\dagger}\psi_L$ and $\overline{\psi}_L\phi e_R$.

Exercise 19: Explicit demonstration of $SU(2)$ invariance (homework)

In the lecture we have raised the question how terms of form $\overline{e}_R e_L + \overline{e}_L e_R$ on one hand can explicitly break local gauge invariance as shown in Exercise 13 and be local gauge invariant in the final formulation of the SM, where they appear in exact the same form. Resolve this mystery by explicitly calculating the $SU(2)$ transformation behavior of the term:

$$-f_e\left(\sqrt{\frac{\mu^2}{2\lambda}} + \frac{H}{\sqrt{2}}\right)\left(\overline{e}_R e_L + \overline{e}_L e_R\right)$$

Make use of the general $SU(2)$ rotation matrices $\mathcal{U}$ and $\mathcal{U}^{\dagger}$ as given for Exercise 13 and transform H and $\overline{e}_L$. What is the transformation behavior of e_R?

Exercise 20: Coupling of Fermions to the Higgs Boson (presence)

In the lecture you have seen how the mass terms of fermions can emerge dynamically from the coupling to the Higgs field ϕ via an explicit *Yukawa* coupling. The corresponding term is:

$$\mathcal{L}^{\text{Yukawa}} = -f_e \left(\sqrt{\frac{\mu^2}{2\lambda}} + \frac{H}{\sqrt{2}} \right) \bar{e} e$$

Express this terms by the fermion mass and the vacuum expectation value $v = \sqrt{\frac{\mu^2}{2\lambda}}$ and show the explicit proportionality of the coupling of the fermion to the Higgs boson field H.