Teilchenphysik II (Higgs-Physik) (SS 2016)

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Exercises Sheet 07

http://www-ekp.physik.uni-karlsruhe.de/~quast/ Release: Thu, 20.05.2016

Exercise 24: *Green*'s Function of the Photon Wave Equation (presence)

In the lecture we have introduced the *Green*'s function $D^{\mu\nu}(x-x')$ for solving of the inhomogeneous photon wave equation (in Lorentz gauge $\partial_\mu A^\mu = 0$):

$$\Box A^\mu = eJ^\mu. \tag{1}$$

where $D^{\mu\nu}(x-x')$ has the property:

$$\Box D^{\mu\nu}(x-x') = g^{\mu\nu}\delta^4(x-x') \tag{2}$$

and $\delta^4(x-x')$ is the four dimensional delta distribution. Proof that due to the property of Equation (2)

$$A^\mu(x) = e\int \mathrm{d}^4x' D^{\mu\nu}(x-x')J_\nu(x') \tag{3}$$

is a solution of Equation (1) in the point x if $J_\nu(x')$ is known in point x'.

Exercise 25: Triviality Bound for the Higgs Boson Mass (presence)

In the lecture we have discussed the solution of the renormalization group equation (at one loop level) for the Higgs self-coupling λ for very large Higgs boson masses ($Q^2, m_t^2 \ll m_H^2$):

$$\lambda(Q^2) = \frac{\lambda(v^2)}{1 - \frac{3}{4\pi^2}\lambda(v^2)\log\left(Q^2/v^2\right)}$$

where $v = \sqrt{\frac{\mu^2}{2\lambda}} = \sqrt{\frac{m_H^2}{4\lambda}} = 246\,\mathrm{GeV}$ is the vacuum expectation value. Calculate the *Landau* pole, where the denominator turns to zero. Calculate upper bounds for m_H for the assumption that the SM is a valid and perturbative theory up to scales of $\Lambda = 10^3\mathrm{GeV}$ ($\Lambda = 10^{16}\mathrm{GeV}$).

Exercise 26: Stability Bound for the Higgs Boson Mass (presence)

In the lecture we have also discussed the solution of the renormalization group equation for the case of $m_H \ll m_t$, where the coupling to the *top*-quark is the dominant part in the loop:

$$\lambda(Q^2) = \lambda(v^2) - \frac{3}{16\pi^2}\frac{m_t^4}{v^4}\log\left(Q^2/v^2\right)$$

Note that $\lambda(v^2) = \frac{m_H^2}{4v^2}$. Derive a formula to estimate a lower limit on m_H. Calculate the lower bounds for the assumption that the SM is a valid and perturbative theory up to scales of $\Lambda = 10^3\mathrm{GeV}$ ($\Lambda = 10^{16}\mathrm{GeV}$).