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Electroweak Sector of the SM

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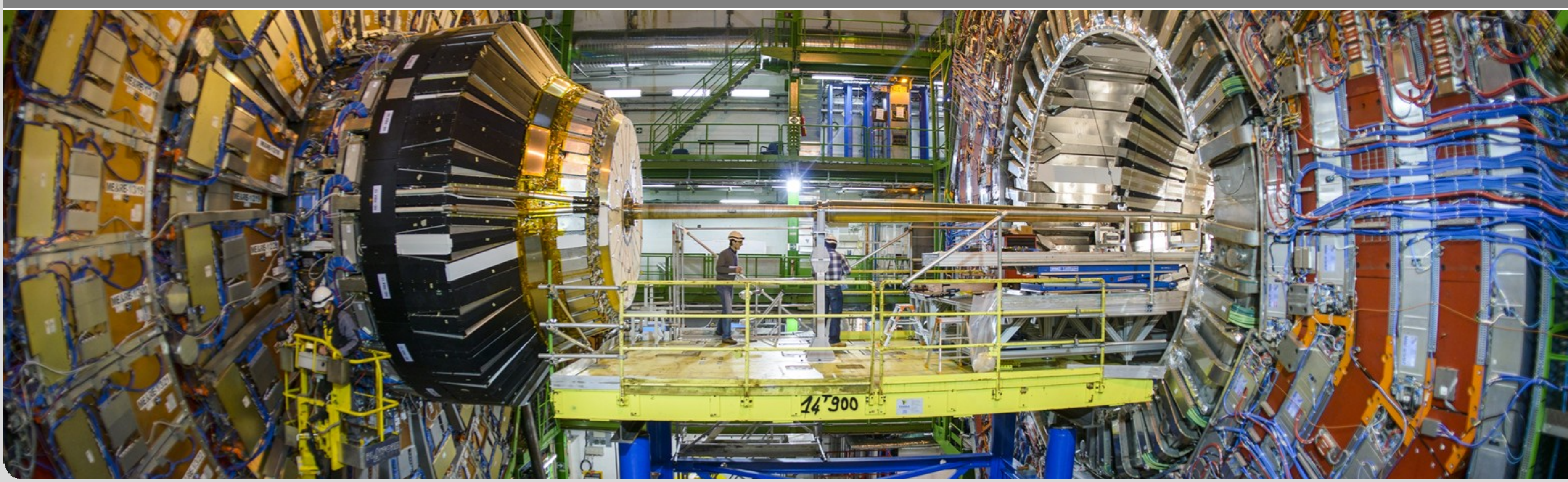
INSTITUTE OF EXPERIMENTAL PARTICLE PHYSICS (IEKP) – PHYSICS FACULTY



Schedule for today

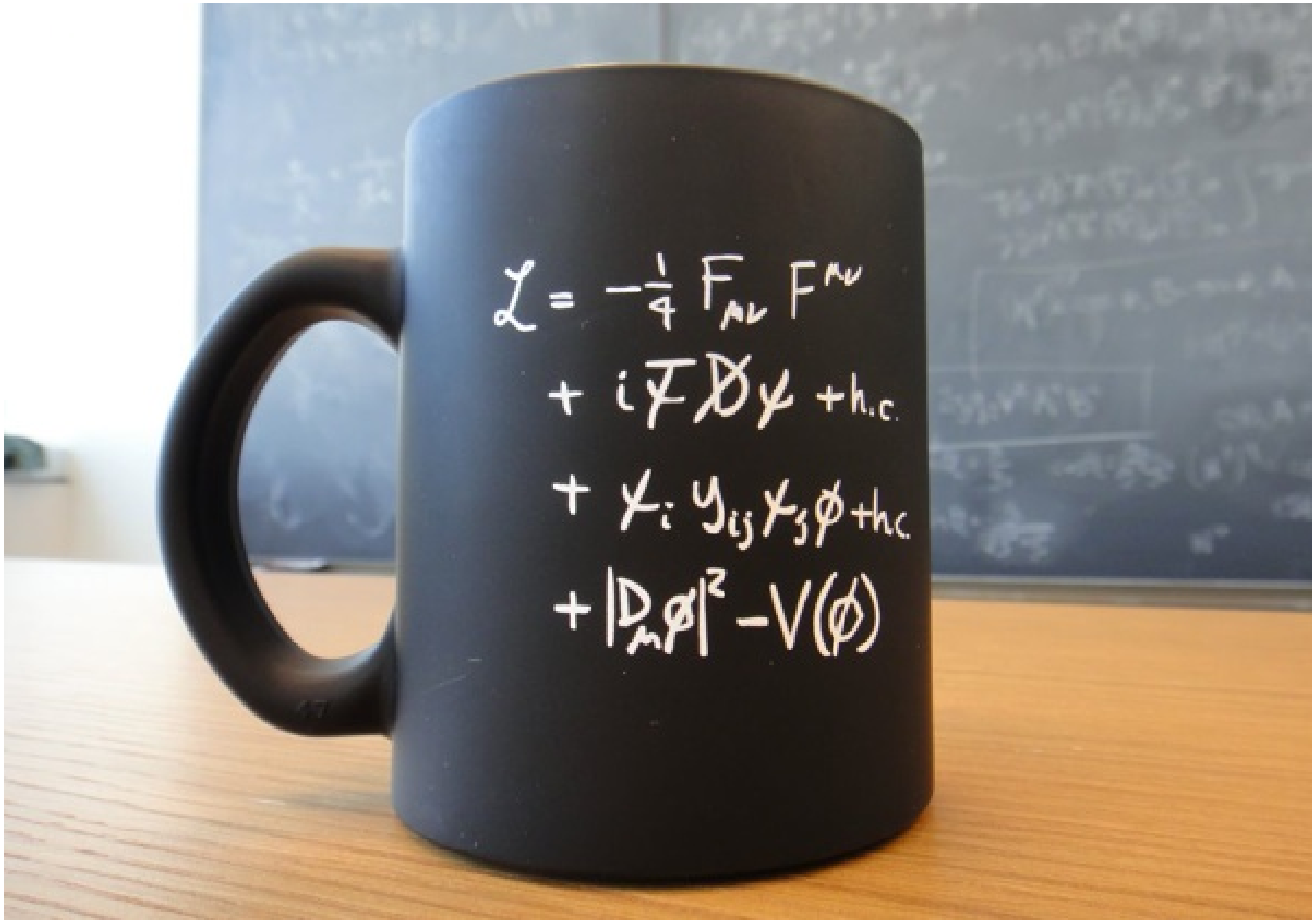
- Does the Z boson couple only to right-handed particles?
- Are the following gauge boson self-couplings allowed: $WWWW$, ZWW?

3 Electroweak gauge symmetry of the SM

2 Weak isospin (left- and right-handedness)

1 Phenomenology of weak interaction

The SM of particle physics



Constituents & interactions of the SM

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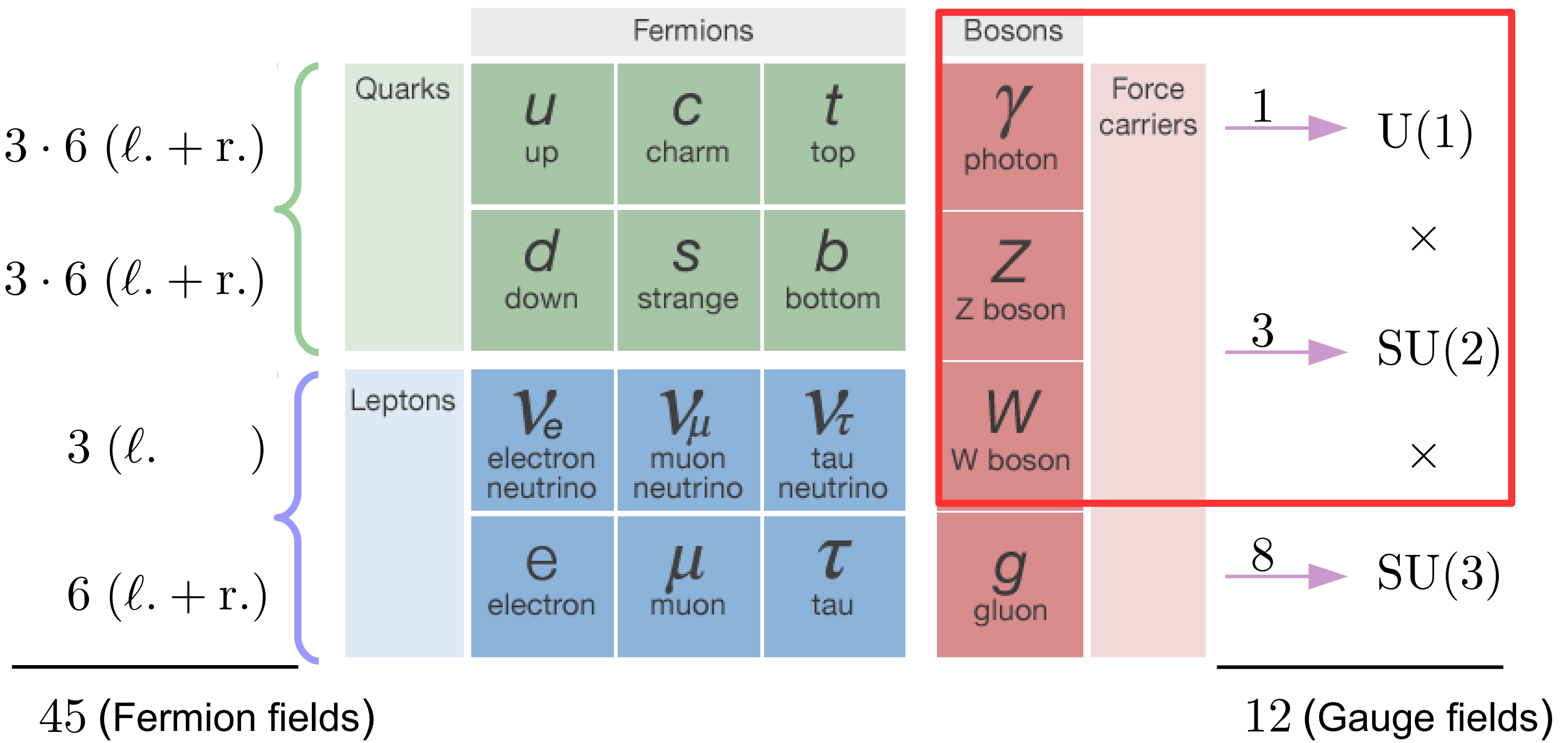
- 18 free parameters



Constituents & interactions of the SM

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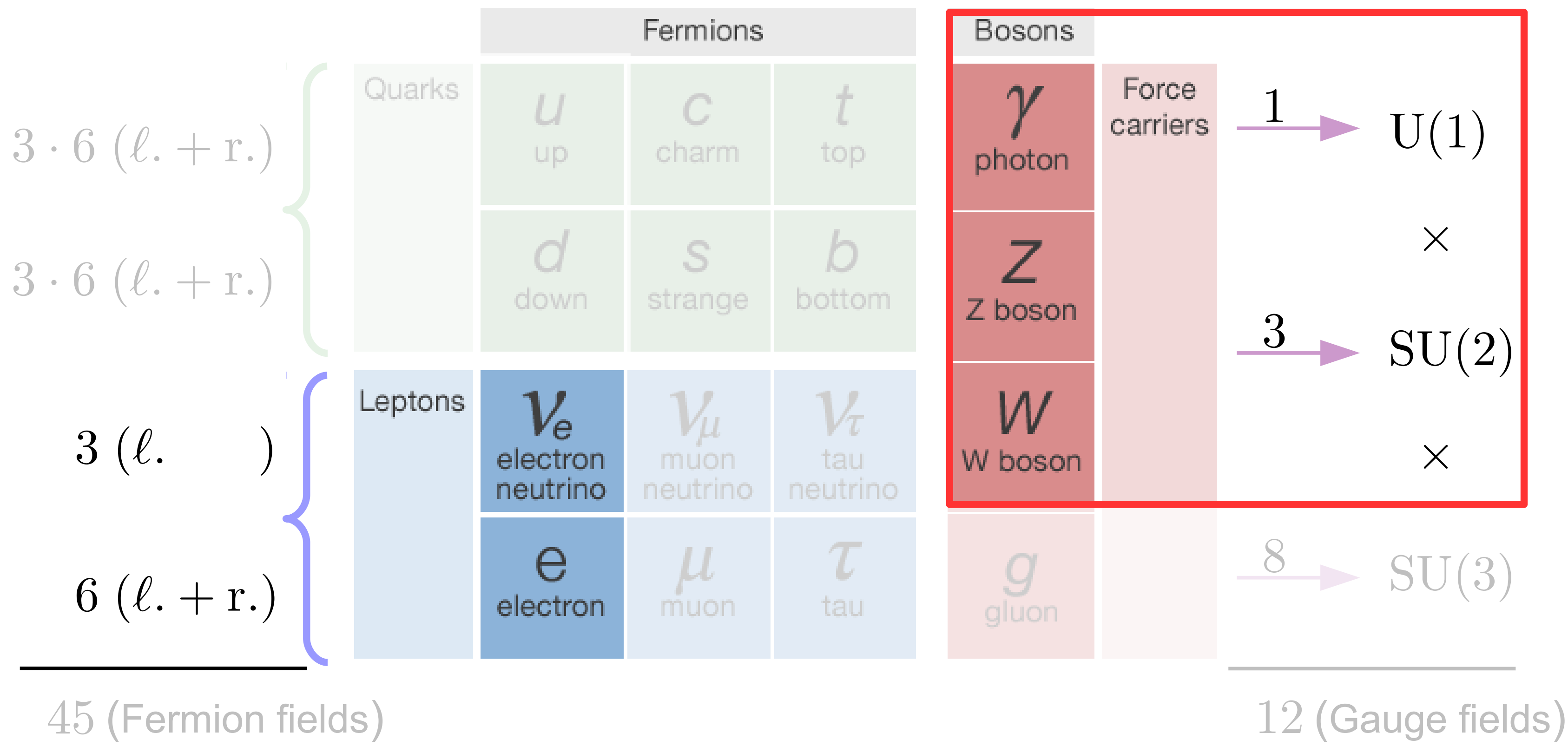
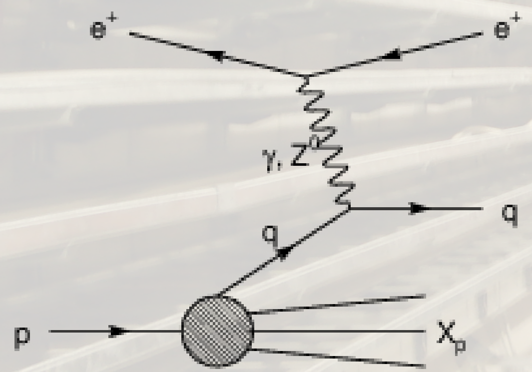
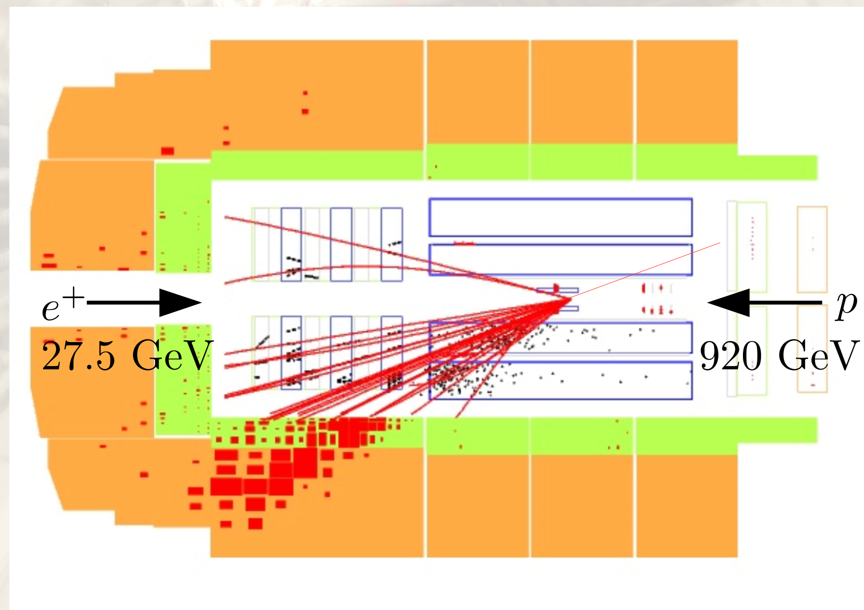


Phenomenology of weak interaction

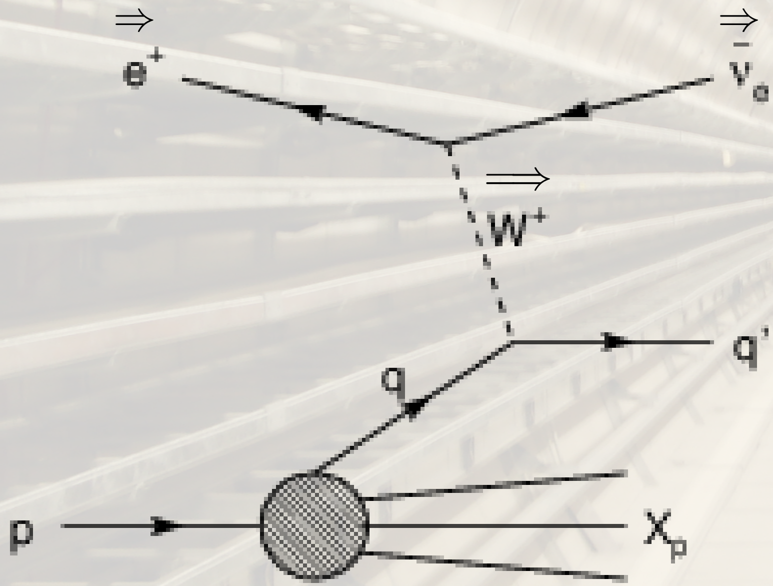
- From the view of a high energy physics scattering experiment:

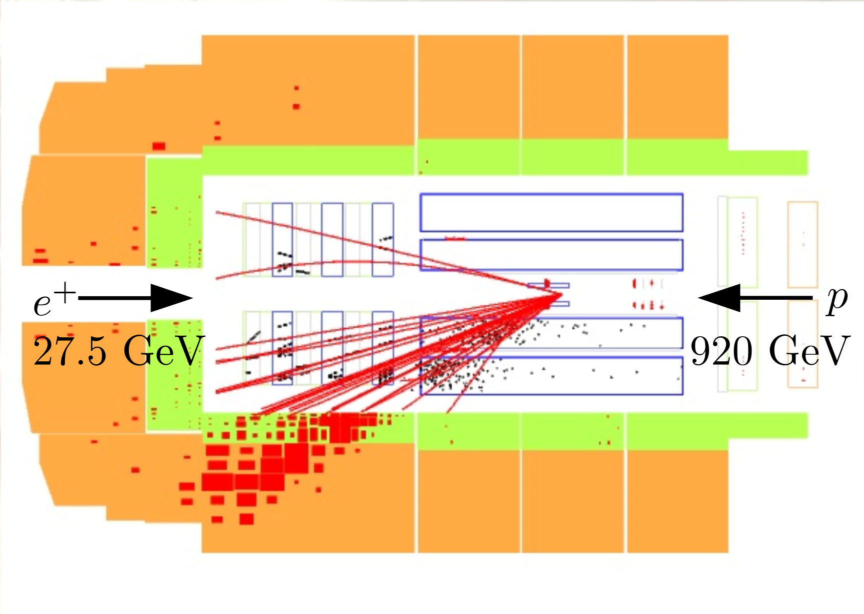


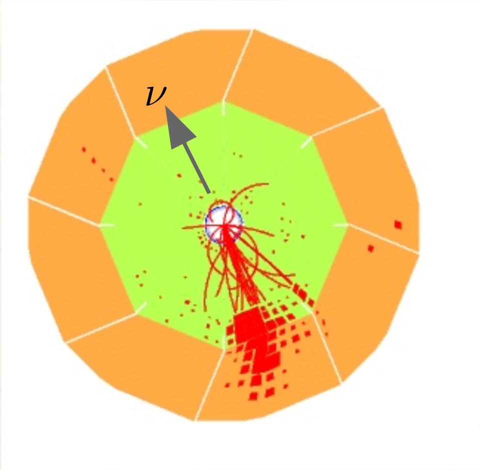


H1 Experiment @ HERA

Change of flavor & charge

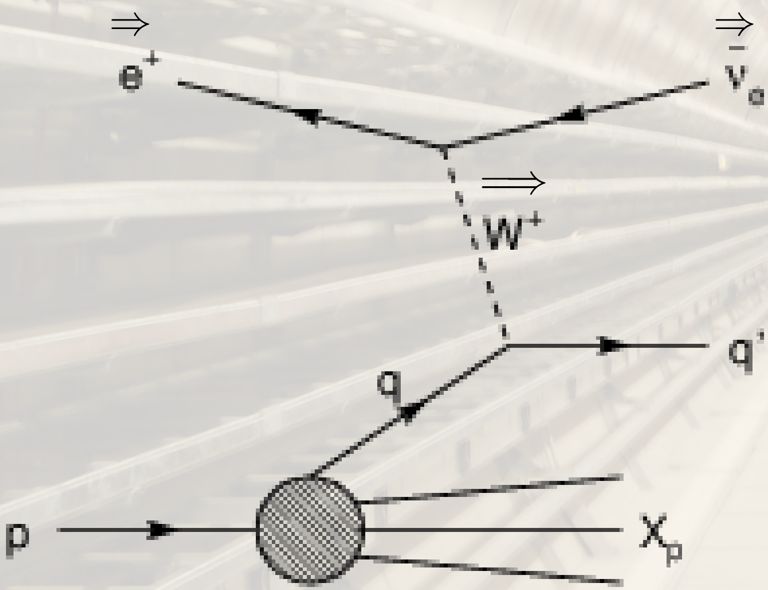


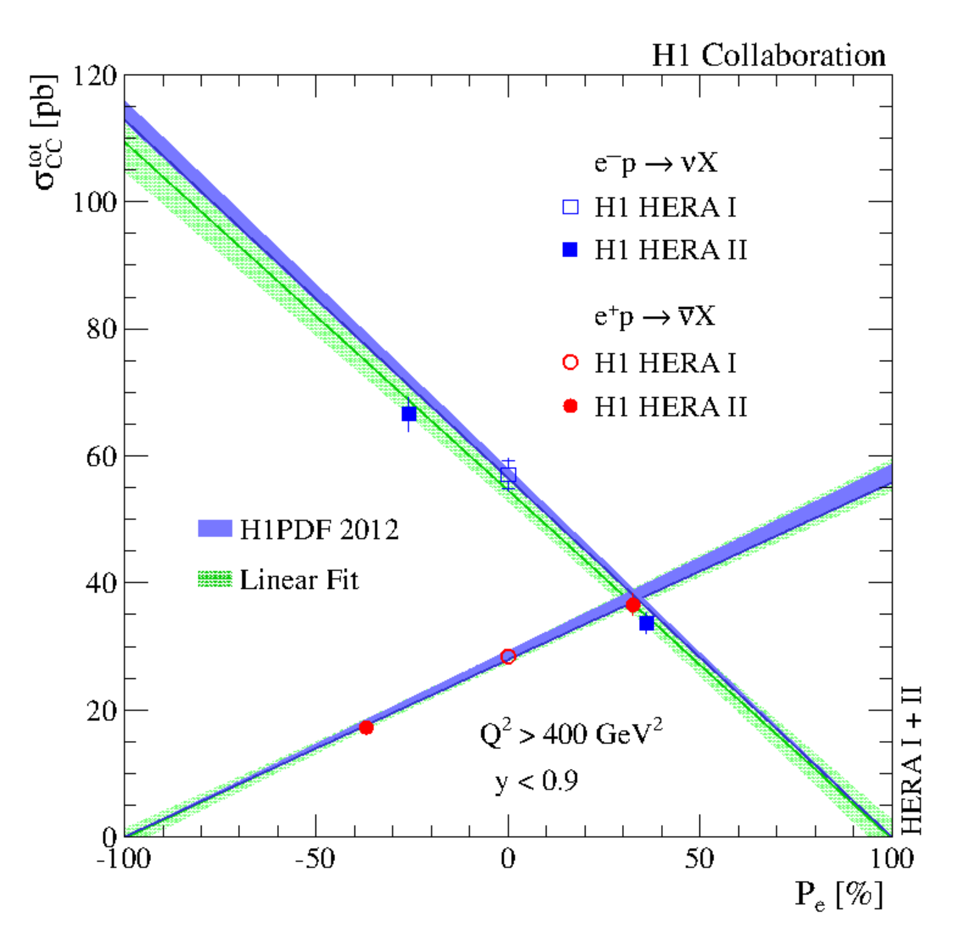




Parity violation

- W bosons couple only to left-handed particles (and right-handed anti-particles):

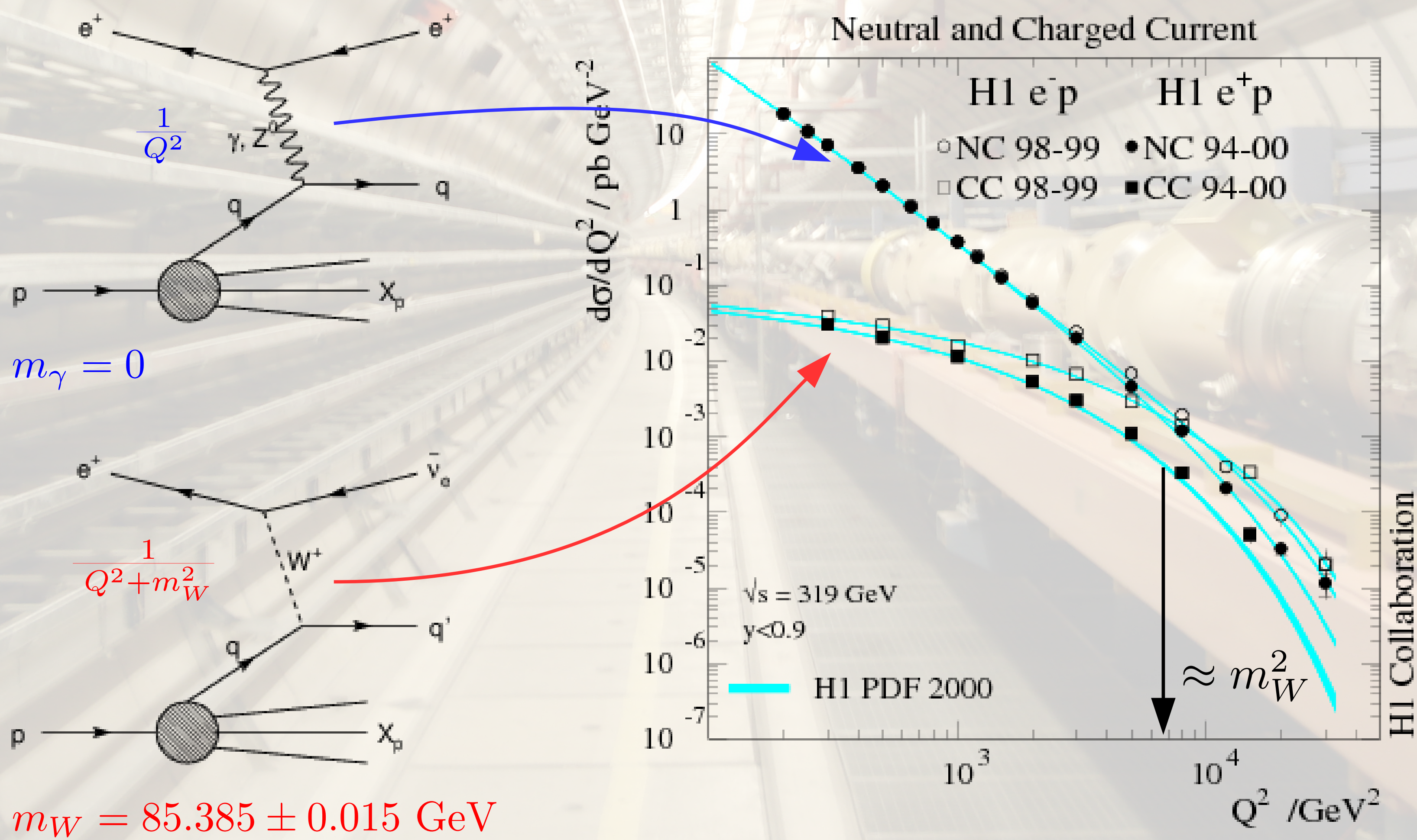




- Maximally parity violating!
- Intrinsically violating CP as well!

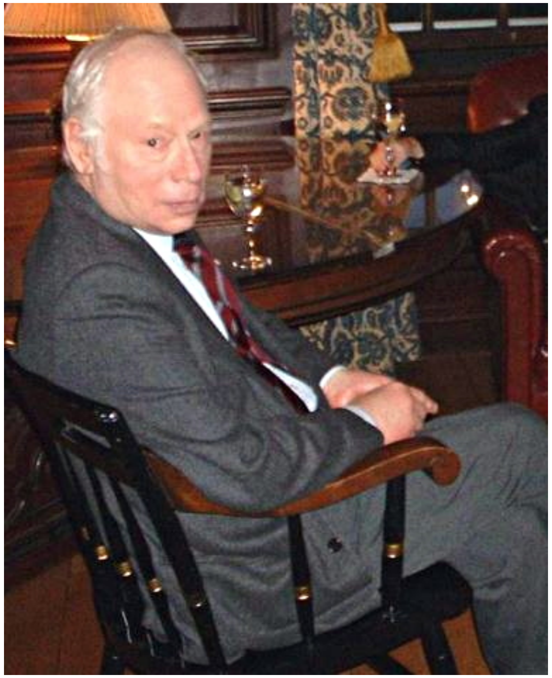
Heavy mediators

- Mediation by heavy gauge bosons:



The model of weak interactions

Sheldon Glashow
(*5. December 1932)



Steven Weinberg
(*3. Mai 1933)

$SU(2)$ space of weak isospin

(*) Transforms like a spin ½ object in space of weak isospin.

- Example:

$$\psi_L = \begin{pmatrix} \nu \\ e \end{pmatrix}_L \longrightarrow$$

- *left-handed* e_L & ν form ***isospin doublet***.(*)

$$e_R \longrightarrow$$

- *right-handed* e_R forms ***isospin singlet***.

- *Left- & right-handed* components of fermions can be projected conveniently:

$$e = e_L + e_R \quad \begin{cases} e_L = \left(\frac{1-\gamma^5}{2}\right) e \\ e_R = \left(\frac{1+\gamma^5}{2}\right) e \end{cases} \qquad \bar{e}\gamma^\mu \left(\frac{1-\gamma^5}{2}\right) \nu = \bar{e}_L \gamma^\mu \nu_L$$

- Lagrangian w/o mass terms can be written in form:

$$\mathcal{L}_0 = \overline{\psi}_L \gamma^\mu \partial_\mu \psi_L + \bar{e}_R \gamma^\mu \partial_\mu e_R = \bar{e}_L \gamma^\mu \partial_\mu e_L + \bar{\nu} \gamma^\mu \partial_\mu \nu + \bar{e}_R \gamma^\mu \partial_\mu e_R$$

Covariant derivative of $SU(2) \times U(1)$

Covariant derivative corresponding to $SU(2)$ acts on *isospin doublet* only.

$$\mathcal{L}_{IA}^{SU(2)\times U(1)} = i\overline{\psi}_L \gamma^\mu \overbrace{\boxed{\left(\partial_\mu \qquad\qquad + igW_\mu^a \mathbf{t}^a\right)} \psi_L \cdots}$$

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$$\mathbf{t}^+ = \mathbf{t}_1 + i\,\mathbf{t}_2 = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$$ (*ascending operator*)

$$\mathbf{t}^- = \mathbf{t}_1 - i\,\mathbf{t}_2 = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$$ (*descending operator*)

$$\mathbf{t}^3 = {}^1\!/_2 \cdot \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$W_\mu^{\mathrm{a}}\mathbf{t}^{\mathrm{a}} = \frac{1}{\sqrt{2}}\left(W_\mu^+\mathbf{t}^+ + W_\mu^-\mathbf{t}^-\right) + W_\mu^3\mathbf{t}^3$$

Covariant derivative of $SU(2) \times U(1)$

Covariant derivative corresponding to $SU(2)$ acts on *isospin doublet* only.

$$\mathcal{L}_{IA}^{SU(2)\times U(1)} = i\overline{\psi}_L\gamma^\mu \boxed{\left(\partial_\mu + i\frac{g'}{2}Y_L B_\mu + igW_\mu^{\mathrm{a}}\mathbf{t}^{\mathrm{a}}\right)} \psi_L + i\overline{e}_R\gamma^\mu \left(\partial_\mu + i\frac{g'}{2}Y_R B_\mu\right) e_R$$

Covariant derivative corresponding to $U(1)$ acts on *isospin doublet* (as a whole) and on *isospin singlet.*

$$W_\mu^{\mathrm{a}}\mathbf{t}^{\mathrm{a}} = \frac{1}{\sqrt{2}}\left(W_\mu^+\mathbf{t}^+ + W_\mu^-\mathbf{t}^-\right) + W_\mu^3\mathbf{t}^3$$

Covariant derivative of $SU(2) \times U(1)$

Covariant derivative corresponding to $SU(2)$ acts on *isospin doublet* only.

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Covariant derivative corresponding to $U(1)$ acts on *isospin doublet* (as a whole) and on *isospin singlet.*

$SU(2)\times U(1)$ Hypercharges			
Particle	$Y_{R/L}$	I_3	Q
ν	-1	$+1/2$	
e_L	-1	$-1/2$	
e_R		0	-1

$$Q = I_3 + \frac{Y}{2} \quad \text{(Gell-Mann Nischijama)}$$

$$W_\mu^a \mathbf{t}^a = \frac{1}{\sqrt{2}}\left(W_\mu^+\mathbf{t}^+ + W_\mu^-\mathbf{t}^-\right) + W_\mu^3\mathbf{t}^3$$

Covariant derivative of $SU(2) \times U(1)$

Covariant derivative corresponding to $SU(2)$ acts on *isospin doublet* only.

$$\mathcal{L}_{IA}^{SU(2)\times U(1)} = i\overline{\psi}_L\gamma^\mu \boxed{\left(\partial_\mu + i\frac{g'}{2}Y_L B_\mu + igW_\mu^a \mathbf{t}^a\right)} \psi_L + i\overline{e}_R\gamma^\mu\left(\partial_\mu + i\frac{g'}{2}Y_R B_\mu\right)e_R$$

Covariant derivative corresponding to $U(1)$ acts on *isospin doublet* (as a whole) and on *isospin singlet.*

$SU(2) \times U(1)$ Hypercharges			
Particle	$Y_{R/L}$	I_3	Q
ν	-1	$+1/2$	0
e_L	-1	$-1/2$	-1
e_R	-2	0	-1

$$Q = I_3 + \frac{Y}{2} \quad \text{(Gell-Mann Nischijama)}$$

$$W_\mu^a \mathbf{t}^a = \frac{1}{\sqrt{2}}\left(W_\mu^+\mathbf{t}^+ + W_\mu^-\mathbf{t}^-\right) + W_\mu^3\mathbf{t}^3$$

$SU(2) \times U(1)$ interactions

- Charged current interaction:

$$\mathcal{L}_{IA}^{CC} = -\frac{g}{\sqrt{2}}\left[\overline{\nu}\left(W_\mu^+\gamma^\mu\right)e_L + \overline{e}_L\left(W_\mu^-\gamma^\mu\right)\nu\right]$$

from $\mathbf{t}^+$ — $e \to \nu$; $\nu \to e$ — from $\mathbf{t}^-$

$$\ldots\left(\overline{\nu} \quad \overline{e}_L\right)\begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} \nu \\ e_L \end{pmatrix}$$

- Neutral current interaction:

$$\mathcal{L}_{IA}^{NC} = -\left(\frac{g}{2}W_\mu^3 - \frac{g'}{2}B_\mu\right)\left(\overline{\nu}\gamma^\mu\nu\right) + \left(\frac{g}{2}W_\mu^3 + \frac{g'}{2}B_\mu\right)\left(\overline{e}_L\gamma^\mu e_L\right) + g' B_\mu\left(\overline{e}_R\gamma^\mu e_R\right)$$

from $\mathbf{t}^3$ — $\propto Z_\mu$

$$W_\mu^a \mathbf{t}^a = \frac{1}{\sqrt{2}}\left(W_\mu^+\mathbf{t}^+ + W_\mu^-\mathbf{t}^-\right) + W_\mu^3\mathbf{t}^3$$

$SU(2) \times U(1)$ interactions

- Charged current interaction:

$$\mathcal{L}_{IA}^{CC} = -\frac{g}{\sqrt{2}}\left[\underbrace{\overline{\nu}\left(W_\mu^+\gamma^\mu\right)e_L}_{e \to \nu} + \underbrace{\overline{e}_L\left(W_\mu^-\gamma^\mu\right)\nu}_{\nu \to e}\right]$$

from t^+

from t^-

$$\ldots\left(\overline{\nu} \quad \overline{e}_L\right)\begin{pmatrix}0 & 0\\ 1 & 0\end{pmatrix}\begin{pmatrix}\nu\\ e_L\end{pmatrix}$$

- Neutral current interaction:

$$\mathcal{L}_{IA}^{NC} = -\underbrace{\left(\frac{g}{2}W_\mu^3 - \frac{g'}{2}B_\mu\right)}_{\propto Z_\mu}(\overline{\nu}\gamma^\mu\nu) + \left(\frac{g}{2}W_\mu^3 + \frac{g'}{2}B_\mu\right)(\overline{e}_L\gamma^\mu e_L) + g'B_\mu\left(\overline{e}_R\gamma^\mu e_R\right)$$

from t^3

$$\begin{pmatrix}Z_\mu\\ A_\mu\end{pmatrix} = \begin{pmatrix}\cos\theta_W & -\sin\theta_W\\ \sin\theta_W & \cos\theta_W\end{pmatrix}\begin{pmatrix}W_\mu^3\\ B_\mu\end{pmatrix}$$

$$\sin\theta_W = \frac{g'}{\sqrt{g^2+g'^2}} \qquad \cos\theta_W = \frac{g}{\sqrt{g^2+g'^2}}$$

(*Weinberg rotation*)

$SU(2) \times U(1)$ interactions

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- Charged current interaction:

$$\mathcal{L}_{IA}^{CC} = -\frac{g}{\sqrt{2}}\left[\underbrace{\overline{\nu}\left(W_\mu^+\gamma^\mu\right)e_L}_{e\,\to\,\nu} + \underbrace{\overline{e}_L\left(W_\mu^-\gamma^\mu\right)\nu}_{\nu\,\to\,e}\right]$$

Desired behavior: A_μ couples to left- and right handed component of e in the same way!

- Neutral current interaction:

$$\begin{aligned}\mathcal{L}_{IA}^{NC} = &-\frac{\sqrt{g^2+g'^2}}{2} Z_\mu \left(\overline{\nu}\gamma_\mu\nu\right) \\ &+\frac{\sqrt{g^2+g'^2}}{2}\left[\left(\cos^2\theta_W - \sin^2\theta_W\right) Z_\mu + 2\sin\theta_W\cos\theta_W A_\mu\right]\left(\overline{e}_L\gamma_\mu e_L\right) \\ &+\frac{\sqrt{g^2+g'^2}}{2}\left[\qquad -2\sin^2\theta_W\, Z_\mu + 2\sin\theta_W\cos\theta_W A_\mu\right]\left(\overline{e}_R\gamma_\mu e_R\right)\end{aligned}$$

What is the expression for e ?

$SU(2) \times U(1)$ **interactions**

- Charged current interaction:

$$\mathcal{L}_{IA}^{CC} = -\frac{g}{\sqrt{2}}\left[\underbrace{\overline{\nu}\left(W_\mu^+\gamma^\mu\right)e_L}_{e\to\nu} + \underbrace{\overline{e}_L\left(W_\mu^-\gamma^\mu\right)\nu}_{\nu\to e}\right]$$

Desired behavior: A_μ couples to left- and right handed component of e in the same way!

- Neutral current interaction:

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What is the expression for e? → $e = \sqrt{g^2+g'^2}\sin\theta_W\cos\theta_W$

Skewness of the $SU(2) \times U(1)$

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- Gauge boson *eigenstates* of the symmetry do not correspond to the *eigenstates* of the IA:

$$\begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta_W & -\sin\theta_W \\ \sin\theta_W & \cos\theta_W \end{pmatrix} \begin{pmatrix} W_\mu^3 \\ B_\mu \end{pmatrix}$$

- Quark *eigenstates* of the $SU(2)$ do not correspond to the quark *eigenstates* of the $SU(3)$(NB: which are the mass *eigenstates*):

$$\mathcal{M}_{\mathrm{CKM}} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

$$= \begin{pmatrix} c_1 & s_1 c_3 & s_1 s_3 \\ -s_1 c_2 & c_1 c_2 c_3 - s_2 s_3 e^{i\delta} & c_1 c_2 s_3 + s_2 c_3 e^{i\delta} \\ -s_1 s_2 & c_1 s_2 c_3 + c_2 s_3 e^{i\delta} & c_1 s_2 s_3 - c_2 c_3 e^{i\delta} \end{pmatrix}$$

$$c_i = \cos\vartheta_i \; ; \; s_i = \sin\vartheta_i \; (i = 1...3)$$

Non-Abelian gauge structure of $SU(2)$

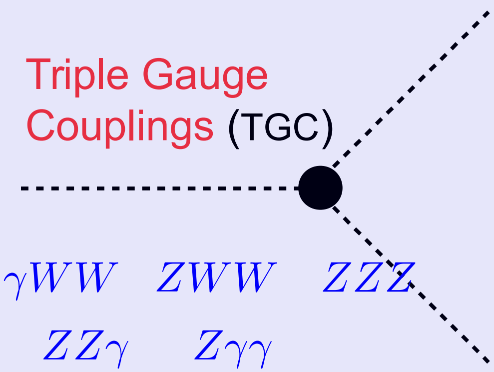
$$\mathcal{L}^{\text{gauge}} = -\frac{1}{2}\text{Tr}\left(W^a_{\mu\nu}W^{a\mu\nu}\right) - \frac{1}{4}B_{\mu\nu}B^{\mu\nu}\Bigg| \quad \begin{matrix} B_\mu \to A_\mu \\ W^3_\mu \to Z_\mu \end{matrix}$$

$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$$

$$W_{\mu\nu} = \partial_\mu W^a_\nu - \partial_\nu W^a_\mu + ig\left[W^a_\mu, W^a_\nu\right]$$

- This part of the Lagrangian density introduces:

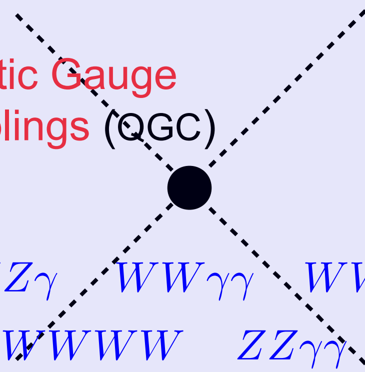
Triple Gauge Couplings (TGC)



γWW ZWW ZZZ

$ZZ\gamma$ $Z\gamma\gamma$

Quartic Gauge Couplings (QGC)



$WWZ\gamma$ $WW\gamma\gamma$ $WWZZ$

$WWWW$ $ZZ\gamma\gamma$

Which couplings are allowed (at tree level), which are not?

Non-Abelian gauge structure of $SU(2)$

$$\mathcal{L}^{\text{gauge}} = -\frac{1}{2}\text{Tr}\left(W^a_{\mu\nu}W^{a\mu\nu}\right) - \frac{1}{4}B_{\mu\nu}B^{\mu\nu}$$

$$B_\mu \to A_\mu$$
$$W^3_\mu \to Z_\mu$$

$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$$

$$W_{\mu\nu} = \partial_\mu W^a_\nu - \partial_\nu W^a_\mu + ig\left[W^a_\mu, W^a_\nu\right]$$

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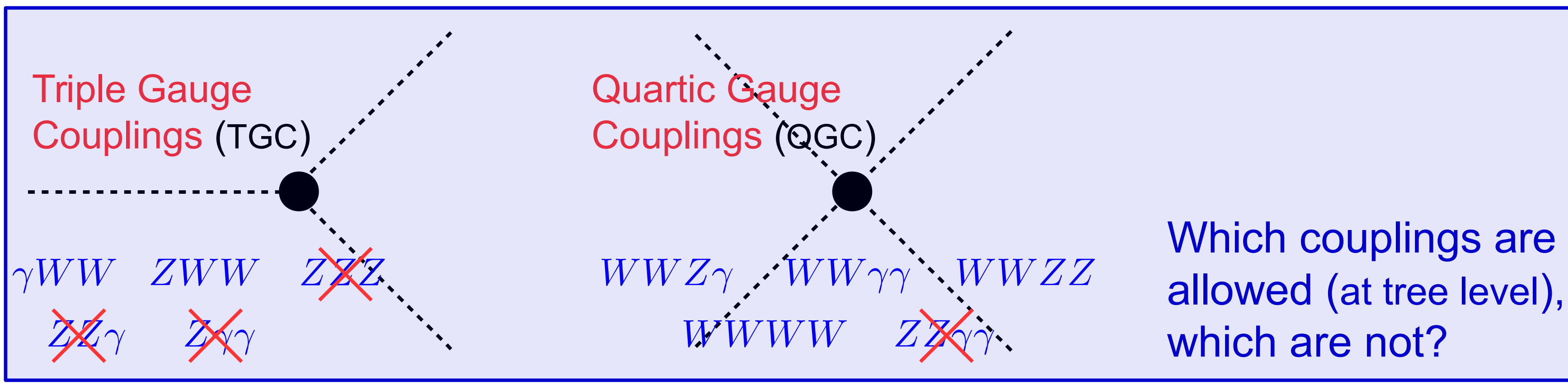

Concluding remarks

- $SU(3) \times SU(2) \times U(1)$ gauge symmetries of the SM are internal continuous symmetries (→ corresponding to Lie-transformations).

- Of those symmetries the “$SU(2)$-part“ has the most peculiar behavior:

- Fermions can *change charge* at IA vertex;
- Fermions can *change flavor* at IA vertex;
- No *parity* conservation;
- No *CP* conservation;
- No “*EWK symmetry* conservation”!
- ...

- Next lecture will discuss the problems of local gauge symmetries with massive particles and the principal of spontaneous symmetry breaking.

- Prepare “*The Higgs Boson Discovery at the Large Hadron Collider*” Section 2.3.

Backup

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