Spontaneous Symmetry Breaking and the Higgs Mechanism

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INSTITUTE OF EXPERIMENTAL PARTICLE PHYSICS (IEKP) – PHYSICS FACULTY

Schedule for today

- Is the following statement true: “all parts of the $SU(3) \times SU(2) \times U(1)$ symmetry have a general problem both with mass terms for gauge bosons and fermions”?

- Is the following statement true: “the Higgs boson is a *Goldstone* boson”?

3 The Higgs mechanism

2 Spontaneous symmetry breaking

1 The problem of masses in the SM

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Mass terms in the SM

The problem of massive gauge bosons

- Example: *Abelian* gauge field theories (→ see first lecture).

- Transformation: $A_\mu \rightarrow A'_\mu = A_\mu + \frac{1}{e}\partial_\mu\vartheta$

- In mass term : $m_A A_\mu A^{\mu *} \rightarrow m_A A'_\mu A'^{\mu *} =$

$$m_A A_\mu A^{\mu *} + \frac{1}{e} m_A \left(A_\mu \partial^\mu \vartheta + A^{\mu *}\partial_\mu \vartheta\right) + m_A \frac{1}{e^2}\partial_\mu\vartheta\partial^\mu\vartheta$$

These terms explicitly break local gauge covariance of $\mathcal{L}$.

- This is a fundamental problem for all gauge field theories.

- Remember: in Lecture-02 we have explicitly shown that the gauge field naturally emerges as a boson with mass zero.

The problem of massive fermions

- Check $U(1)$: ✓

- Transformation: $\psi \to \psi' = e^{i\vartheta} \psi$ $\quad \overline{\psi} \to \overline{\psi}' = \overline{\psi} e^{-i\vartheta}$
- In mass term : $m_\psi \overline{\psi}\psi \to m_{\psi'} \overline{\psi'}\psi' = m_\psi \overline{\psi}\psi$

- No obvious problem with fermion masses here. So is it a problem of *non-abelian* gauge symmetries?

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- Check $SU(3)$: ✓

  Similarly no problem in $SU(3) \to$ no problem of non-Abelian gauge field theories.

The problem of massive fermions

- Check $U(1)$:

- Transformation: $\psi \to \psi' = e^{i\vartheta}\ \psi$ $\qquad \overline{\psi} \to \overline{\psi}' = \overline{\psi} e^{-i\vartheta}$
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- What is the problem of $SU(2)$ in the SM?

The problem of massive fermions

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  - In mass term : $m_\psi \overline{\psi}\psi \rightarrow m_{\psi'} \overline{\psi'}\psi' = m_\psi \overline{\psi}\psi$

- What is the problem of $SU(2)$ in the SM?

- It is the distinction between left- (ψ_L) and right-handed (e_R) fermions, with different coupling structure:

$$m_e \bar{e} e = m_e \overline{(e_L + e_R)}(e_L + e_R) = \overbrace{m_e \bar{e}_R e_L + m_e \bar{e}_L e_R}^{(1)}$$

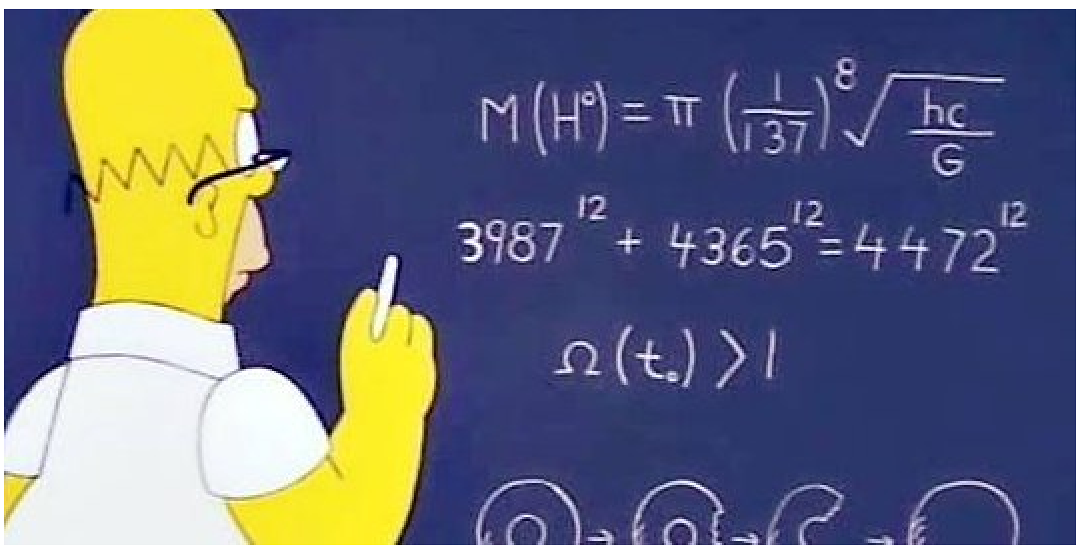
$SU(2)$ singlet

lower component of $SU(2)$ doublet.

(1) check Exercise 9 e).

Success of the $SU(2) \times U(1)$

- Can motivate structure of interactions between elementary particles.
- Gives geometrical interpretation for the presence of gauge bosons (transport of phase information from one space point to another).
- Predicts non-trivial self-interactions and couplings of Z boson to left- and right-handed fermions.



Dilemma of the $SU(2) \times U(1)$

- Can motivate structure of interactions betwee…
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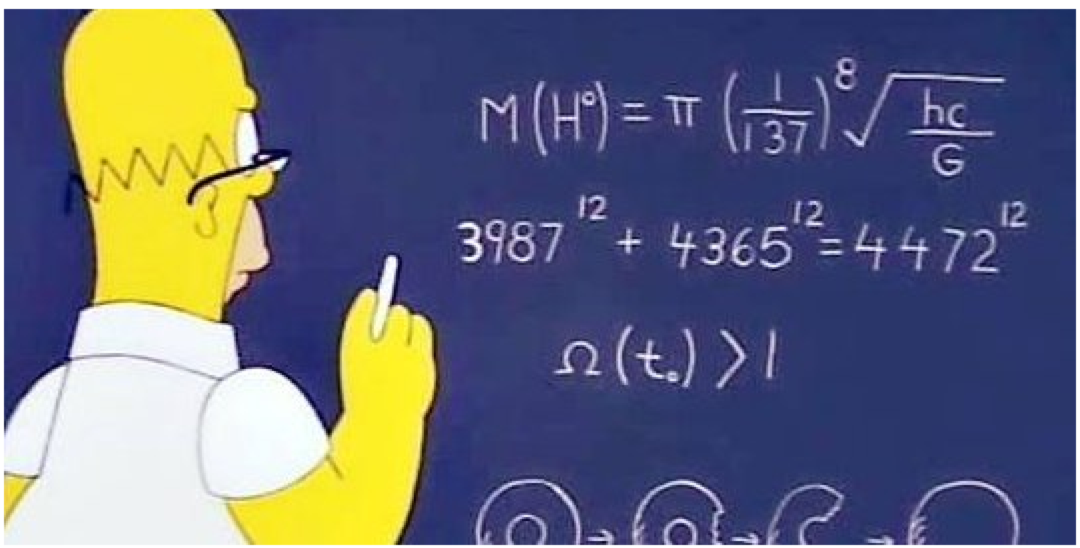
Explicitly incompatible with (naive) incorporation of mass terms in Lagrangian density.



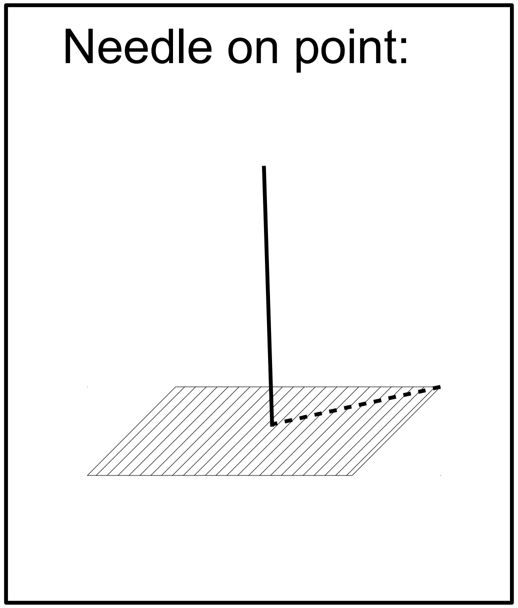
The remedy


Bach
ORIGINAL
FLOWER REMEDIES
RESCUE
REMEDY
TO COMFORT
& REASSURE
10ml

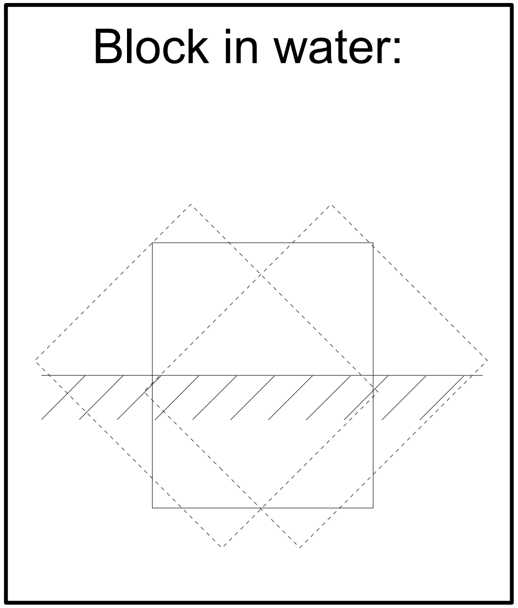

NDC# 057687-245-50
Bach
ORIGINAL
FLOWER REMEDIES
RESCUE
REMEDY
PASTILLES
NATURAL
STRESS RELIEF
ALCOHOL FREE
SUGAR FREE
50g PASTILLES
(1.7 oz)

Spontaneous symmetry breaking

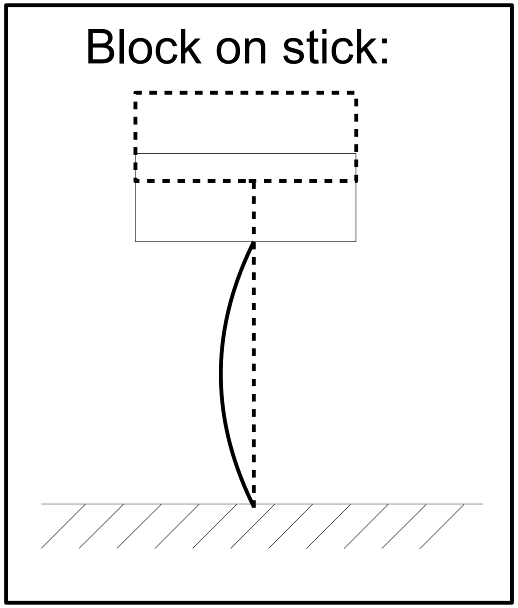
- Symmetry is present in the system (i.e. in the Lagrangian density $\mathcal{L}$).
- BUT it is broken in the ground state (i.e. in the quantum vacuum).
- Three examples (from classical mechanics):



φ symmetry



axis-symmetry

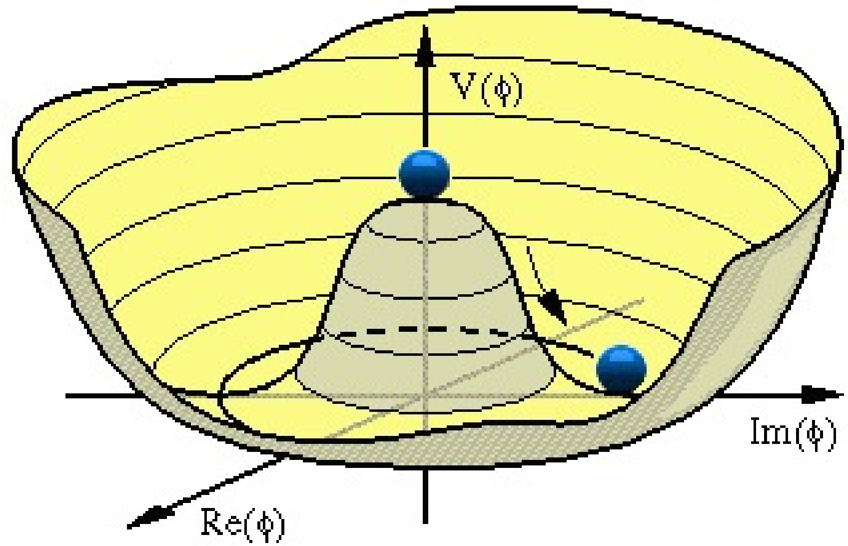


φ symmetry

Incorporation in particle physics

- *Goldstone* Potential:

$$\phi = \frac{1}{\sqrt{2}}(\phi_1 + i\phi_2)$$
$$V(\phi) = -\mu^2|\phi|^2 + \lambda|\phi|^4$$
$$\mathcal{L}(\phi) = \partial_\mu\phi\partial^\mu\phi^* - V(\phi)$$



- invariant under $U(1)$ transformations (i.e. φ symmetric).
- metastable in $\phi = 0$.
- ground state breaks $U(1)$ symmetry, BUT at the same time all ground states are in-distinguishable in φ .

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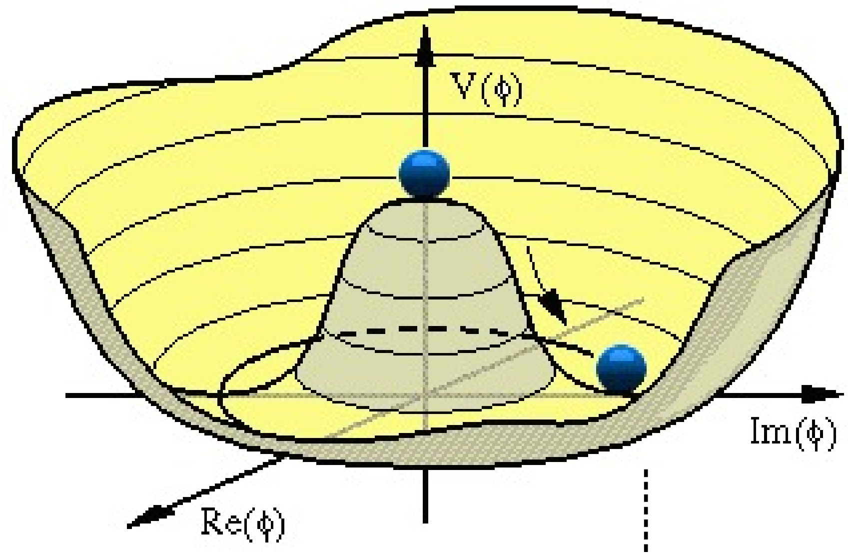
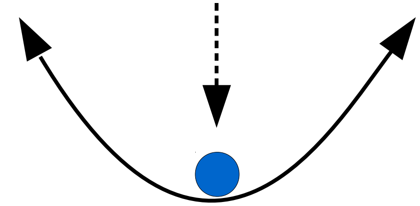


- ϕ has radial excitations in the potential $V(\phi)$.



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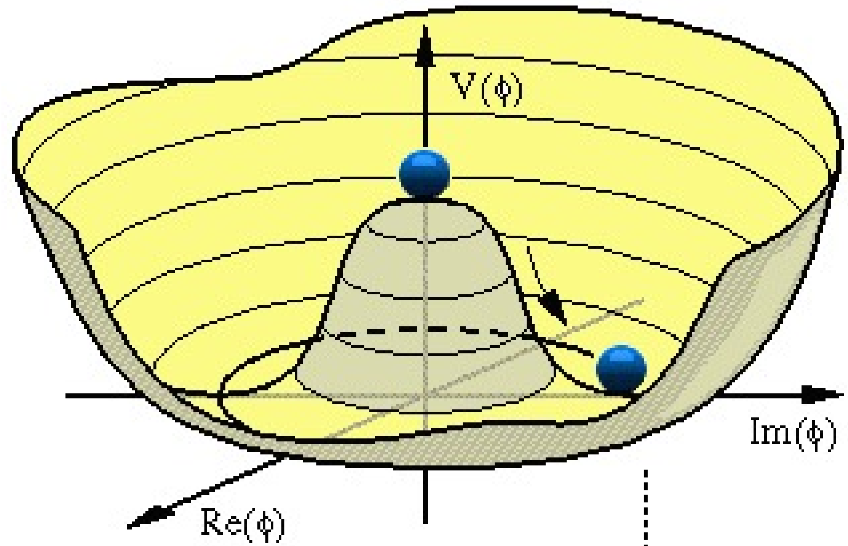
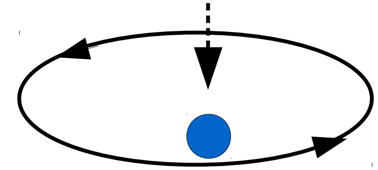


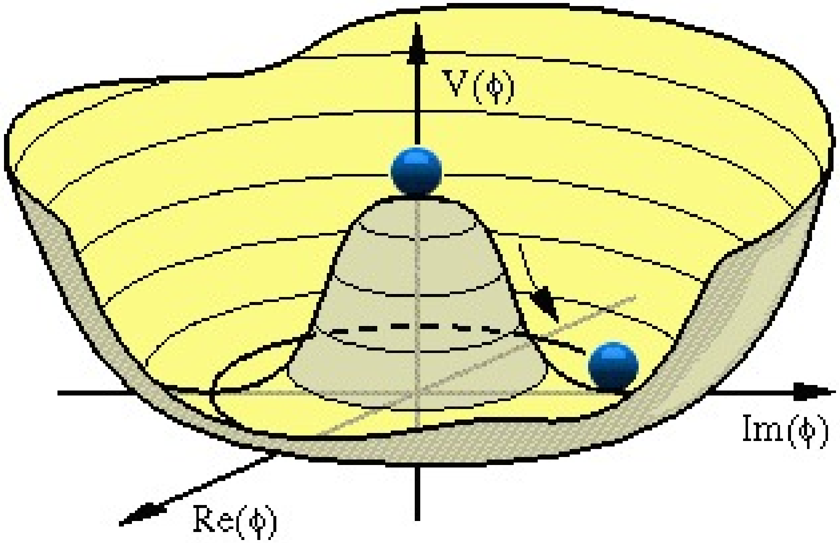
- invariant under $U(1)$ transformations (i.e. φ symmetric).
- metastable in $\phi = 0$.
- ground state breaks $U(1)$ symmetry, BUT at the same time all ground states are in-distinguishable in φ .
- ϕ can "move freely" in the circle that corresponds to the minimum of $V(\phi)$.



The *Goldstone* theorem

- In particle physics this is formalized in the *Goldstone* theorem:

> In a relativistic covariant quantum field theory with spontaneously broken symmetries massless particles (=*Goldstone* bosons) are created.



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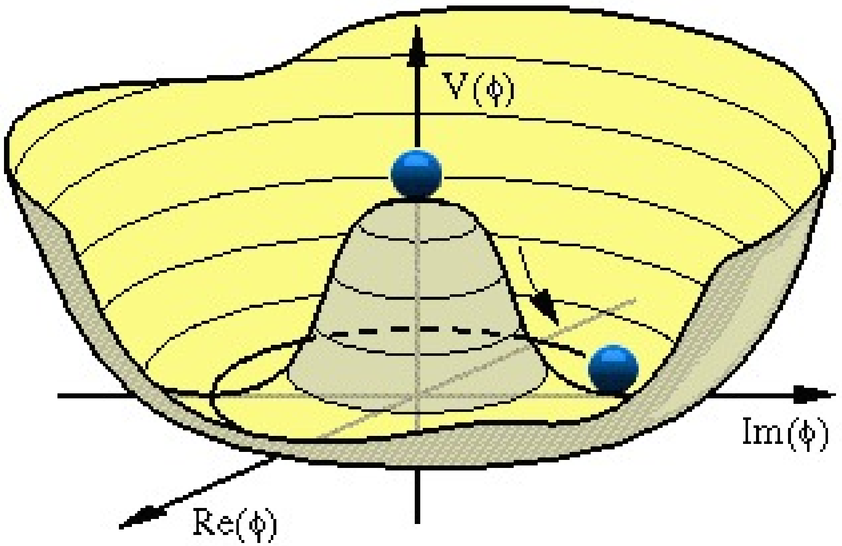


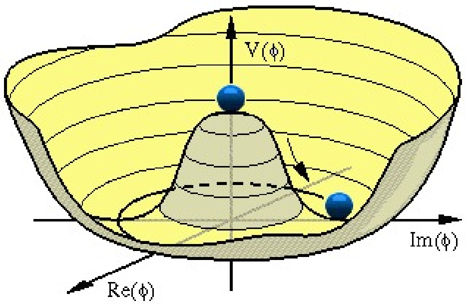
- *Goldstone* Bosons can be:
  - Elementary fields, which are already part of $\mathcal{L}$.
  - Bound states, which are created by the theory (e.g. the H-atom, Cooper-pairs, ...).
  - Unphysical or gauge degrees of freedom, which can be removed by appropriate boundary conditions.

Analyzing the energy ground state

- The energy ground state is where the *Hameltonian* operator

$$\mathcal{H} = \frac{\partial L}{\partial\left(\partial_0 \phi\right)} \partial_0 \phi - \mathcal{L} = \partial_0 \phi \partial^0 \phi^* + \partial_j \phi \partial^j \phi^* + V(\phi)$$

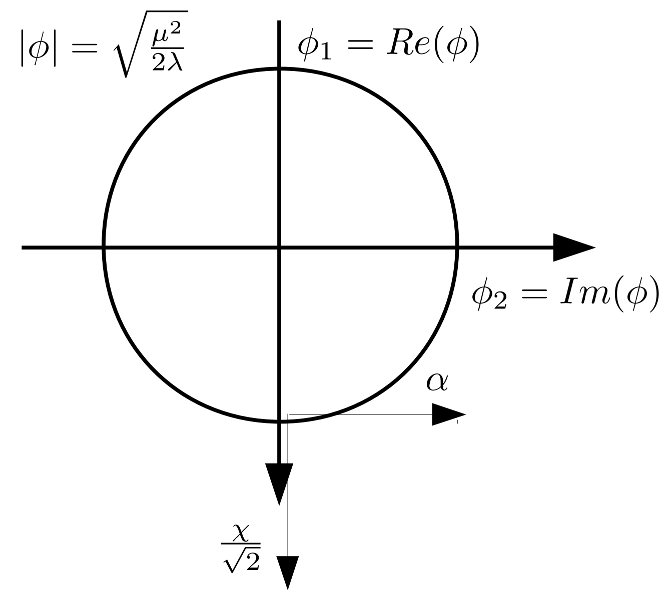
is minimal. This is the case for $|\phi| = \sqrt{\frac{\mu^2}{2\lambda}}$



- To analyze the system in its physical ground state we can make an expansion in an arbitrary point on this cycle:

$$\phi(\chi, \alpha) = e^{i\alpha} \left(\sqrt{\frac{\mu^2}{2\lambda}} + \frac{\chi}{\sqrt{2}} \right)$$

Top view on *Goldstone* potential



Dynamic mass terms (part-1)

- An expansion in the ground state in cylindrical coordinates leads to:

$$\mathcal{L} = \Big[\partial_\mu\phi\partial^\mu\phi^* - V(\phi)\Big]_{\phi(\chi,\alpha)} = \frac{1}{2}\partial_\mu\chi\partial^\mu\chi + \left(\sqrt{\frac{\mu^2}{2\lambda}} + \frac{\chi}{\sqrt{2}}\right)^2 \partial_\mu\alpha\partial^\mu\alpha - V'(\chi)$$

$$V'(\chi) = \Big[-\mu^2|\phi|^2 + \lambda|\phi|^4\Big]_{\phi(\chi)} = -\frac{\mu^4}{4\lambda} + \mu^2\chi^2 + \mu\sqrt{\lambda}\chi^3 + \frac{\lambda}{4}\chi^4$$

const.

dynamic mass term

self-couplings

- Why is there no linear term in χ ?

Dynamic mass terms (part-1)

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$-\frac{\mu^4}{4\lambda}$: *const.*

$\mu^2\chi^2$: *dynamic mass term*

$\mu\sqrt{\lambda}\chi^3 + \frac{\lambda}{4}\chi^4$: *self-couplings*

- Why is there no linear term in χ ?

- We have performed a *Taylor* expansion in the minimum. By construction there cannot be any linear terms in there.

Dynamic mass terms (part-1)

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- **Remarks:**
  - The mass term is acquired for the field χ along the radial excitation, which leads out of the minimum of $V(\chi)$. It is the term at lowest order in the *Taylor* expansion in the minimum, and therefore independent from the concrete form of $V(\chi)$ in the minimum.
  - The field α, which does not lead out of the minimum of $V(\chi)$ does not acquire a mass term. It corresponds to the *Goldstone* boson.

Extension to a gauge field theory

- For simplicity reasons shown for an *Abelian* model:

Introduce covariant derivative

$$\phi(\chi,\alpha) = e^{i\alpha}\left(\sqrt{\frac{\mu^2}{2\lambda}} + \frac{\chi}{\sqrt{2}}\right)$$

$$\mathcal{L} = \left[\left(\partial_\mu + ieA_\mu\right)\phi\right]\left[\left(\partial^\mu + ieA^\mu\right)\phi\right]^* - V(\phi) - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

$$= \left|\frac{1}{\sqrt{2}}\partial_\mu\chi e^{i\alpha} + ie^{i\alpha}\left(\sqrt{\frac{\mu^2}{\lambda}} + \frac{\chi}{\sqrt{2}}\right)\left(eA_\mu + \partial_\mu\alpha\right)\right|^2 - V'(\chi) - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

$$= \frac{1}{2}\partial_\mu\chi\partial^\mu\chi + \left(\left(\sqrt{\frac{\mu^2}{2\lambda}} + \frac{\chi}{\sqrt{2}}\right)\underbrace{\left(eA_\mu + \partial_\mu\alpha\right)}\right)^2 - V'(\chi) - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

Remove by proper gauge:

$$A'_\mu = A_\mu - \frac{1}{e}\partial_\mu\vartheta$$

How does this gauge look like?

Extension to a gauge field theory

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Remove by proper gauge:

$A'_\mu = A_\mu - \frac{1}{e}\partial_\mu\vartheta$

How does this gauge look like? $\vartheta = -\alpha$

Extension to a gauge field theory

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$$= \frac{1}{2}\partial_\mu\chi\partial^\mu\chi + \left(\left(\sqrt{\frac{\mu^2}{2\lambda}} + \frac{\chi}{\sqrt{2}}\right) eA'_\mu \right)^2 - V'(\chi) - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

Mass term for A'_μ: $\frac{e^2\mu^2}{2\lambda}A'_\mu A'^{\mu*}$

Quartic and tri-linear couplings with χ.

The Higgs mechanism

- The expansion of $\phi \rightarrow \phi(\chi, \alpha)$ in the energy ground state of the *Goldstone* potential has generated a mass term $\frac{e^2\mu^2}{2\lambda}A'_\mu A^{\mu *}$ for the gauge field A'_μ from the bare coupling $e^2\left|\phi\right|^2 A'_\mu A'^{\mu *}$.

- χ is a real field, α has been absorbed into A'_μ. It seems as if one degree of freedom were lost. This is not the case:

  - as a massless particle A'_μ has only two degrees of freedom (±1 helicity states).

  - as a massive particle it gains one additional degree of freedom (±1-helicity states + 0-helicity state).

The Higgs mechanism

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> "The gauge boson has eaten up the *Goldstone* boson and has become fat on it".

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This shuffle of degrees of freedom from *Goldstone* boson(s) to gauge boson(s) is called equivalence principle of the Higgs mechanism.

Notes on the *Goldstone* potential (part-1)

- The choice of the *Goldstone* potential has the following properties:
  - it leads to spontaneous symmetry breaking.
  - it does not distinguish any direction in space (→ i.e. only depends on $|\phi|$).
  - it is bound from below and does not lead to infinite negative energies, which is a prerequisite for a stable theory.
  - it is the simplest potential with these features.

Notes on the *Goldstone* potential (part-2)

- The potential has been chosen to be cut at the order of $|\phi|^4$. This can be motivated by a dimensional analysis:
  - Due to gauge invariance ϕ has to appear in even order (c.f. transformation behavior of objects in Lecture-01).
  - What is the dimension of $\mathcal{L}$?

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  - What is the dimension of ϕ ?
  - What is the dimension of μ ?
  - What is the dimension of λ ?

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  - What is the dimension of $\mathcal{L}$? $[\mathcal{L}] = \mathrm{GeV}^4$
  - What is the dimension of ϕ ? $[\phi] = \mathrm{GeV}^1$
  - What is the dimension of μ ? $[\mu] = \mathrm{GeV}^1$
  - What is the dimension of λ ? $[\lambda] = \mathrm{GeV}^0$
- **NB:** it would be possible to extend the potential to higher dimensions of ϕ, but couplings with negative dimension will turn the theory non-renormalizable.

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Concluding remarks

- Today we have discussed the problem of mass terms in the SM.
- Keep in mind that the SM has two problems of masses with different origin.
- We have introduced the principles of spontaneous symmetry breaking and how it translates into particle physics as the *Goldstone* theorem.
- Finally we have implemented the concept of spontaneous symmetry breaking into an (*Abelian*) gauge field theory to see how the Higgs mechanism works.
- Next we will go through the implementation of the Higgs mechanism into the SM. This step will complete the SM as a theory.
- Prepare "*The Higgs Boson Discovery at the Large Hadron Collider*" Section 2.4.

Backup