Karlsruhe Institute of Technology

The Higgs Mechanism in the SM

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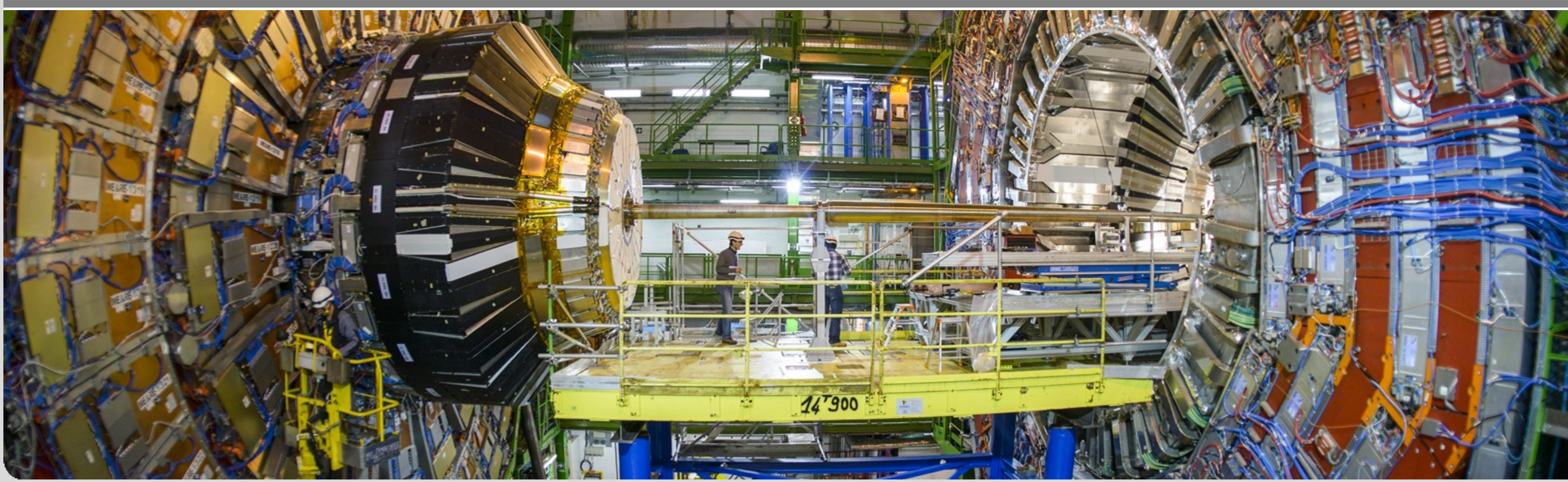
INSTITUTE OF EXPERIMENTAL PARTICLE PHYSICS (IEKP) – PHYSICS FACULTY



Schedule for today

- Is the following statement true: "the Higgs boson couples always proportional to the mass of the particle"?
- How many and which symmetries in the SM are broken?

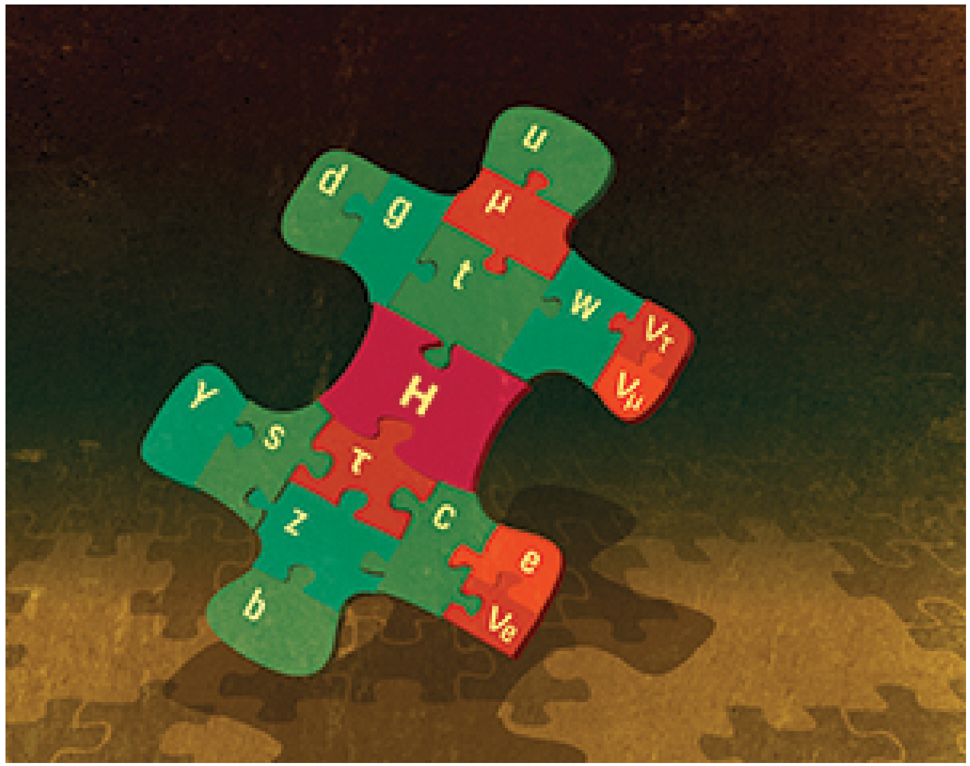
3 Obtaining massive fermions

2 Obtaining massive gauge bosons

1 Reprise: SM w/o masses

Institute of Experimental Particle Physics (IEKP)

The final construction of the SM



SM without mass terms

- Compilation of the last two lectures:

Fermion kinematics

$$\mathcal{L}^{SU(2)\times U(1)} = \mathcal{L}^{\text{kin}} + \mathcal{L}^{CC} + \mathcal{L}^{NC} + \mathcal{L}^{\text{gauge}}$$

$$\mathcal{L}^{\text{kin}} = i\bar{e}\gamma^{\mu}\partial_{\mu}e + i\bar{\nu}\gamma^{\mu}\partial_{\mu}\nu$$

$$\mathcal{L}^{CC} = -\frac{e}{\sqrt{2}\sin\theta_W}\left[W_{\mu}^{+}\bar{\nu}\gamma_{\mu}e_L + W_{\mu}^{-}\bar{e}_L\gamma_{\mu}\nu\right]$$

$$\mathcal{L}^{NC} = -\frac{e}{2\sin\theta_W\cos\theta_W}Z_{\mu}\left[(\bar{\nu}\gamma_{\mu}\nu) + (\bar{e}_L\gamma_{\mu}e_L)\right] - e\left[A_{\mu} + \tan\theta_W Z_{\mu}\right](\bar{e}\gamma_{\mu}e)$$

$$\mathcal{L}^{\text{gauge}} = -\frac{1}{2}Tr\left(W_{\mu\nu}^{a}W^{a\mu\nu}\right) - \frac{1}{4}B_{\mu\nu}B^{\mu\nu}\Bigg|_{\substack{B_{\mu}\rightarrow A_{\mu}\\ W_{\mu}^{3}\rightarrow Z_{\mu}}}$$

SM without mass terms

- Compilation of the last two lectures:

Charged current IA

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Fermion kinematics — Charged current IA — Neutral current IA

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SM without mass terms

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Fermion kinematics — Charged current IA — Neutral current IA — Gauge field kinematic

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Extension by a new field ϕ

- SM does not allow for naive introduction of mass terms for gauge bosons nor fermions.

- But possible to create mass terms dynamically via the Higgs mechanism. Requires that the symmetry in energy ground state must be spontaneously broken.

- All fields we have introduced so far do obey all symmetries, also in their energy ground state. → Need new field with self-interaction that leads to spontaneously symmetry breaking (*Goldstone*) potential.

The new field ϕ

- Add ϕ as $SU(2)$ doublet field:

$$\phi = \begin{pmatrix} \phi_+ \\ \phi_0 \end{pmatrix}$$

Transformation behavior:

$$\phi \rightarrow \phi' = e^{i\vartheta'} G\phi$$

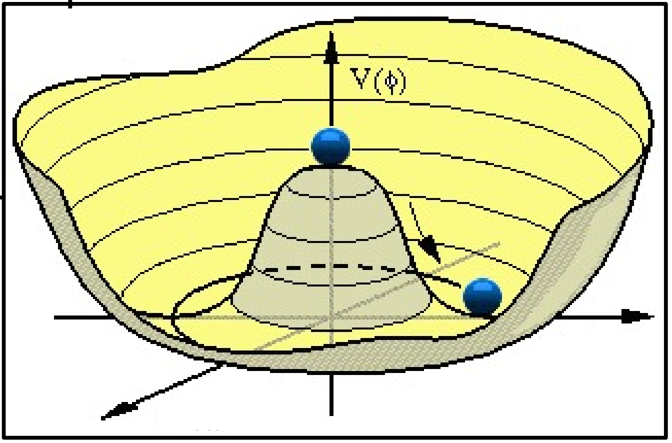
$$\phi^\dagger \rightarrow \phi'^\dagger = \phi^\dagger G^\dagger e^{-i\vartheta'}$$

$$G = e^{i\vartheta^a t^a} \in SU(2) \quad \vartheta^a, \vartheta' \in \mathbb{R}$$

$$\mathcal{L}^{SU(2)\times U(1)} = \mathcal{L}^{\text{kin}} + \mathcal{L}^{CC} + \mathcal{L}^{NC} + \mathcal{L}^{\text{gauge}} + \mathcal{L}^{\text{Higgs}}$$

$$\mathcal{L}^{\text{Higgs}} = \partial_\mu \phi^\dagger \partial^\mu \phi - V(\phi)$$

$$V(\phi) = -\mu^2 \phi^\dagger \phi + \lambda \left(\phi^\dagger \phi\right)^2$$



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Can you point to the *Goldstone* bosons?

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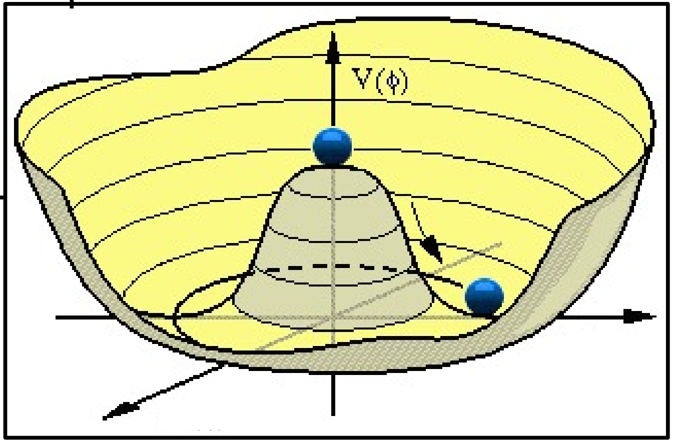

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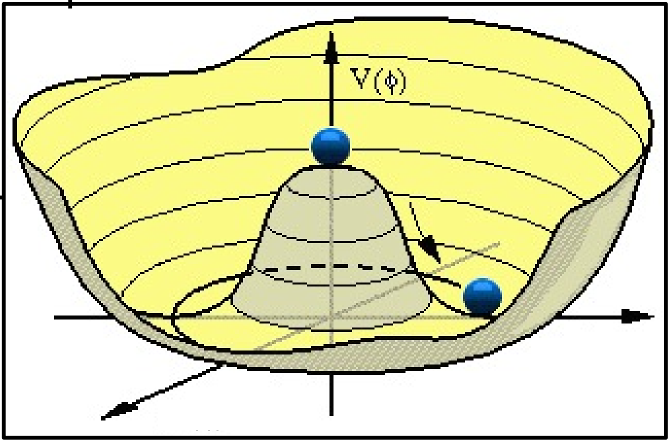


Expansion close to energy ground state

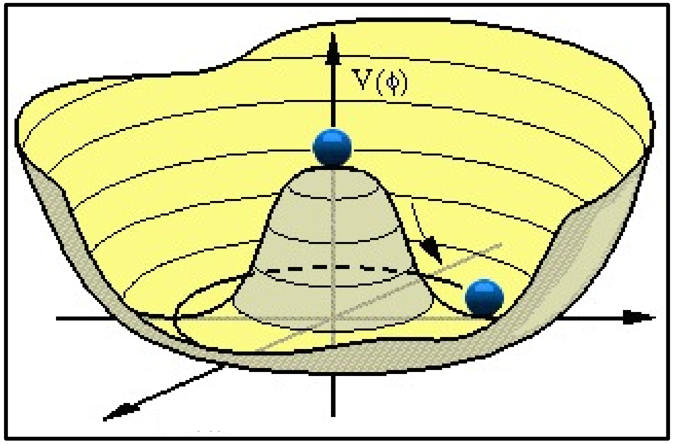
- Develop ϕ in its energy ground state at $|\phi| = \sqrt{\frac{\mu^2}{2\lambda}}$:

$$\phi = \begin{pmatrix} \phi_+ \\ \phi_0 \end{pmatrix} \xrightarrow{|\phi| = \sqrt{\frac{\mu^2}{2\lambda}}} \phi = \begin{pmatrix} 0 \\ \sqrt{\frac{\mu^2}{2\lambda}} + \frac{H}{\sqrt{2}} \end{pmatrix}$$

NB: In principle this can be done anywhere in the minimum. For a consistent model it is done in the lower component of ϕ.

Non-zero vacuum expectation value $v = \sqrt{\frac{\mu^2}{2\lambda}}$.

Radial excitation field. → This is the Higgs boson.



Enforcing local gauge invariance for ϕ

- Develop ϕ in its energy ground state at $|\phi| = \sqrt{\frac{\mu^2}{2\lambda}}$:

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- $\mathcal{L}^{\mathrm{Higgs}}$ is covariant under global $SU(2)$ transformations. Introduce covariant derivative D_μ to enforce local gauge invariance:

$$\partial_\mu \rightarrow D_\mu = \partial_\mu + ig' \frac{Y_\phi}{2} B_\mu + igW_\mu^a \mathbf{t}^a$$

$SU(2) \times U(1)$ Hypercharges			
Field	Y_ϕ	I_3	Q
ϕ_+	$+1$	$+1/2$	$+1$
ϕ_0		$-1/2$	0

$Q = I_3 + \frac{Y}{2}$

Dynamic term of Lagrangian density

- Couple to gauge fields:

$$\phi = \begin{pmatrix} \phi_+ \\ \phi_0 \end{pmatrix} \xrightarrow{|\phi| = \sqrt{\frac{\mu^2}{2\lambda}}} \phi = \begin{pmatrix} 0 \\ \sqrt{\frac{\mu^2}{2\lambda}} + \frac{H}{\sqrt{2}} \end{pmatrix}$$

$$D_\mu \phi^\dagger D^\mu \phi = \left| \left[\frac{1}{\sqrt{2}} \partial_\mu H + \left(\sqrt{\frac{\mu^2}{2\lambda}} + \frac{H}{\sqrt{2}} \right) \left(i g' \frac{Y_\phi}{2} B_\mu + i g W_\mu^a \mathbf{t}^a \right) \right] \begin{pmatrix} 0 \\ 1 \end{pmatrix} \right|^2$$

$$D_\mu = \partial_\mu + i g' \frac{Y_\phi}{2} B_\mu + i g W_\mu^a \mathbf{t}^a$$ (covariant derivative)

Dynamic term of Lagrangian density

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(covariant derivative)

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- Resolve products of *Pauli* matrices ($\mathbf{t}^a \equiv \frac{1}{2}\sigma_a$):

$$D_\mu\phi^\dagger D^\mu\phi = \left|\left[\frac{1}{\sqrt{2}}\partial_\mu H + \left(\sqrt{\frac{\mu^2}{2\lambda}} + \frac{H}{\sqrt{2}}\right)\left(ig'\frac{Y_\phi}{2}B_\mu + igW_\mu^a\mathbf{t}^a\right)\right]\begin{pmatrix} 0 \\ 1 \end{pmatrix}\right|^2$$

$$D_\mu\phi^\dagger D^\mu\phi = \left|\left[\frac{1}{\sqrt{2}}\partial_\mu H - \frac{i}{2}\left(\sqrt{\frac{\mu^2}{2\lambda}} + \frac{H}{\sqrt{2}}\right)(gW_\mu^3 - g'B_\mu)\right]\begin{pmatrix} 0 \\ 1 \end{pmatrix}\right|^2 + \left|\left[\frac{i}{2}\left(\sqrt{\frac{\mu^2}{2\lambda}} + \frac{H}{\sqrt{2}}\right)gW_\mu^+\right]\begin{pmatrix} 1 \\ 0 \end{pmatrix}\right|^2$$

Dynamic term of Lagrangian density

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- Resolve products of *Pauli* matrices ($\mathbf{t}^a \equiv \frac{1}{2}\sigma_a$):
  - Ascending operator $\mathbf{t}^+$ (of W_μ^+) shifts unit vector of the down component up.

$$D_\mu\phi^\dagger D^\mu\phi = \left|\left[\frac{1}{\sqrt{2}}\partial_\mu H + \left(\sqrt{\frac{\mu^2}{2\lambda}} + \frac{H}{\sqrt{2}}\right)\left(ig'\frac{Y_\phi}{2}B_\mu + igW_\mu^a\mathbf{t}^a\right)\right]\begin{pmatrix}0\\1\end{pmatrix}\right|^2$$

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Dynamic term of Lagrangian density

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  - Ascending operator $\mathbf{t}^+$(of W_μ^+) shifts unit vector of the down component up.
  - Descending operator $\mathbf{t}^-$(of W_μ^-) “destroys” unit vector of the down component.

$$D_\mu\phi^\dagger D^\mu\phi = \left| \left[\frac{1}{\sqrt{2}}\partial_\mu H + \left(\sqrt{\frac{\mu^2}{2\lambda}} + \frac{H}{\sqrt{2}} \right) \left(ig'\frac{Y_\phi}{2}B_\mu + igW_\mu^a \mathbf{t}^a \right) \right] \begin{pmatrix} 0 \\ 1 \end{pmatrix} \right|^2$$

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  - Operator $\mathbf{t}^3$ switches sign for unit vector of down component.

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Dynamic term of Lagrangian density

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- Evaluate components of absolute value squared:

$$D_\mu\phi^\dagger D^\mu\phi = \tfrac{1}{2}\partial_\mu H\partial^\mu H + \tfrac{g^2+g'^2}{4}\left(\sqrt{\tfrac{\mu^2}{2\lambda}}+\tfrac{H}{\sqrt{2}}\right)^2 Z_\mu Z^\mu + \tfrac{g^2}{4}\left(\sqrt{\tfrac{\mu^2}{2\lambda}}+\tfrac{H}{\sqrt{2}}\right)^2 W_\mu^+ W^{\mu-}$$

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Masses for Gauge Bosons

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- By introducing ϕ as a $SU(2)$ doublet with a non-zero energy ground state we have obtained:

$$D_\mu \phi^\dagger D^\mu \phi = \frac{1}{2}\partial_\mu H \partial^\mu H + \underbrace{\frac{g^2+g'^2}{4}\left(\sqrt{\frac{\mu^2}{2\lambda}} + \frac{H}{\sqrt{2}}\right)^2}_{} Z_\mu Z^\mu + \underbrace{\frac{g^2}{4}\left(\sqrt{\frac{\mu^2}{2\lambda}} + \frac{H}{\sqrt{2}}\right)^2}_{} W^+_\mu W^{\mu -}$$

- Masses: $m_Z^2 \equiv \frac{(g^2+g'^2)\mu^2}{8\lambda}$ $\quad m_W^2 \equiv \frac{g^2 \mu^2}{8\lambda}$

- Characteristic tri-linear and quartic couplings of the gauge bosons to the Higgs field.

- A solid prediction of the SM on the masses of the gauge bosons:

$$\rho = \frac{m_W}{m_Z \cdot \cos^2\theta_W} \equiv 1 \longrightarrow m_Z > m_W$$

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$$D_\mu\phi^\dagger D^\mu\phi = \tfrac{1}{2}\partial_\mu H\partial^\mu H + m_Z^2 Z_\mu Z^\mu + \frac{2m_Z^2}{v} Z_\mu Z^\mu \frac{H}{\sqrt{2}} + \frac{m_Z^2}{v^2} Z_\mu Z^\mu \frac{H}{\sqrt{2}}\frac{H}{\sqrt{2}} + m_W^2 W_\mu^+ W^{\mu-} + \frac{2m_W^2}{v} W_\mu^+ W^{\mu-} \frac{H}{\sqrt{2}} + \frac{m_W^2}{v^2} W_\mu^+ W^{\mu-} \frac{H}{\sqrt{2}}\frac{H}{\sqrt{2}}$$

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Vacuum expectation value

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- We can obtain a precise estimate for the vacuum expectation value, $v = \sqrt{\frac{\mu^2}{2\lambda}}$, via its relation to m_W.

$m_W^2 = \left(\frac{g}{2}\right)^2 v^2$ (from Higgs mechanism, c.f. slide 25)

$m_W^2 = \frac{\sqrt{2} g^2}{8 G_F}$ (from Fermi theory)

Fermi constant:

$$G_F = (1.16639 \pm 0.00002) \times 10^{-5}\ \text{GeV}^{-2}$$

(determined from muon lifetime measurements)

$$v = \frac{1}{\sqrt{\sqrt{2} G_F}} = 246.22\ \text{GeV}$$

- Sets the scale of electroweak symmetry breaking.

Gauge Degrees of Freedom

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- We have discussed how gauge bosons obtain mass by a gauge that absorbs the *Goldstone* bosons in the theory.
- As a complex $SU(2)$ doublet ϕ has four degrees of freedom.
- In the final formulation only the radial excitation H of ϕ remains. The *Goldstone* bosons (ϑ^a) have been absorbed into the gauge fields $W_\mu^\pm$ & Z_μ, which have obtained their masses in this way.

Congratulations – you got it...

...almost

We are still left with the problem of fermion masses.

Coupling to fermions

- The Higgs mechanism can also help to obtain mass terms for fermions, by coupling the fermions to ϕ .

$$\mathcal{L}^{\text{Yukawa}} = -f_e \left(\bar{e}_R \phi^\dagger \psi_L\right) - f_e^* \left(\overline{\psi}_L \phi e_R\right)$$

- check the $SU(2)$ and $U(1)$ behavior of $\mathcal{L}^{\text{Yukawa}}$.

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- check the $SU(2)$ and $U(1)$ behavior of $\mathcal{L}^{\text{Yukawa}}$.

$$\begin{aligned}
\overline{e'}_R \phi'^\dagger \psi'_L &= \overline{e}_R e^{-iY_R/2\vartheta'} \phi^\dagger G^\dagger e^{-iY_\phi/2\vartheta'} e^{iY_L/2\vartheta'} G \psi_L \\
&= e^{i(Y_L - Y_\phi - Y_R)\vartheta'/2} \overline{e}_R \phi^\dagger G^\dagger G \psi_L \\
&= e^{i((-1)-(+1)-(-2))\vartheta'/2} \overline{e}_R \phi^\dagger \psi_L \\
&= \overline{e}_R \phi^\dagger \psi_L \\
\overline{\psi'}_L \phi' e_R &= \text{analog}
\end{aligned}$$

$\mathcal{L}^{\text{Yukawa}}$ is manifest gauge invariant.

- NB: $f_e = f_e^*$ can be chosen real. Residual phases can be re-defined in e_R.

Expansion close to energy ground state

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- Develop ϕ in its energy ground state at $|\phi| = \sqrt{\frac{\mu^2}{2\lambda}}$:

$$\phi = \begin{pmatrix} \phi_+ \\ \phi_0 \end{pmatrix} \xrightarrow{|\phi| = \sqrt{\frac{\mu^2}{2\lambda}}} \phi = \begin{pmatrix} 0 \\ \sqrt{\frac{\mu^2}{2\lambda}} + \frac{H}{\sqrt{2}} \end{pmatrix}$$

$$\begin{aligned}
\mathcal{L}^{\text{Yukawa}} &= -f_e \left(\bar{e}_R \phi^\dagger \psi_L\right) - f_e^* \left(\overline{\psi}_L \phi e_R\right) \\
&= -f_e \left(\bar{e}_R \begin{pmatrix} 0 & \sqrt{\frac{\mu^2}{2\lambda}} + \frac{H}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} \nu \\ e_L \end{pmatrix} + \begin{pmatrix} \bar{\nu} & \bar{e}_L \end{pmatrix} \begin{pmatrix} 0 \\ \sqrt{\frac{\mu^2}{2\lambda}} + \frac{H}{\sqrt{2}} \end{pmatrix} e_R \right) \\
&= \underbrace{-f_e \left(\sqrt{\frac{\mu^2}{2\lambda}} + \frac{H}{\sqrt{2}} \right)}_{m_e \equiv f_e \sqrt{\frac{\mu^2}{2\lambda}}} \underbrace{\left(\bar{e}_R e_L + \bar{e}_L e_R\right)}_{\overset{(1)}{\equiv} \bar{e}e} = -f_e \left(\sqrt{\frac{\mu^2}{2\lambda}} + \frac{H}{\sqrt{2}} \right) \bar{e}e
\end{aligned}$$

$$\mathcal{L}^{\text{Yukawa}} = -f_e \left(\sqrt{\frac{\mu^2}{2\lambda}} + \frac{H}{\sqrt{2}} \right) \left(\bar{e}_R e_L + \bar{e}_L e_R\right)$$

Give the explicit coupling structure.

(1) check Exercise 9 e).

Expansion close to energy ground state

- Develop ϕ in its energy ground state at $|\phi| = \sqrt{\frac{\mu^2}{2\lambda}}$:

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\end{aligned}$$

$$\mathcal{L}^{\text{Yukawa}} = -f_e \left(\sqrt{\frac{\mu^2}{2\lambda}} + \frac{H}{\sqrt{2}} \right) \left(\bar{e}_R e_L + \bar{e}_L e_R\right) = -m_e \bar{e}e - \frac{m_e}{v} \frac{H}{\sqrt{2}} \bar{e}e$$

[1] check Exercise 9 e).

Here comes the 64 billion $ question

- In the beginning of Lecture-04 I explicitly showed to you that terms of type $\bar{e}e = \bar{e}_R e_L + \bar{e}_L e_R$ break gauge invariance. Now I tell you the opposite. Did I lie to you? When yes, when?

Here comes the 64 billion $ question

- In the beginning of Lecture-04 I explicitly showed to you that terms of type $\overline{e}e = \overline{e}_R e_L + \overline{e}_L e_R$ break gauge invariance. Now I tell you the opposite. Did I lie to you? When yes, when?

Of course I would never lie to you. Single terms of this type do indeed break gauge invariance. It is the combination w/ the coupling to the Higgs boson field, which restores the gauge invariance.

Full SM Lagrangian (first lepton generation)

$$\mathcal{L}^{\mathrm{SM}} = \mathcal{L}_{\mathrm{kin}}^{\mathrm{Lepton}} + \mathcal{L}_{\mathrm{IA}}^{CC} + \mathcal{L}_{\mathrm{IA}}^{NC} + \mathcal{L}_{\mathrm{kin}}^{\mathrm{Gauge}} + \mathcal{L}_{kin}^{\mathrm{Higgs}} + \mathcal{L}_{V(\phi)}^{\mathrm{Higgs}} + \mathcal{L}_{\mathrm{Yukawa}}^{\mathrm{Higgs}}$$

$$\mathcal{L}_{\mathrm{kin}}^{\mathrm{Lepton}} = i\bar{e}\gamma^{\mu}\partial_{\mu}e + i\bar{\nu}\gamma^{\mu}\partial_{\mu}\nu$$

$$\mathcal{L}_{\mathrm{IA}}^{CC} = -\frac{e}{\sqrt{2}\sin\theta_W}\left[W_{\mu}^{+}\bar{\nu}\gamma_{\mu}e_L + W_{\mu}^{-}\bar{e}_L\gamma_{\mu}\nu\right]$$

$$\mathcal{L}_{\mathrm{IA}}^{NC} = -\frac{e}{2\sin\theta_W\cos\theta_W}Z_{\mu}\left[(\bar{\nu}\gamma_{\mu}\nu) + (\bar{e}_L\gamma_{\mu}e_L)\right] - e\left[A_{\mu} + \tan\theta_W Z_{\mu}\right](\bar{e}\gamma_{\mu}e)$$

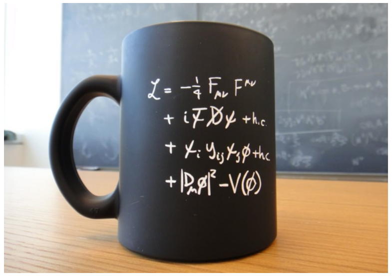
$$\mathcal{L}_{\mathrm{kin}}^{\mathrm{Gauge}} = -\frac{1}{2}Tr\left(W_{\mu\nu}^{a}W^{a\mu\nu}\right) - \frac{1}{4}B_{\mu\nu}B^{\mu\nu}\Big|$$

$$B_{\mu} \to A_{\mu}$$
$$W_{\mu}^{3} \to Z_{\mu}$$

$$\mathcal{L}_{\mathrm{kin}}^{\mathrm{Higgs}} = \frac{1}{2}\partial_{\mu}H\partial^{\mu}H + \left(1 + \frac{1}{v}\frac{H}{\sqrt{2}}\right)^{2} m_W^2 W_{\mu}^{+}W^{\mu-} + \left(1 + \frac{1}{v}\frac{H}{\sqrt{2}}\right)^{2} m_Z^2 Z_{\mu}Z^{\mu}$$

$$\mathcal{L}_{V(\phi)}^{\mathrm{Higgs}} = -\frac{m_H^2 v^2}{4} + \frac{m_H^2}{2}\left(\frac{H}{\sqrt{2}}\right)^{2} + \frac{m_H^2}{v}\left(\frac{H}{\sqrt{2}}\right)^{3} + \frac{m_H^2}{4v^2}\left(\frac{H}{\sqrt{2}}\right)^{4}$$

$$\mathcal{L}_{\mathrm{Yukawa}}^{\mathrm{Higgs}} = -\left(1 + \frac{1}{v}\frac{H}{\sqrt{2}}\right)m_e\bar{e}e$$



A word on masses...

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- On our way we have witnessed three mechanisms of mass generation in the SM:
  - Via gauge coupling: $\left(\frac{(g^2+g'^2)}{2}\right)^2 v^2 Z_\mu Z^\mu$ $\quad \left(\frac{g^2}{2}\right)^2 v^2 W_\mu^+ W^{\mu -}$
  - Via *Yukawa* coupling: $f_e v \bar{e} e$
  - Via *Goldstone* potential (→ self-coupling): $\frac{m_H^2}{2}\left(\frac{H}{\sqrt{2}}\right)^2$

Mass generation via coupling to non-vanishing vacuum.

A word on couplings...

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- We summarize the couplings of fermions and bosons to the Higgs boson (according to Feynman rules):

$f_{H\to ff} = i\frac{m_f}{v}$	(Fermions)	combinatoric factor	
$f_{H\to VV} = i\frac{2m_V^2}{v}$	(Heavy Bosons trilinear)		
$f_{HH\to VV} = i\frac{2m_V^2}{v^2}$	(Heavy Bosons quartic)		$2!$
$f_{H\to HH} = i\frac{3m_H^2}{v}$	(H Boson trilinear)		$3!/2$
$f_{HH\to HH} = i\frac{3m_H^2}{v^2}$	(H Boson quartic)		$4!/2$

- The couplings can be read off from the Lagrangian density (c.f. slide 37), times i .

- It has to be taken into account that H is an indistinguishable particle. It therefore contributes with a combinatoric factor for all amplitudes with in- and out-going H fields, wherever the H appears more then once in an interaction vertex.

Concluding Remarks

- Higgs mechanism = incorporation of spontaneous symmetry breaking into a gauge field theory. Leads to the fact that gauge bosons eat up *Goldstone* bosons in the system and gain mass.

- → Higgs boson obtains mass from the *Goldstone* potential.
→ Gauge bosons obtain mass from their coupling to ϕ via the covariant derivative.
→ Fermions obtain mass via direct *Yukawa* coupling to ϕ.

- Gauge bosons couple to the Higgs like $\propto m_V^2$, fermion fields couple to the Higgs like $\propto m_f$.

- Next week we will recapitulate how to get from a prediction in a Lagrangian density to an observable cross section (→ Feynman rules). For this I will follow the book "*Feynman-Graphen und Eichtheorien für Experimentalphysiker*" by Peter Schmüser.

Backup