Karlsruhe Institute of Technology

# Moderne Experimentalphysik III: Kerne und Teilchen (Physik VI)

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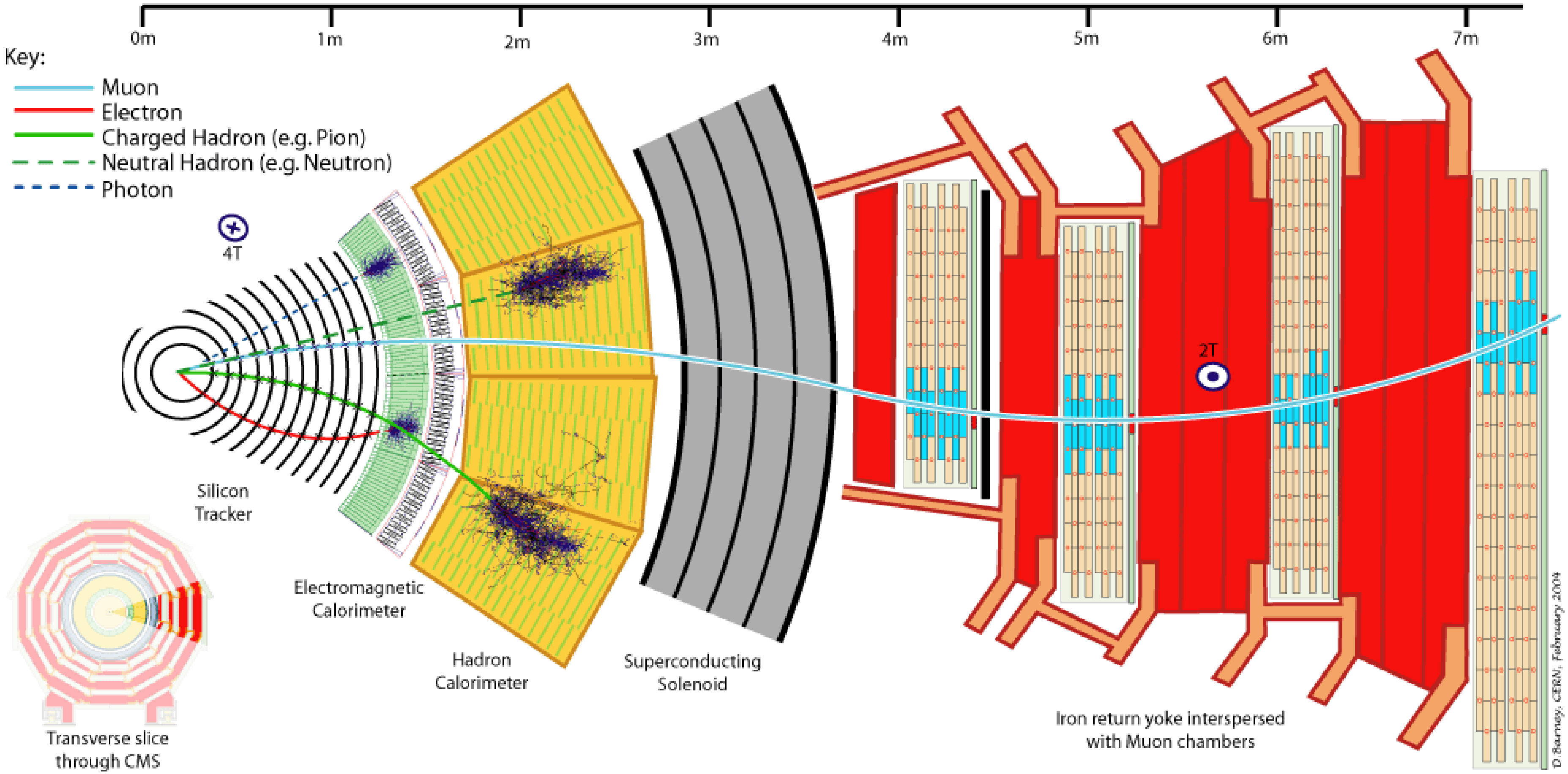
INSTITUTE OF EXPERIMENTAL PARTICLE PHYSICS (IEKP) – PHYSICS FACULTY

# Kapitel 2.5: Detektorsysteme in der Teilchenphysik

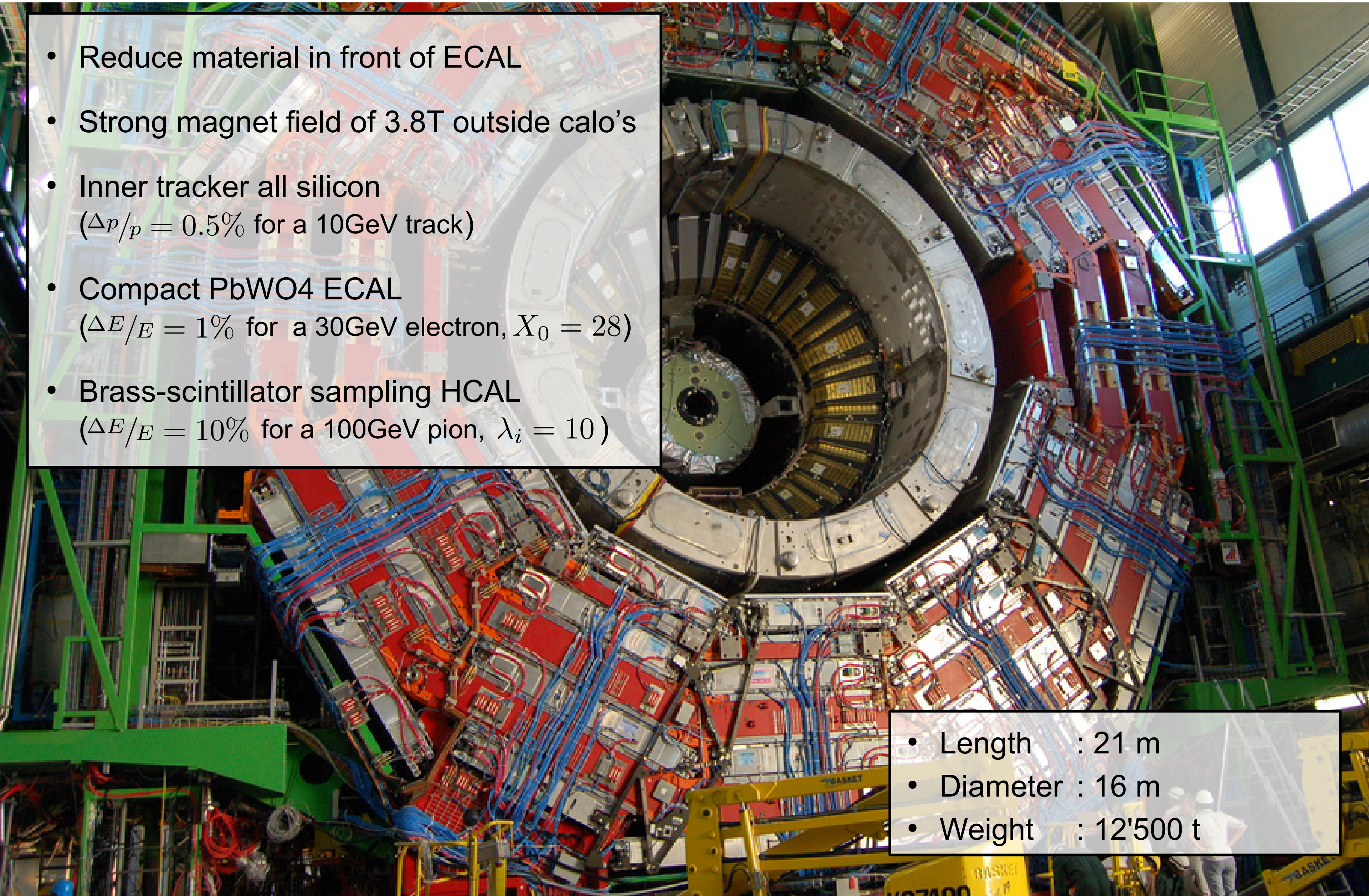
# Teilchennachweis bei CMS

- Putting things together…




- **Schlüsselanforderungen:** optimale Impuls- und Energiebestimmung, möglichst alle erzeugten Teilchen in aktivem Detektormaterial stoppen

# The Compact Solution (CMS)

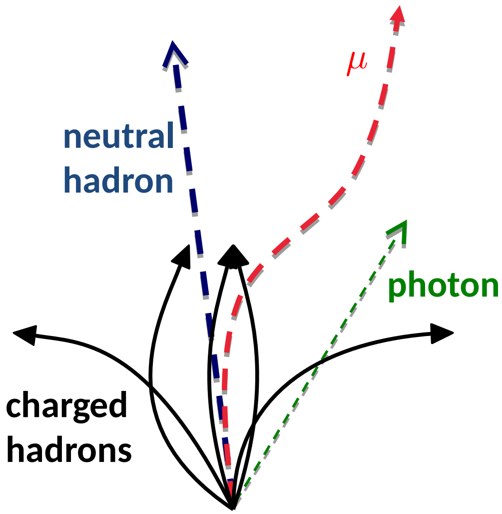


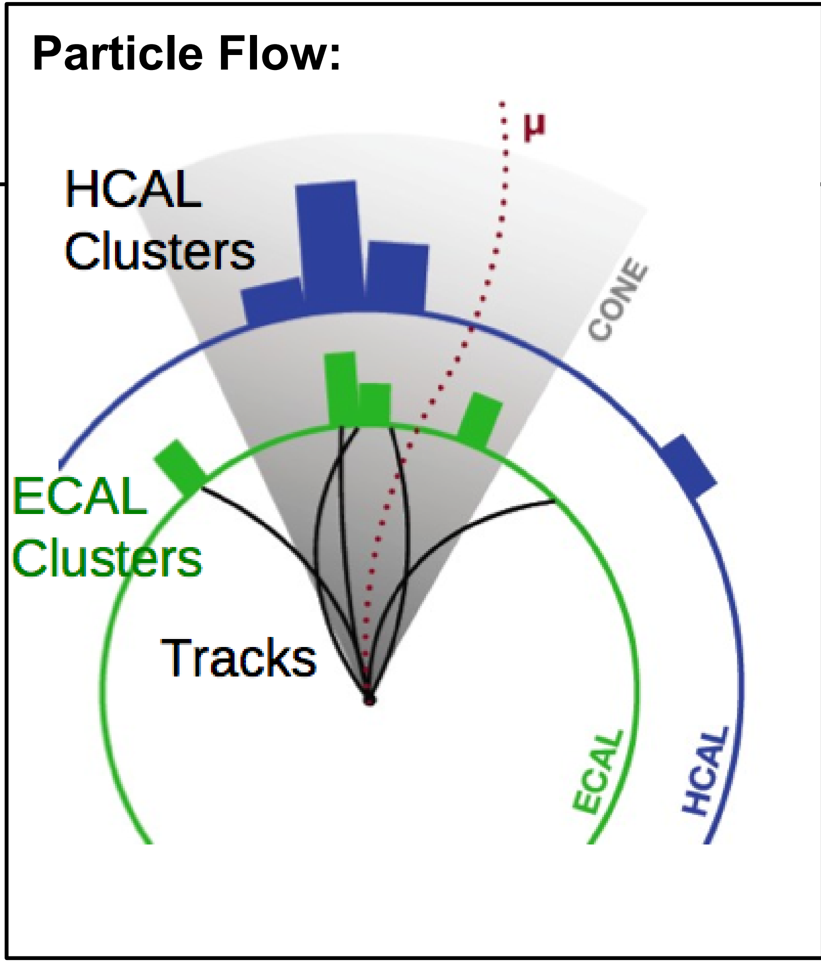
- Reduce material in front of ECAL
- Strong magnet field of 3.8T outside calo's
- Inner tracker all silicon
  ($\Delta p/p = 0.5\%$ for a 10GeV track)
- Compact PbWO4 ECAL
  ($\Delta E/E = 1\%$ for a 30GeV electron, $X_0 = 28$)
- Brass-scintillator sampling HCAL
  ($\Delta E/E = 10\%$ for a 100GeV pion, $\lambda_i = 10$)

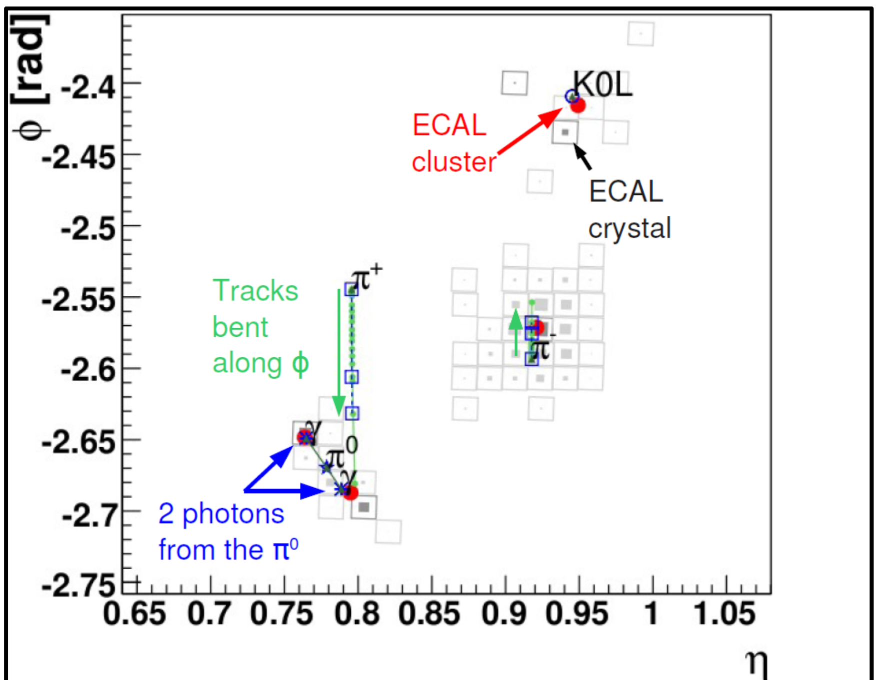
- Length : 21 m
- Diameter : 16 m
- Weight : 12'500 t

# High level object reconstruction

- Combine all energy deposits in detector to a unique event description (→ stable particle level)




- Unambiguous list of stable particles: muons, electrons, photons, charged & neutral hadrons

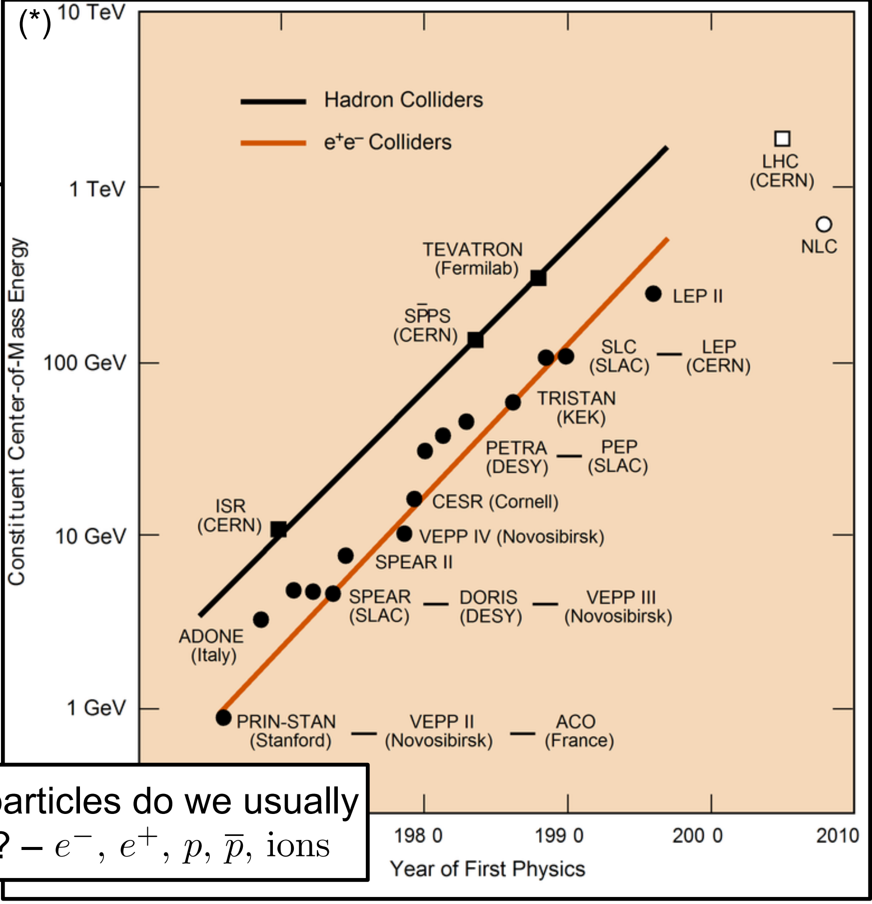
# Kapitel 2.6: Beschleuniger in der Teilchenphysik

# What is a particle accelerator?

M. S. Livingston (1905 – 1986):

A particle accelerator is a machine that uses electromagnetic fields to propel charged particles to nearly light speed and to contain them in well-defined beams.

- Colliding beams are our laboratory
- Reach out to highest energies (→ resolve smallest structures, Heisenberg uncertainty principle)
- Provide as many collisions per second as possible (→ observe rarest events)




What particles do we usually collide? – $e^-$, $e^+$, $p$, $\bar{p}$, ions

Livingston plot

Vergleiche mit VL-03 Folie 17

**Cross section:**

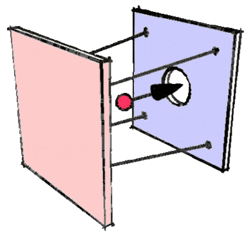
$$\sigma = \frac{N_{obs} - N_{BG}}{\mathcal{L} \cdot \epsilon \cdot A} \frac{1}{T}$$

$N_{obs}$ : N observed reactions.
$N_{BG}$ : N expected BG reactions.
$\mathcal{L}$ : luminosity.
$\epsilon$ : detection efficiency.
$A$ : detector acceptance.
$T$ : observation time.

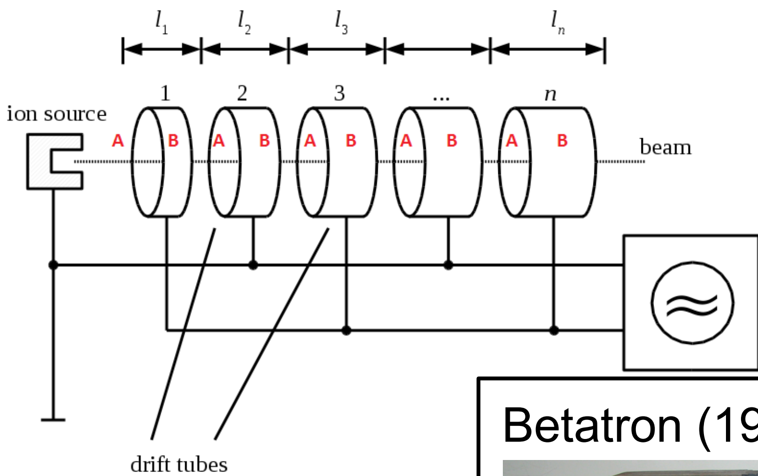
(*)
Aufgetragen ist colliding beam energy, für den LHC sind das nominell 7TeV (wird durch den plot nur von der Größenordnung her getroffen)

# Different accelerators

## Electrostatic acceleration:

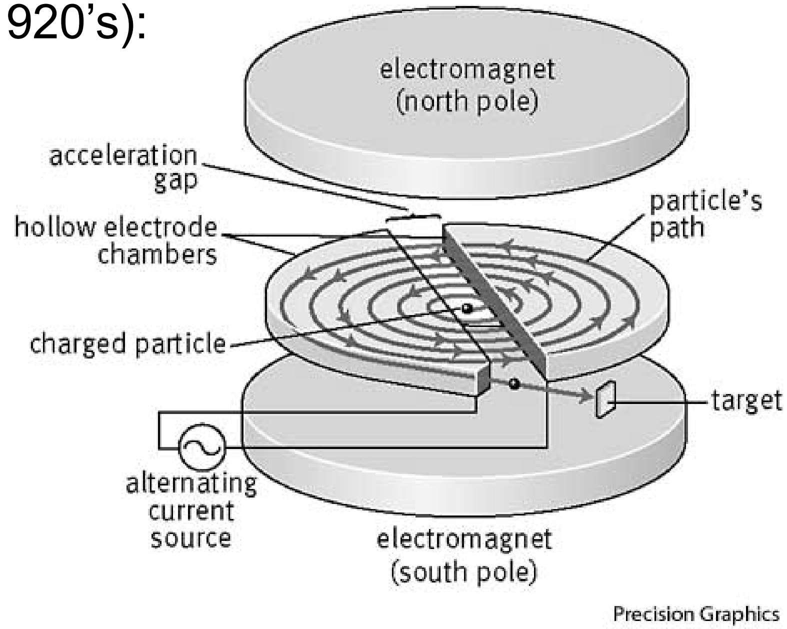


## Linear accelerator:




## Cyclotron (1920's):

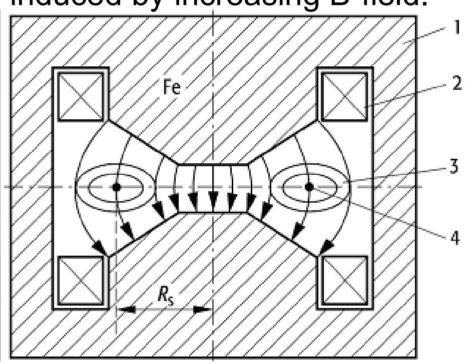
Radius increases with beam energy.




## Betatron (1920's):

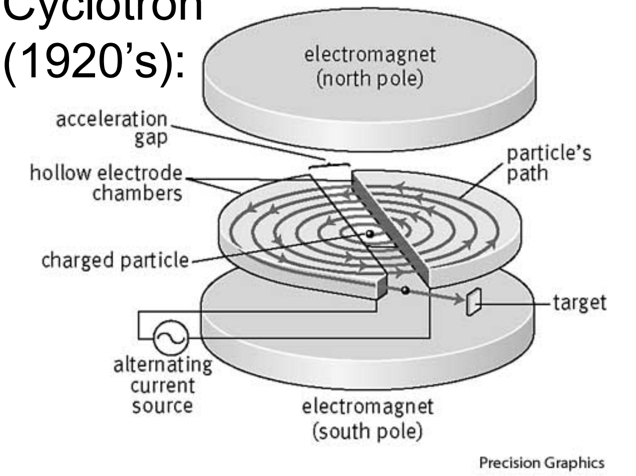
Radius const. – accelerating field induced by increasing B-field.

6MeV Betatron 1942–44
Siemens-Museum München



# Zyklotronfrequenz

Cyclotron (1920's):




- Klassisch:

$$m\frac{v^2}{r} = q\,v\,B$$

$$\omega = \frac{v}{r} = \frac{q\,B}{m} \qquad \text{(Zyklotronfrequenz)}$$

→ unabhängig vom Impuls

- Reativitstisch:

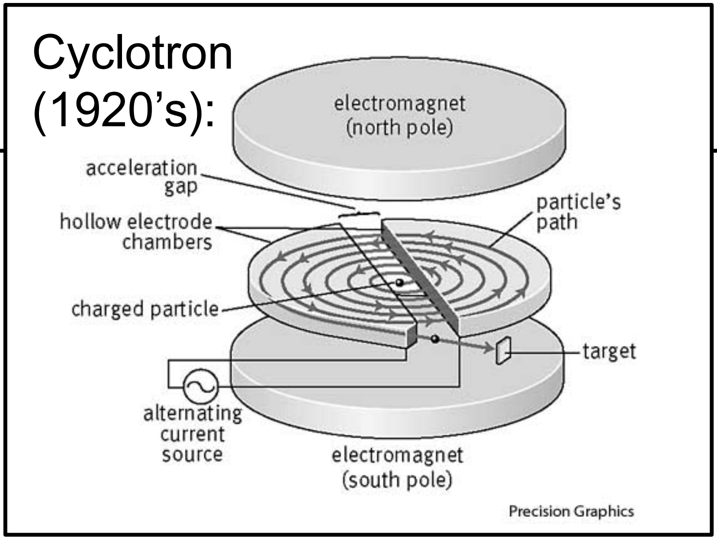
$$\omega = \frac{q\,B}{\gamma\,m}$$

$$r = \frac{\beta\,c}{\omega} = \frac{m\,\gamma\,\beta\,c}{q\,B} = \frac{p}{q\,B} \qquad \text{(Zyklotronradius)}$$

→ analog zu Impulsbestimmung bei vorgegebenem Radius (s. VL-06 Folie 24)

- **Beispiel:** Bahnradius für ein Proton, dass mit einem Zyklotron auf 30 MeV beschleunigt wurde?

# Zyklotronfrequenz

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$\rightarrow$ analog zu Impulsbestim-mung bei vorgegebenem Radius (s. VL-06 Folie 24)

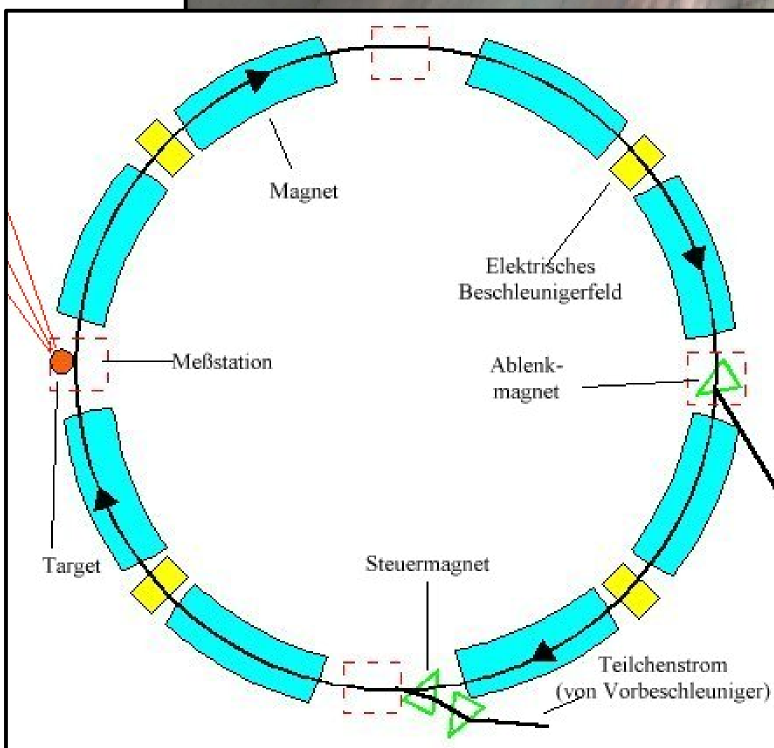
- **Beispiel:** Bahnradius für ein Proton, dass mit einem Zyklotron auf 30 MeV beschleunigt wurde?

$$r\,[\mathrm{m}] = \frac{p\,[\mathrm{GeV}]}{0.3 \cdot B\,[\mathrm{T}]} = 0.1\,\mathrm{m}$$

# Synchrotron

Radius const. – B-field increased synchronously w/ beam energy

LHC, CERN 2010

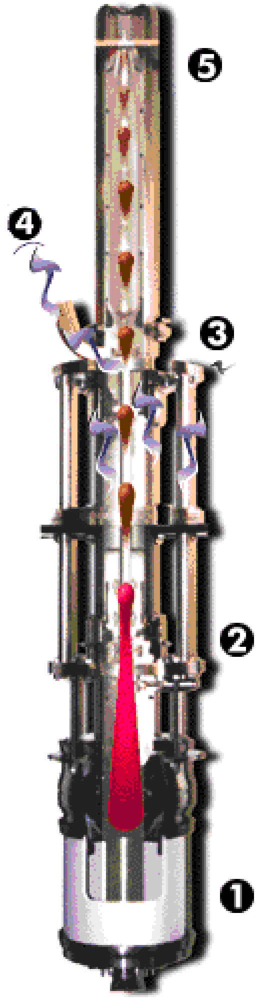


# Accelerating power

- Acceleration happens via UHF in Klystrons:

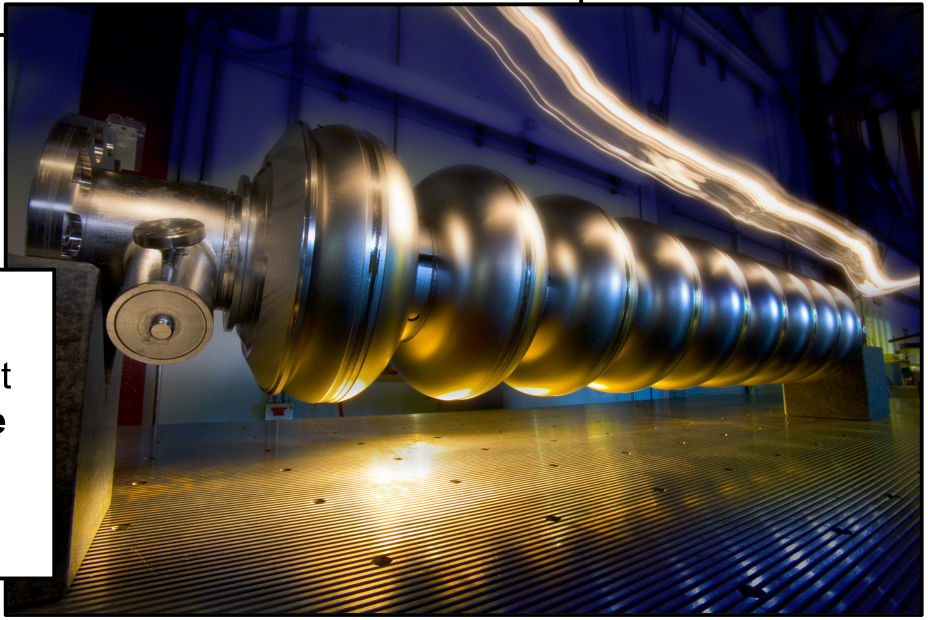
  - Acceleration of electrons (1)
  - Density modulations in electron beam implied by external field (2)
  - Due to these modulations electromagnetic wave travels through first cavity (3)
  - Exit hole at end of cavity. The passing wave induces resonant wave in the surface of hole which damps electron beam and couples energy out to second cavity (4)

(1) source
(2) first cavity
(3) UHF created by electron bunches
(4) exit to second cavity
(5) electron beam dump




Such cavities have to stand $50 - 80$ MeV/m without discharges.

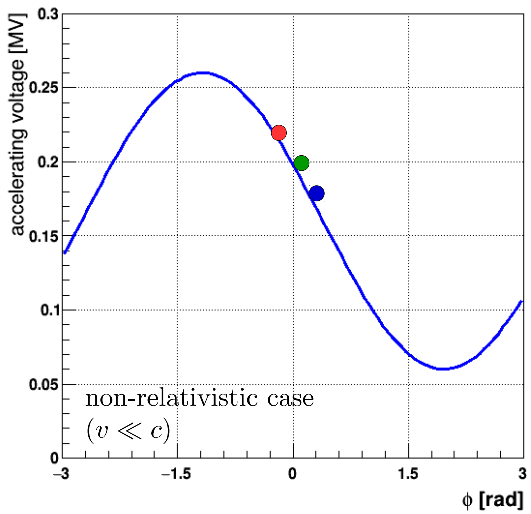
**Anmerkung:**
In der beschleunigenden Kavität bildet sich eine **stehende Welle** aus, die in ihrer Form auch der Stuktur der Kavität entspricht

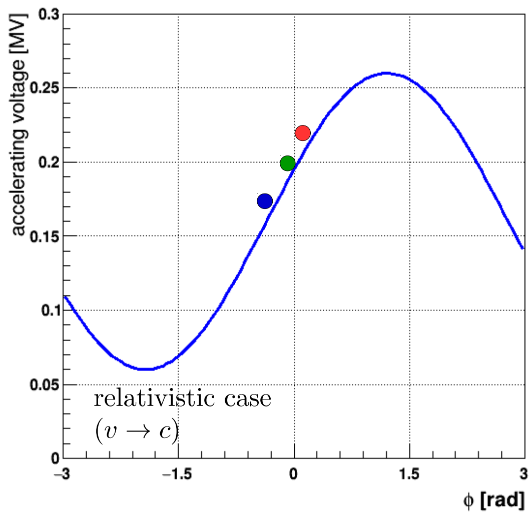


TESLA 9-cell 1.5 GHz SRF cavities from ACCEL Corp. Germany for the ILC

# Phase focusing

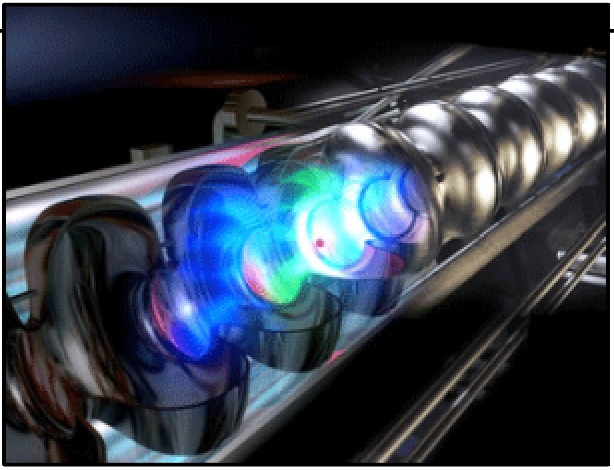
- Energy focusing achieved by proper choice of phase of accelerating wave:




- energy lower → more acceleration
- energy exact → nominal acceleration
- energy higher → less acceleration

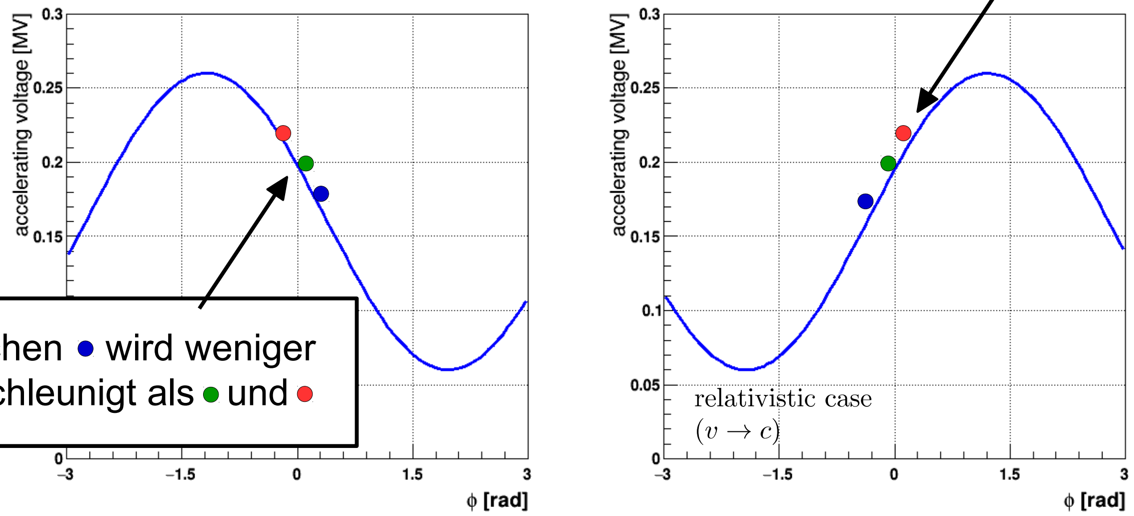
- This kind of acceleration leads to **bunching of projectiles**.
- Beams are brought to collision in bunches



# Phase focusing

- Energy focusing achieved by proper choice of accelerating wave:

c=const! Aber ● bewegt sich auf größerem Radius durch den Beschleuniger. Jetzt werden ● und ● mehr beschleunigt und vergößern ihren Radius entsprechend




Teilchen ● wird weniger beschleunigt als ● und ●

- energy lower → more acceleration
- energy exact → nominal acceleration
- energy higher → less acceleration

- This kind of acceleration leads to **bunching of projectiles**.
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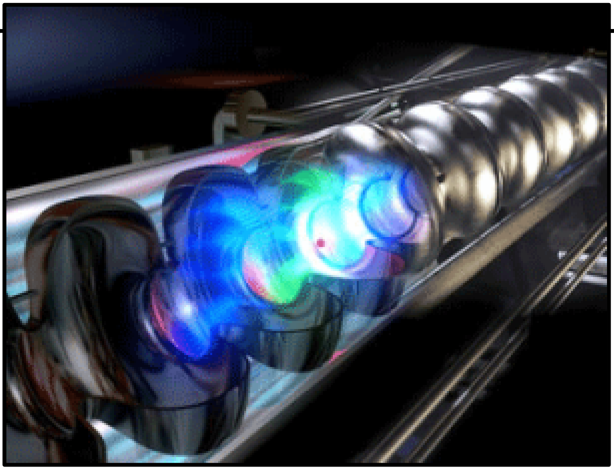
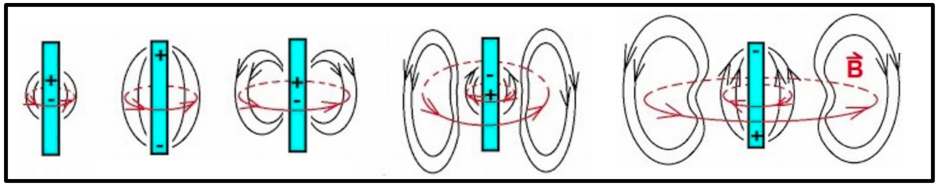


# Synchrotron radiation

**Advantage of circular structures:** acceleration infrastructure can be recycled.

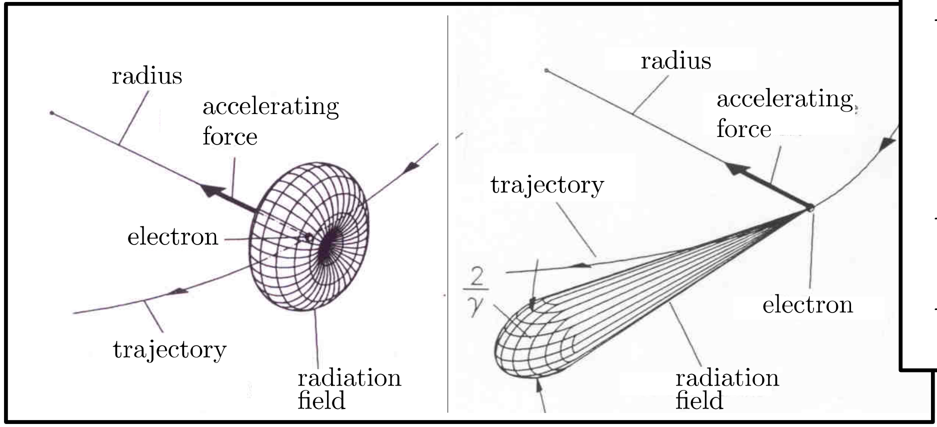
**Disadvantage:** need acceleration energy only to keep particles on track.




Radiation pattern of a dipole antenna.

electron center of mass frame:

laboratory frame:




Radiation pattern of a circular accelerated electron.

Energy radiated off per rotation cycle:

$$P = \frac{e^2}{6\pi\epsilon_0 c}|\vec{\beta}|^2\gamma^4 = \frac{e^2 c}{6\pi\epsilon_0 R^2}\gamma^4 \quad (**)$$

$$= \frac{e^4}{6\pi\epsilon_0 R^2}\frac{E^2 B^2}{m^4}$$

$$P(p|_{m_p=1\ \mathrm{GeV}}) \overset{(*)}{=} 280\ \mu\mathrm{W}$$

$$P(e|_{m_e=0.511\ \mathrm{MeV}}) \overset{(*)}{=} 450\ \mathrm{kW}$$

(*) using LHC parameters.

(**)
R: der Radius der Kreisbahn

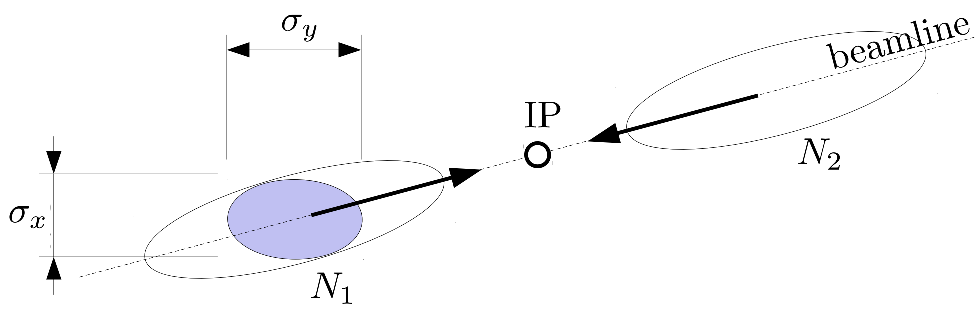
# Beam quality parameters

- Energy should be high, accurate and stable (→ chromaticity).
- Particle flux should be high (→ “brightness of source”):

**Luminosity:**

$$\mathcal{L} = \frac{1}{4\pi}\frac{\nu N_1 N_2}{\sigma_x \sigma_y}$$

$\nu$ : repetition frequency.
$N_{1,2}$ : number of projectiles in beam $i$.
$\sigma_{x,y}$ : beam dimension in x, y.




- In experiment $\mathcal{L}$ correlated against quantities that can be easily monitored (→ hits in pixel, energy in low angle calorimeter)
- Most accurate value obtained from reference processes.

- Particles must be kept on track to achieve and sustain highest luminosity.

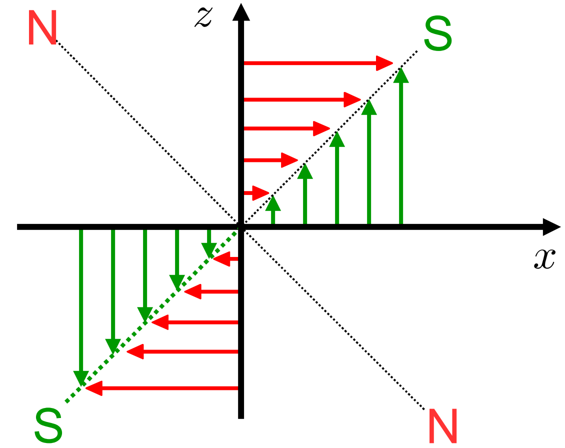
# Weak & strong focusing

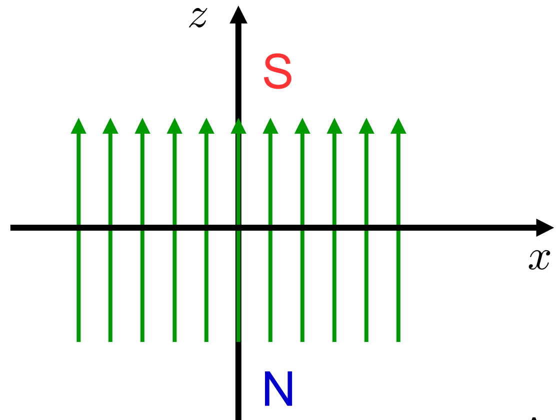
- Projectiles enter acceleration chain with different opening angles.
- Restrict opening angle from beginning (→ collimators).

## Strong focusing:

Quadrupole field:

increasing linearly with x, z. Used for focussing.



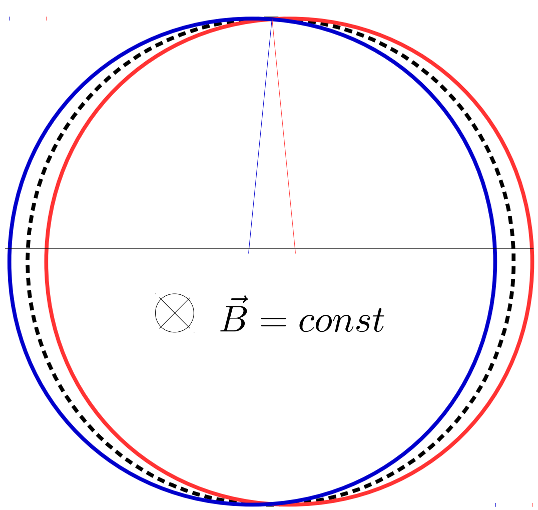

Dipole field:

constant field. Used for bending.

Assume proton moving into projected plain

## Weak focusing:

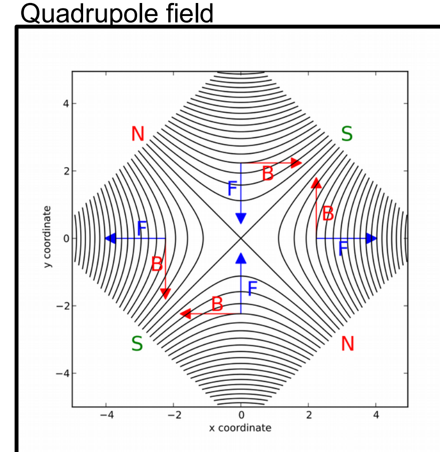
Two particles with small opening angle meet any half cycle.




Imagine opening angle of 1 mrad accelerator radius of $R = 1000$ m:

What maximal distance between the two particles do you expect?

Quadrupole field

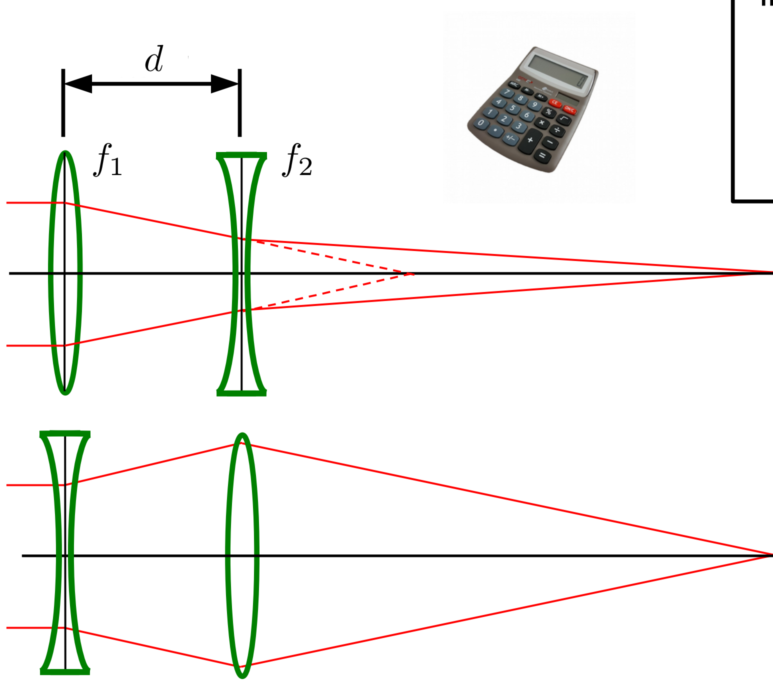


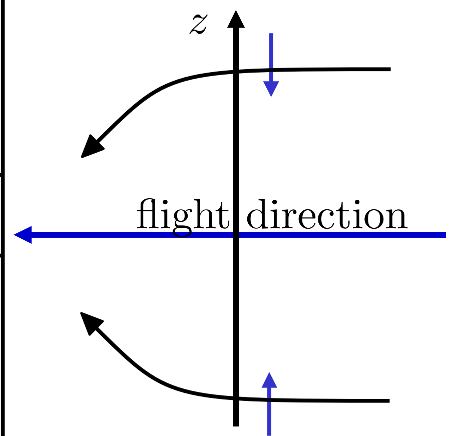
# Quadrupole focusing

Arrange system of “lenses” to achieve focusing in both planes:

$$f_{12} = \frac{1}{\frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}}$$

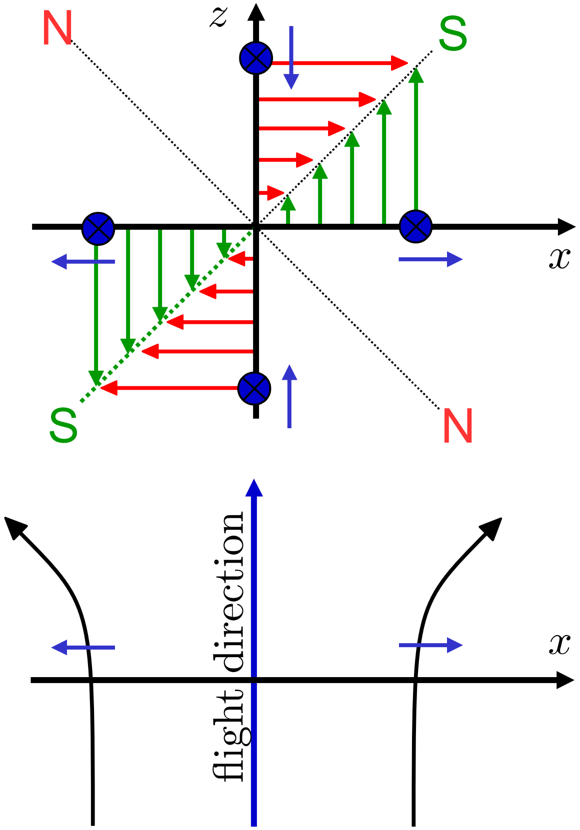
Calculate $f_{12}$ for the simple example:
$f_1 = -f_2 = 100$ m $\quad d = 50$ m



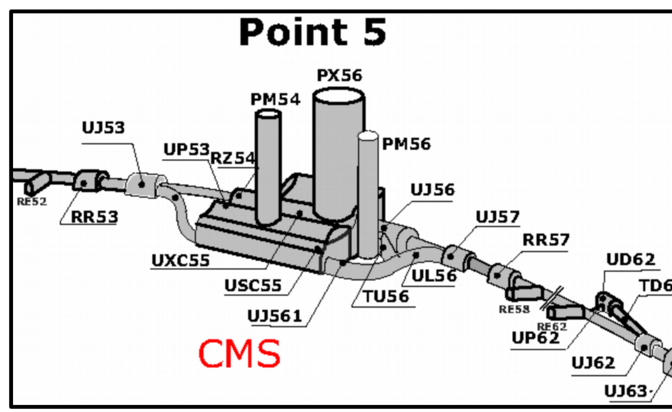

Side-view: focusing

Quadrupole acts like an optical lens focusing in one plane, defocussing in the other.




Up-view: defocussing

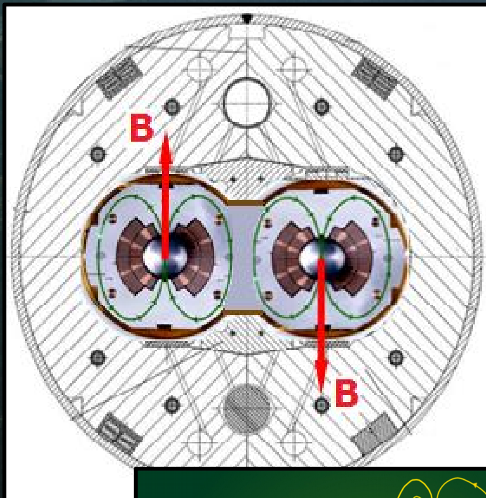
Trajectory of traversing proton



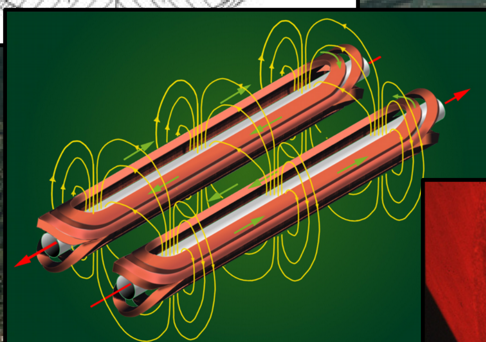

LHC beamline close to CMS

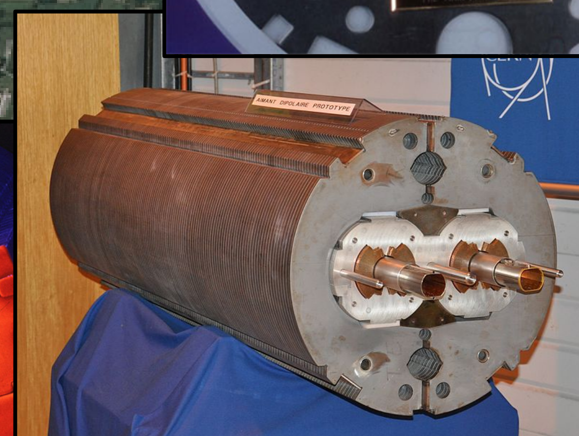
# The Large Hadron Collider

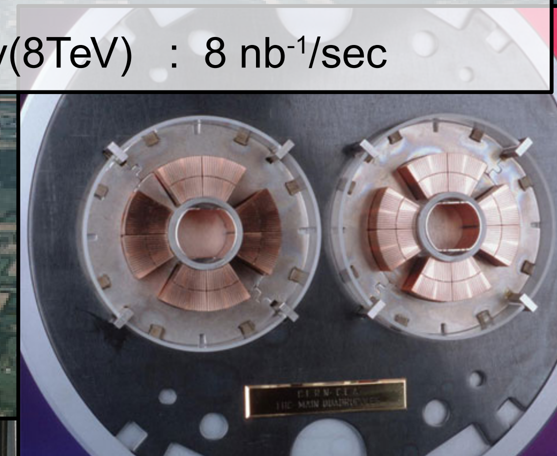
- Construction costs: 4.1 billion $
- Construction time : 14 years
- Circumference : 27 km
- No of dipoles : 1232
- Power : 120 MW
- Luminosity(8TeV) : 8 nb$^{-1}$/sec




- 8.3 T
- 11.8 kA
- 160 cyc



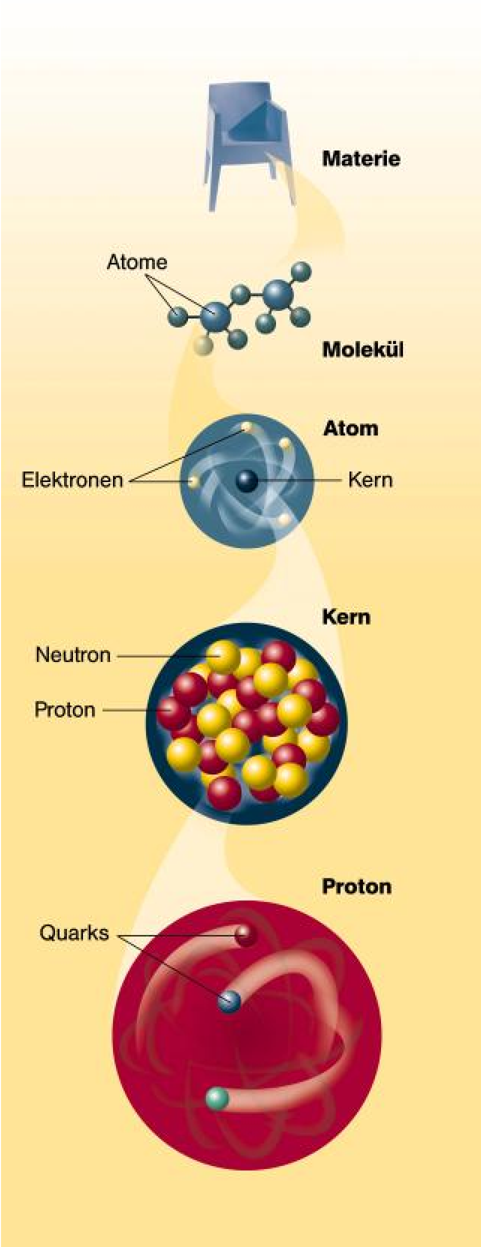




Eine Animation des LHC Beschleunigerkomplexes können Sie unter diesem link sehen

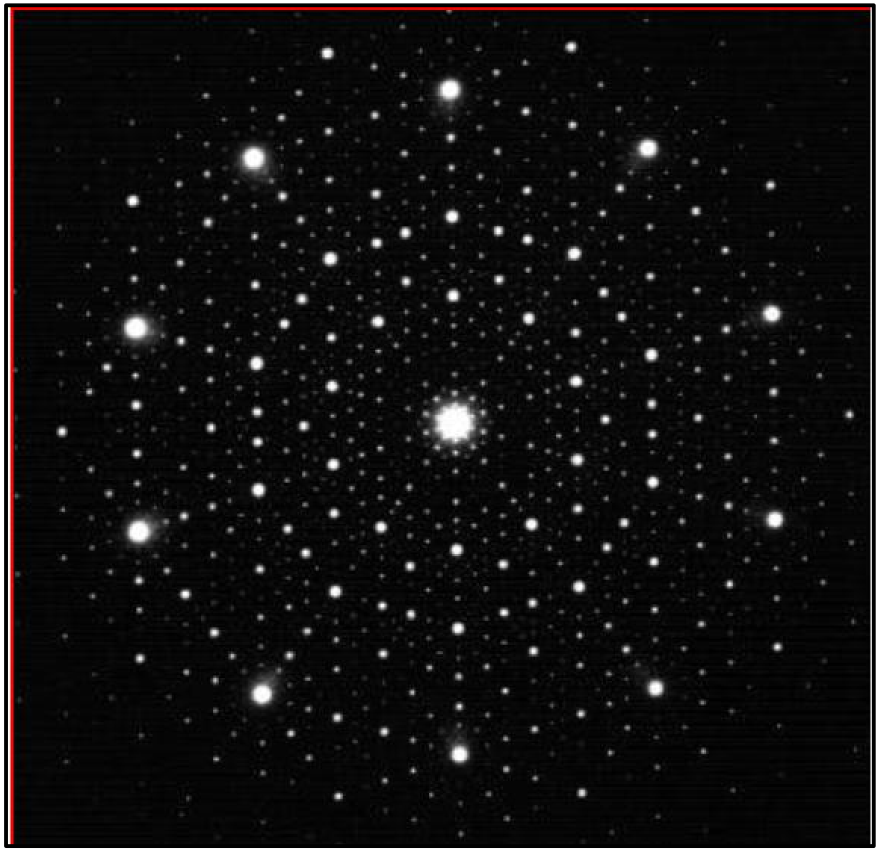
- Energy density 500 kJ/m
- Tension 200'000 t/m

# Kapitel 3: Stuktur der Materie



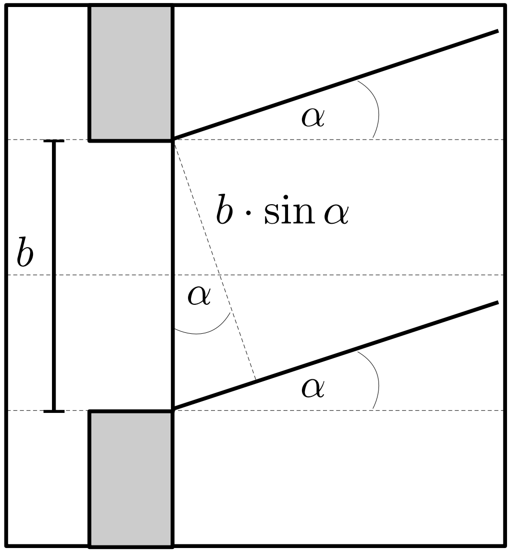
# Strukturanalyse – Streuexperiment – Beugungsbild

- Strukturanalyse erfolgt mit Hilfe von Streuexperimenten
- Für kleine Strukturen: **Beugungsbild** (→ erlaubt Rückschlüsse auf räuml. Beschaffenheit des untersuchten Objekts)



# Beugung am Spalt

- Beispiel **Beugung am Spalt**:

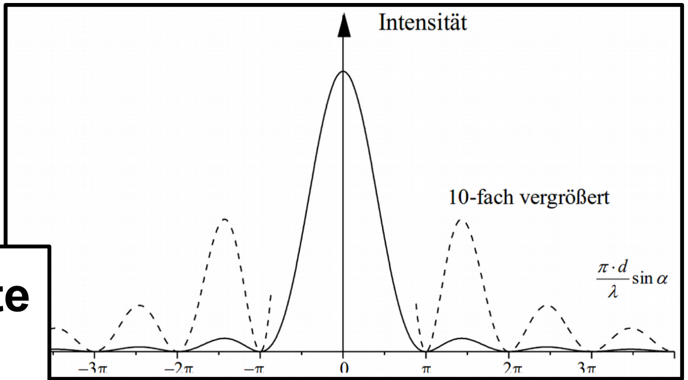



$$\psi(\alpha) = \psi_0\, e^{ikx} = \sum_{j\,<\,b\sin\alpha/\Delta x} \frac{\Delta x}{b\sin\alpha}\psi_0\, e^{ikx_j}$$

im Grenzübergang $\Delta x \to 0$:

$$\psi(\alpha) = \int_{-b\sin\alpha/2}^{+b\sin\alpha/2} \frac{\mathrm{d}x}{b\sin\alpha}\psi_0\, e^{ikx} = \left[\frac{\psi_0}{b\sin\alpha}\,\frac{e^{ikx}}{ik}\right]_{-b\sin\alpha/2}^{+b\sin\alpha/2}$$

$$= \frac{\psi_0}{b\sin\alpha/2}\left(\frac{e^{ik\,b\sin\alpha/2} - e^{-ik\,b\sin\alpha/2}}{2ik}\right) = \psi_0 \cdot \frac{\sin(k\,b\sin\alpha/2)}{k\,b\sin\alpha/2} \quad \text{(Spaltfunktion)}$$



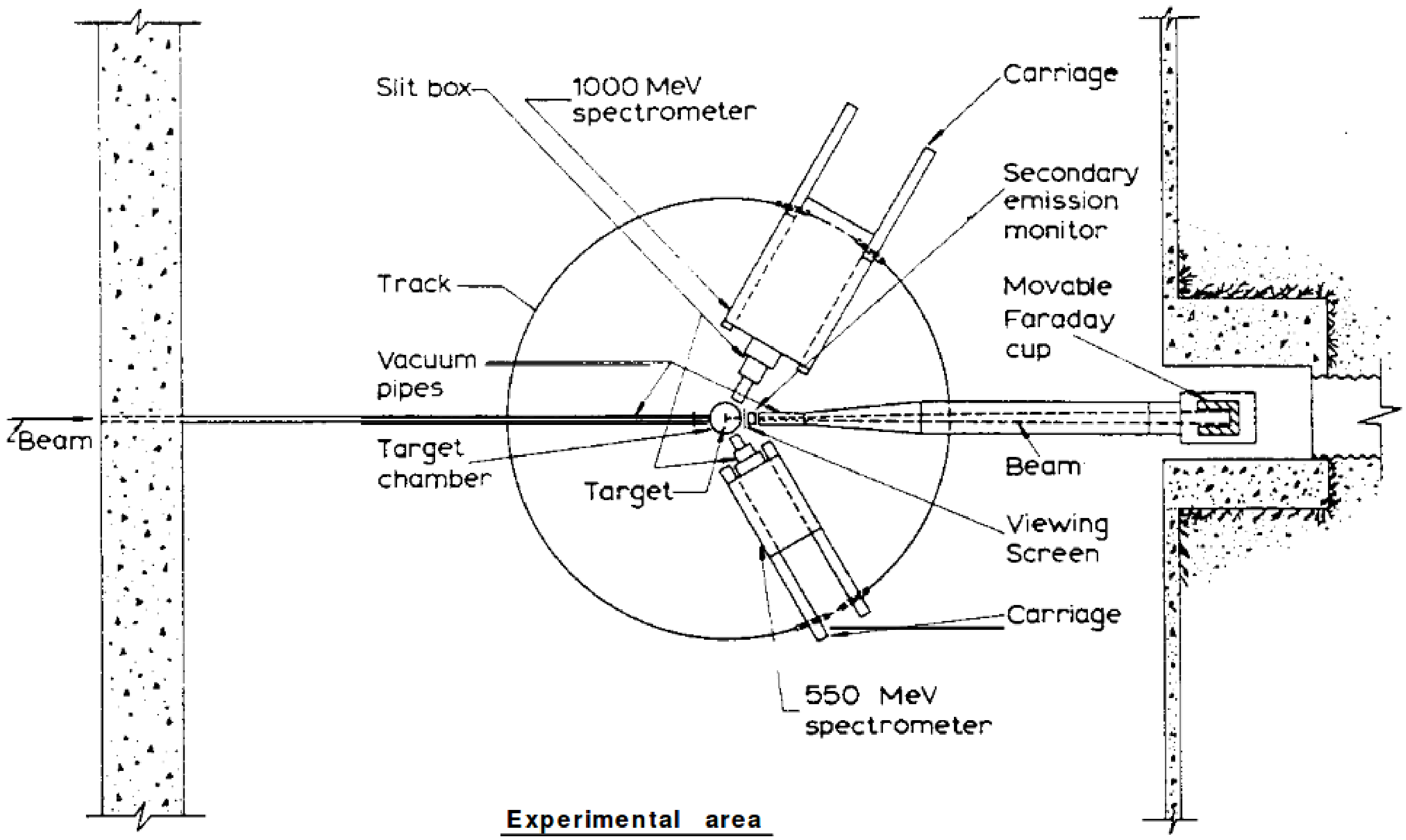

Inverse Fouriertransformation der Spaltfunktion erlaubt Rückschlüsse auf Aussehen des Spalts

**Fouriertransformierte** der Rechteckfunktion

# Kapitel 3.1: Kernradien und Formfaktoren

# Typische Experimente

- Streuung hochenergetischer Elektronen an ruhenden Kernen (fixed target) z.B. am Stanford Linear Accelerator (SLAC):



# Mott-Wirkungsquerschnitt

- Klassische Herleitung Rutherford-Wirkungsquerschnitt (s. VL-03 Folien 9–13, ohne Spin)

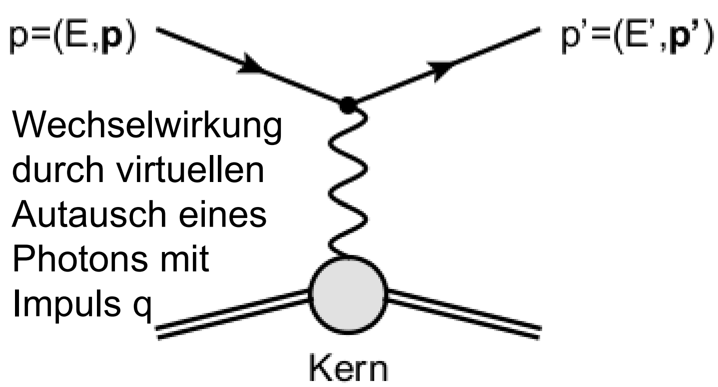
$$\left(\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\right)_{\text{Rutherford}} = \frac{\left(2\,z\,Z\,\alpha(\hbar\,c)\right)^2 E^2}{q^4 c^4}$$

Im Fall von Rückstreuung: $E \to E_f$

- Für **Streuung relativistischer Elektronen**: Modifikationen durch Kernrückstoß und v.a. Spin des Elektrons:

$$\left(\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\right)_{\text{Mott}} = \left(\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\right)_{\text{Rutherford}} \cdot \frac{E_f}{E} \cdot \underbrace{\left(1 - \beta \sin^2\left(\frac{\theta}{2}\right)\right)}_{\cos^2\left(\frac{\theta}{2}\right) \text{ für } \beta \to 1}$$

(Mott-Wirkungsquerschnitt)



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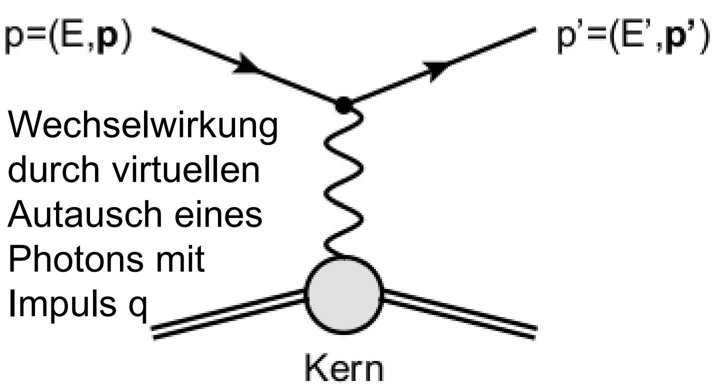
(Mott-Wirkungsquerschnitt)

- Kernrückstoß



# Mott-Wirkungsquerschnitt

- Klassische Herleitung Rutherford-Wirkungsquerschnitt (s. VL-03 Folien 9–13, ohne Spin)

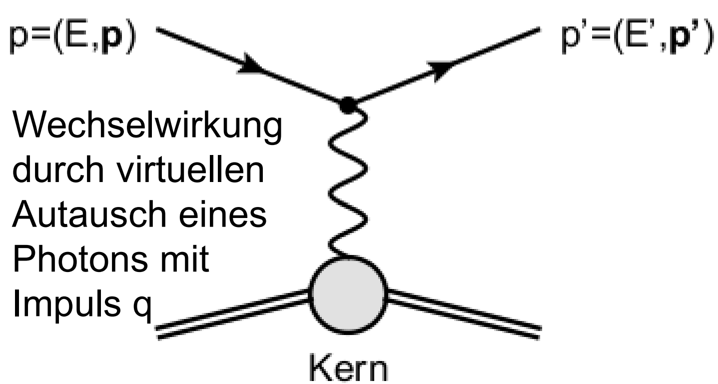
$$\left(\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\right)_{\text{Rutherford}} = \frac{\left(2\,z\,Z\,\alpha(\hbar\,c)\right)^2 E^2}{q^4 c^4}$$

Im Fall von Rückstreuung:
$E \to E_f$

- Für **Streuung relativistischer Elektronen**: Modifikationen durch Kernrückstoß und v.a. Spin des Elektrons:

$$\left(\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\right)_{\text{Mott}} = \left(\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\right)_{\text{Rutherford}} \cdot \frac{E_f}{E} \cdot \underbrace{\left(1 - \beta \sin^2\left(\frac{\theta}{2}\right)\right)}_{\cos^2\left(\frac{\theta}{2}\right)\ \text{für}\ \beta \to 1}$$

(Mott-Wirkungsquerschnitt)




- Kernrückstoß
- Elektronen-Spin
- Für $\beta = 1$ & Kerne ohne Spin Rückstreuung nicht möglich!

# Helizität

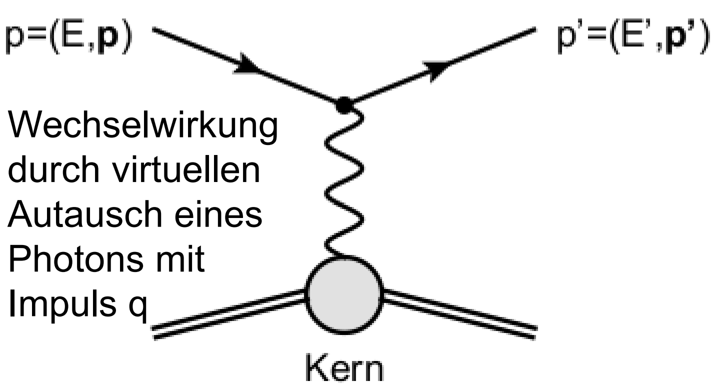
- **Helizität** – Projektion des Spins auf Bewegungsrichtung:

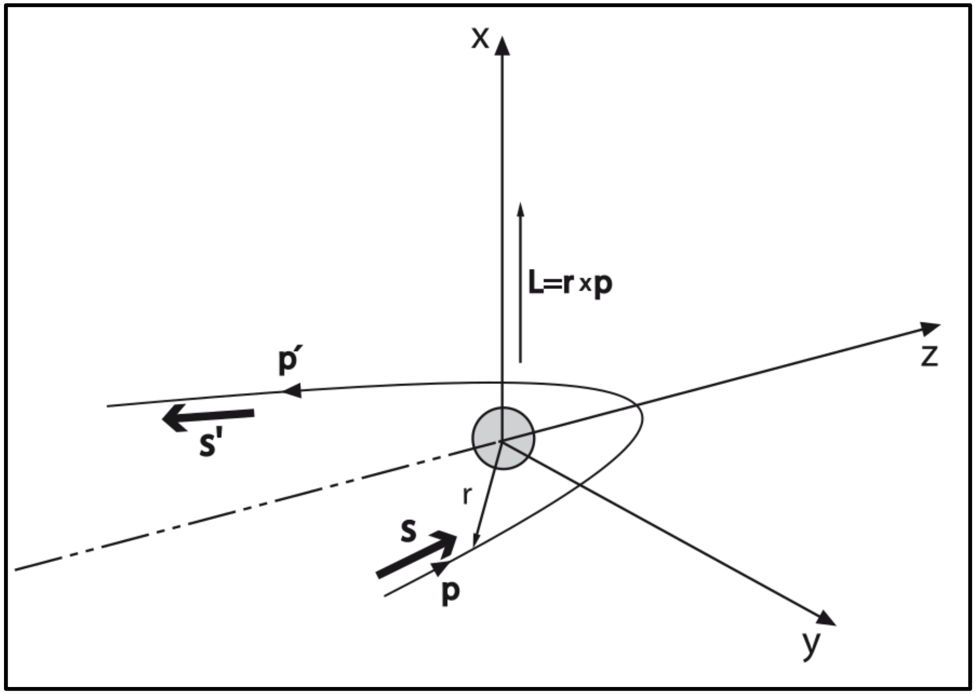
  $$\boxed{h \equiv \frac{\vec{s} \cdot \vec{p}}{|\vec{s}|\,|\vec{p}|}}$$

  Für $\beta \to 1$ ist Helizität eine **Erhaltungsgröße** (folgt aus Dirac-Gleichung)

- Bei Rückstreuung ($\theta = 180°$) müßte Spin aufgrund von Helizitätserhaltung "umklappen"

Bahndrehimpuls senkrecht zu Streuebene. "Umklappen" ohne Spin-Kern-Wechselwirkung nicht möglich

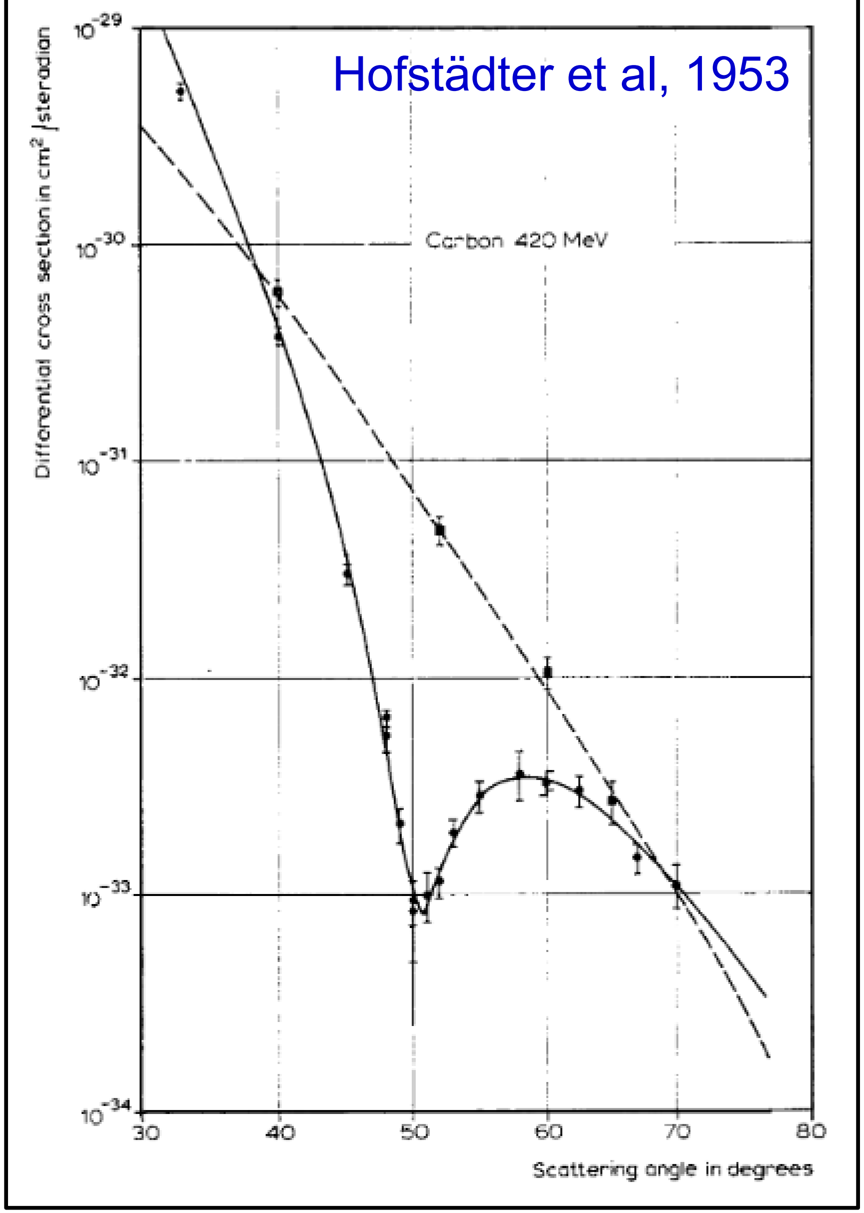


# Beobachtung bei Elektron-Kern-Streuung

- Wirkungsquerschnitt **fällt schneller ab**, als für Mott-Wirkungsquerschnitt erwartet

- Ausgeprägte Minima und Maxima (erinnert an Beugung an Lochblende)




- **Heuristische Erklärung:**
  - Auflösung der Elektronen steigt mit Impulsübertag des virtuellen Photons (Heisenberg: $\Delta x \geq \hbar/q$ )
  - Kern besitzt ausgedehnte Ladungsverteilung. Elektron tastet nur Teil der Ladung ab → geringerer Wirkungsquerschnitt

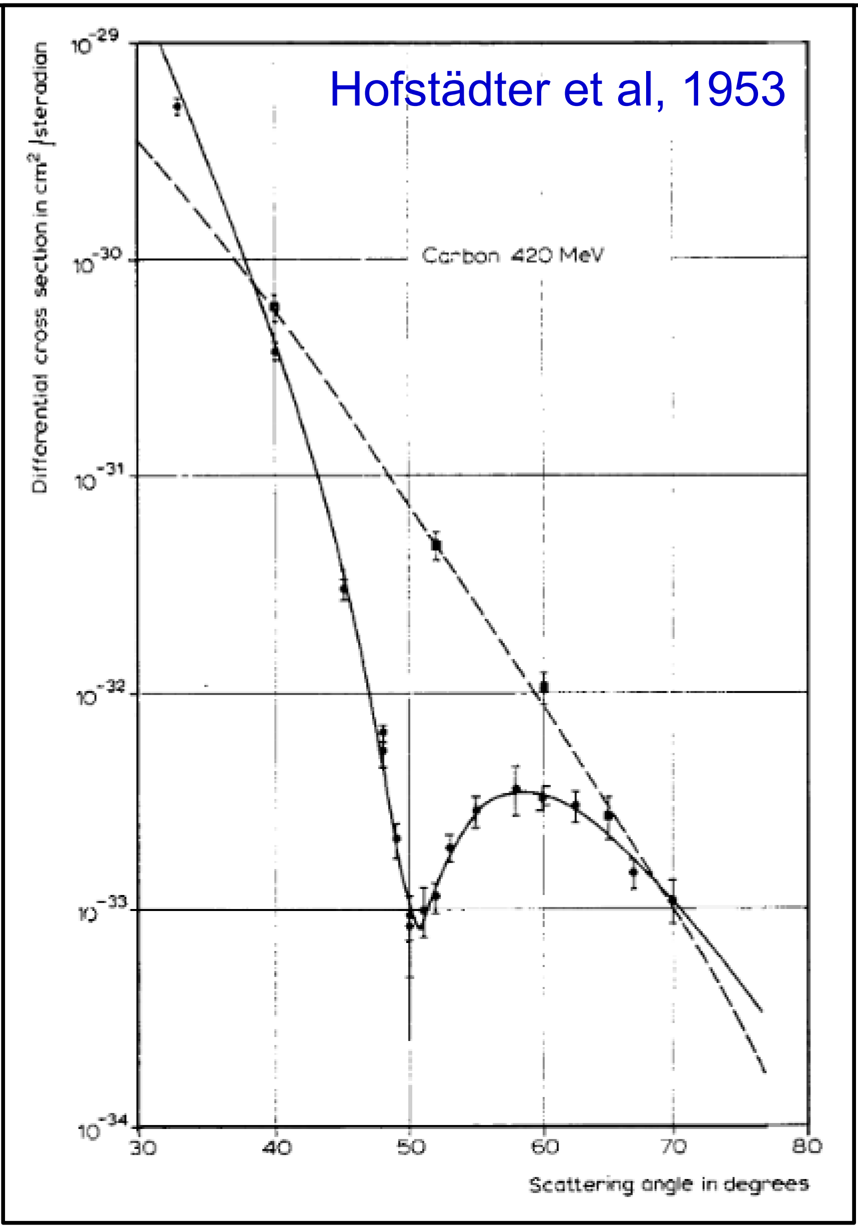
# Beobachtung bei Elektron-Kern-Streuung

- Beschreibung durch **Formfaktor**:

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \left(\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\right)_{\mathrm{Mott}} \cdot |F(\vec{q})|^2$$

(vgl mit Beugungsmuster am Spalt)

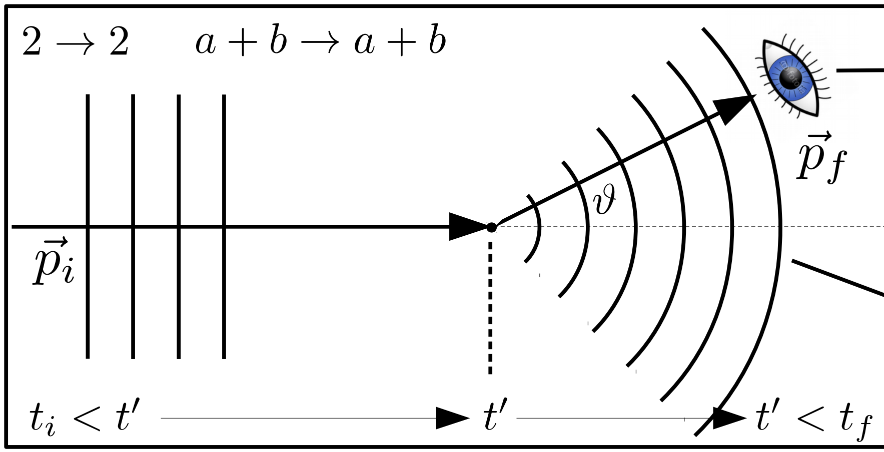
- Um diesen Zusammenhang besser zu verstehen leiten wir im folgenden den Rutherford-Wirkungsquerschnitt noch einmal quantenmechanisch her



(VL-03 Folie 14)

# Rutherford-Wirkungsqueschnitt (Erinnerung WQ in QM)

- Imagine a continuous flux of (small) incident particles $a$ impinging on a target particle $b$ at rest and the elastic reaction $a + b \rightarrow a + b$:




**Observation** (in $\Delta\Omega$): projection of plain wave $\psi_f$ out of spherical scattering wave $\psi_{\mathrm{scat}}$.

Spherical scattering wave $\psi_{\mathrm{scat}}$.

Initial particle: described by plain wave $\psi_i$.

Localized potential.

Observation probability:

$$\mathcal{S}_{fi} = \psi_f^\dagger \cdot \psi_{\mathrm{scat}}$$
$$= \psi_f^\dagger \cdot \mathcal{S} \cdot \psi_i$$

Scattering matrix $\mathcal{S}$ transforms initial state wave function $\psi_i$ into scattering wave $\psi_{\mathrm{scat}}$ ($\psi_{\mathrm{scat}} = \mathcal{S} \cdot \psi_i$).

**Fermi's golden rule:**

$$W = \frac{2\pi}{\hbar} \left|\mathcal{S}_{fi}\right|^2 \rho_f$$

$$\rho_f = \frac{\mathrm{d}}{\mathrm{d}E_f} \int \frac{\mathrm{d}^3 p_f \, \mathrm{d}^3 x_f}{(2\pi\hbar)^3}$$

phasespace factor for final state products.

# Rutherford-Wirkungsqueschnitt (QM) (I)

- **Fermi's Goldene Regel:**

$$W = \frac{2\pi}{\hbar} \cdot \left|\langle\psi_f|\mathcal{H}_{int}|\psi_i\rangle\right|^2 \cdot \rho_f$$ (Streurate, vgl VL-03 Folie 14)

$$W = \mathcal{L} \cdot \sigma = n_a \cdot v \cdot \sigma = \frac{v}{V} \cdot \sigma$$ (Relation zu Wirkungsquerschnitt, vgl VL-03 Folie 4)

**Anmerkung:**
QM Rechnung für elastische Streuung und OHNE Berücksichtigung von Spin

# Rutherford-Wirkungsqueschnitt (QM) (I)

- **Fermi's Goldene Regel:**

$$W = \frac{2\pi}{\hbar} \cdot |\langle\psi_f|\mathcal{H}_{int}|\psi_i\rangle|^2 \cdot \rho_f$$ (Streurate, vgl VL-03 Folie 14)

$$W = \mathcal{L} \cdot \sigma = n_a \cdot v \cdot \sigma = \frac{v}{V} \cdot \sigma$$ (Relation zu Wirkungsquerschnitt, vgl VL-03 Folie 4)

- **Phasenraumfaktor:**

$$\rho_f = \frac{\mathrm{d}}{\mathrm{d}E_f} \int \frac{\mathrm{d}^3 p_f \, \mathrm{d}^3 x_f}{(2\pi\,\hbar)^3} = \frac{4\pi\, p_f^2\, V}{(2\pi\,\hbar)^3} \frac{\mathrm{d}p_f}{\mathrm{d}E_f}$$

mit: $v \approx c \quad p_f = E_f/c \quad \mathrm{d}p_f = \mathrm{d}E_f/c$ (Impuls des gestreuten Teilchens im Endzustand)

$$\mathrm{d}\sigma \frac{v}{V} = \frac{2\pi}{\hbar} \cdot |\langle\psi_f|\mathcal{H}_{int}|\psi_i\rangle|^2 \cdot \frac{4\pi\, p_f^2\, V}{(2\pi\,\hbar)^3} \frac{\mathrm{d}p_f}{\mathrm{d}E_f} \cdot \frac{\mathrm{d}\Omega}{4\pi}$$

$$\boxed{\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \frac{V^2 E_f^2}{(2\pi)^2(\hbar\, c)^4} |\langle\psi_f|\mathcal{H}_{int}|\psi_i\rangle|^2}$$

# Rutherford-Wirkungsqueschnitt (QM) (II)

- **Streuamplitude/Matrixelement (**“in führender Ordnung”**):**

$$\langle\psi_f|\mathcal{H}_{int}|\psi_i\rangle = \frac{z\,e}{V}\int\limits_V e^{-i\vec{p}_f\vec{x}/\hbar}\,\phi(\vec{x})\,e^{i\vec{p}_i\vec{x}/\hbar}\,\mathrm{d}^3x = \int\limits_V \phi(\vec{x})\,e^{i\vec{q}\vec{x}/\hbar}\,\mathrm{d}^3x$$

mit:

$$\mathcal{H}_{int} = z\,e\,\phi(\vec{x}) \qquad \vec{q} = \vec{p}_i - \vec{p}_f \qquad \psi_{i,f} = \frac{1}{\sqrt{V}}e^{i\vec{p}_{i,f}\vec{x}/\hbar}$$

Als nächstes verwenden wir die Identität: $\int\limits_V (u\Delta v - v\Delta u)\,\mathrm{d}^3x = 0$

$$\boxed{\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \frac{V^2E_f^2}{(2\pi)^2(\hbar\,c)^4}\,|\langle\psi_f|\mathcal{H}_{int}|\psi_i\rangle|^2}$$

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Als nächstes verwenden wir die Identität: $\int\limits_V (u\Delta v - v\Delta u)\,\mathrm{d}^3x = 0$

$$\int\limits_V -\frac{q^2}{\hbar^2}\phi(\vec{x})\, e^{i\vec{q}\vec{x}/\hbar}\mathrm{d}^3x = \int\limits_V \Delta\phi(\vec{x})\, e^{i\vec{q}\vec{x}/\hbar}\mathrm{d}^3x$$

$$\int\limits_V \phi(\vec{x})\, e^{i\vec{q}\vec{x}/\hbar}\mathrm{d}^3x = -\frac{\hbar^2}{q^2}\int\limits_V \Delta\phi(\vec{x})\, e^{i\vec{q}\vec{x}/\hbar}\mathrm{d}^3x = -\frac{\hbar^2}{q^2}\int\limits_V \frac{\rho(\vec{x})}{\epsilon_0} e^{i\vec{q}\vec{x}/\hbar}\mathrm{d}^3x$$

$$\rho(\vec{x}) = Z\,e\,f(\vec{x})$$

$$\boxed{\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \frac{V^2 E_f^2}{(2\pi)^2(\hbar\,c)^4}\,\left|\langle \psi_f | \mathcal{H}_{int} | \psi_i \rangle\right|^2}$$

$$\boxed{\langle \psi_f | \mathcal{H}_{int} | \psi_i \rangle = \frac{z\,Z\,e^2\hbar^2}{\epsilon_0\,q^2\,V}\int\limits_V f(\vec{x}) e^{i\vec{q}\vec{x}/\hbar}\mathrm{d}^3x}$$

# Rutherford-Wirkungsqueschnitt (QM) (III)

- **Wirkungsquerschnitt (**“alles zusammengefaßt”**):**

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \frac{V^2 E_f^2}{(2\pi)^2(\hbar\, c)^4} \left| \frac{z\, Z\, e^2 \hbar^2}{\epsilon_0\, q^2\, V} \int_V f(\vec{x}) e^{i\vec{q}\vec{x}/\hbar} \mathrm{d}^3 x \right|^2$$

$$\alpha = \frac{e^2}{4\pi\epsilon_0\hbar c} = \frac{1}{137}$$

(vgl VL-03 Folie 12)

$$= \frac{V^2 E_f^2}{(2\pi)^2(\hbar\, c)^4} \left( \frac{4\,\pi\, z\, Z\, \alpha\, \hbar^3\, c}{q^2\, V} \right)^2 \left| \int_V f(\vec{x}) e^{i\vec{q}\vec{x}/\hbar} \mathrm{d}^3 x \right|^2$$

$$= \frac{\left(2\, z\, Z\, \alpha\, (\hbar\, c)\right)^2 E_f^2}{q^4\, c^4} \left| \underbrace{\int f(\vec{x}) e^{i\vec{q}\vec{x}/\hbar} \mathrm{d}^3 x}_{\equiv 1 \text{ für } f(\vec{x}) = \delta(\vec{x})} \right|^2$$

(vgl VL-03 Folie 13)

$$\boxed{\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \frac{V^2 E_f^2}{(2\pi)^2(\hbar\, c)^4} \left| \langle \psi_f | \mathcal{H}_{int} | \psi_i \rangle \right|^2}$$

$$\boxed{\langle \psi_f | \mathcal{H}_{int} | \psi_i \rangle = \frac{z\, Z\, e^2 \hbar^2}{\epsilon_0\, q^2\, V} \int\limits_V f(\vec{x}) e^{i\vec{q}\vec{x}/\hbar} \mathrm{d}^3 x}$$

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$$= \frac{\left(2\, z\, Z\, \alpha\, (\hbar\, c)\right)^2 E_f^2}{q^4\, c^4} \left| \underbrace{\int f(\vec{x}) e^{i\vec{q}\vec{x}/\hbar} \mathrm{d}^3 x}_{\equiv 1 \text{ für } f(\vec{x}) = \delta(\vec{x})} \right|^2$$

(vgl VL-03 Folie 13)

- **Formfaktor:**

$$F(\vec{q}) = \int_V f(\vec{x}) e^{i\vec{q}\vec{x}/\hbar} \mathrm{d}^3 x \qquad \text{(Formfaktor)}$$

$$F(q) = 4\,\pi \int_0^\infty f(r) \frac{\sin(|q|\, r/\hbar)}{|q|\, r/\hbar}\, r^2 \mathrm{d}r \qquad \text{(für radialsymmetrische Ladungsverteilungen)}$$

**Allgemeine Eigenschaft** aller Streuexperimente (→ Spalt, Gitter, Kern, Nukleon, … )

# Gestallt der Kerne

- Aus Rücktransformation des Formfaktors → Dichte der Ladungsverteilung

$$f(\vec{x}) = \frac{1}{(2\pi)^3} \int_V F(\vec{q})\, e^{i\vec{q}\vec{x}/\hbar} \mathrm{d}^3 q$$

ABER: in Praxis nur begrenzte Bereiche in q meßbar (warum?)

- Daher üblicherweise Modellanpassung

# Gliederung der Vorlesung

YOU ARE HERE

KW-17



KW-18

KW-19



KW-20