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Teilchenphysik 2 — W/Z/Higgs an Collidern

Sommersemester 2019

Exercises No. 1 Discussion on May 8, 2019

Exercise 1: Higgs-Boson Production at Hadron Colliders
Consider on-shell Higgs-boson production at hadron colliders. Assuming the Higgs boson is produced at rest in the laboratory frame, what is the rapidity of the Higgs boson? Determine the momentum fraction x of the initial partons (quarks or gluons) in the production process at the Tevatron and the LHC.

Which parton has the highest probability at this x? You can draw the parton distribution functions (PDFs) using the applet at `http://hepdata.cedar.ac.uk/pdf/pdf3.html`. Which value of the momentum transfer Q^2 do you have to use?

From this, can you motivate the dominant Higgs-boson production channel?

Without the assumption that the Higgs boson is produced at rest, what is the minimal value of x to produce a Higgs boson at the Tevatron and the LHC?

Exercise 2: Equations of Motion
The equations of motion of a system described by the field $\Phi(x)$ can be derived from the Lagrange density $\mathcal{L}$ using the Euler-Lagrange equations

$$\partial_\mu \frac{\partial \mathcal{L}}{\partial(\partial_\mu \Phi(x))} - \frac{\partial \mathcal{L}}{\partial \Phi(x)} = 0\,. \tag{1}$$

Show that the Lagrange density

$$\mathcal{L} = \overline{\psi}\,(i\gamma^\mu \partial_\mu - m)\,\psi \tag{2}$$

describes a fermion field ψ, by deriving the Dirac equation using (1). Perfom the calcuation for both $\Phi = \overline{\psi}$ and $\Phi = \psi$.

Analogously, derive the equations of motion for a complex scalar field $\phi = \frac{1}{\sqrt{2}}(\phi_1 + i\phi_2)$ from the Lagrange density

$$\mathcal{L} = \tfrac{1}{2}\left[(\partial_\mu \phi)(\partial^\mu \phi)^* - m^2\phi^2\right] .$$

What is the interpretation of the two obtained equations of motion?

<u>Exercise 3</u>: QED Gauge Field

Invariance of the Lagrangian (2) of a free fermion ψ field under local U(1) phase transformations $\mathrm{e}^{i\alpha(x)}$ can be achieved by replacing the partial derivative with the covariant derivative

$$\partial_\mu \to D_\mu = \partial_\mu + ieA_\mu . \qquad (3)$$

The gauge field A_μ can be interpreted as the photon of QED, the coupling constant e as the electric charge. As a consequence, the fermion is no longer free but interacts with the photon field.

In order to achieve local gauge invariance, the covariant derivative is required to transform as

$$D_\mu \to D'_\mu = D_\mu - i\partial_\mu \alpha(x) .$$

Show explicitly why this particular transformation behaviour is required. What is the required transformation behaviour of the gauge field A_μ?

For a consistent theory of QED, a kinetic term $\mathcal{L}_{\text{kin}}$ for the gauge field needs to be added to (2) in addition to the replacement (3). In the lecture, we have used

$$\mathcal{L}_{\text{kin}} = \tfrac{1}{4}F^{\mu\nu}F_{\mu\nu} \quad \text{with} \quad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu .$$

Using the transformation behaviour of the gauge field A_μ obtained above, prove that the field-strength tensor $F_{\mu\nu}$ is locally gauge invariant, i. e. that

$$F'_{\mu\nu} = F_{\mu\nu} .$$

As a consequence, $\mathcal{L}_{\text{kin}}$ is also gauge invariant.

In order to further investigate the association of the gauge field A_μ with the QED photon, derive the equations of motion of A_μ by applying the Euler-Lagrange equations (1) to $\mathcal{L}_{\text{kin}}$. Show that this leads to the Proca equation when using the Lorenz gauge $\partial_\mu A^\mu = 0$ of electrodynamics.

Bonus: Show that the field-strength tensor can be conveniently written as

$$F_{\mu\nu} = -\tfrac{i}{e}[D_\mu, D_\nu] .$$

<u>Exercise 4</u>: Chiral Symmetry

The transformation $\chi : \psi \to \gamma^5\psi$ is called *chiral* transformation. It turns e. g. axial vectors into vectors and vice versa.

a) What is the adjoint of the transformed spinor?

b) Show that e_L and e_R are *eigentstates* of the chiral transformation with the *eigenvalues* -1 and $+1$, respectively.

c) Show that terms of type $\overline{\psi}\gamma^\mu\partial_\mu\psi$ are covariant under chiral transformations, while terms of type $\overline{\psi}m\psi$ are not. As a consequence the presence of light particles is a small perturbation of a chiral symmetry in the SM Lagrangian density.