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Teilchenphysik 2 — W/Z/Higgs an Collidern

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Exercise 1: Masses for the Gauge Bosons

In the Standard Model, the mass terms for the gauge bosons $\mathrm{W}^{\pm}$ and Z emerge dynamically from their coupling to the Higgs field via the covariant derivative. We want to study this in the following.

The Higgs field ϕ of the Standard Model is a weak-isospin doublet, and its covariant derivative is

$$D_\mu\phi = \left[\partial_\mu + i\tfrac{g}{2}\tau_a \mathrm{W}^a_\mu + i\tfrac{g'}{2}Y_\phi \mathrm{B}_\mu\right]\phi$$

with the three $\mathrm{SU}(2)_\mathrm{L}$ gauge bosons W^a, the $\mathrm{U}(1)_\mathrm{Y}$ gauge boson B, the three Pauli matrices τ^a, and the weak hypercharge $Y_\phi = +1$ of the Higgs field. After electroweak symmetry breaking, the ground state ϕ_0 of the Higgs field can be chosen as

$$\phi_0 = \tfrac{1}{\sqrt{2}}\begin{pmatrix}0\\ v\end{pmatrix}, \qquad v = \sqrt{-\tfrac{\mu^2}{\lambda}}\,. \tag{1}$$

As a first step, the Higgs field is expanded around its ground state by a small perturbation $\mathrm{H}(x) \equiv \mathrm{H}$, identified with the Higgs boson, such that ϕ becomes

$$\phi = \tfrac{1}{\sqrt{2}}\begin{pmatrix}0\\ v+\mathrm{H}\end{pmatrix}. \tag{2}$$

(Note that ϕ has two components because it is an isospin doublet.)

Show that, with Eq. (2), the covariant derivative and its conjugate of the Higgs field become

$$D_\mu\phi = \tfrac{1}{\sqrt{2}}\begin{pmatrix}0\\ \partial_\mu\mathrm{H}\end{pmatrix} + \tfrac{i}{\sqrt{8}}\begin{pmatrix}g(\mathrm{W}^1_\mu - i\mathrm{W}^2_\mu)\\ -g\mathrm{W}^3_\mu + g'\mathrm{B}_\mu\end{pmatrix}(v+\mathrm{H})$$

$$D^\mu\phi^\dagger = \tfrac{1}{\sqrt{2}}\begin{pmatrix}0 & \partial^\mu\mathrm{H}\end{pmatrix} - \tfrac{i}{\sqrt{8}}\begin{pmatrix}g(\mathrm{W}^{1,\mu} + i\mathrm{W}^{2,\mu}) & -g\mathrm{W}^{3,\mu} + g'\mathrm{B}^\mu\end{pmatrix}(v+\mathrm{H})$$

and that the dynamic term in the Higgs Lagrangian becomes

$$D^\mu\phi^\dagger D_\mu\phi = \tfrac{1}{2}\partial^\mu \mathrm{H}\partial_\mu \mathrm{H} + \tfrac{1}{8}g^2\left(|\mathrm{W}^1|^2+|\mathrm{W}^2|^2\right)(v+\mathrm{H})^2 + \tfrac{1}{8}\left(-g\mathrm{W}^3_\mu + g'\mathrm{B}_\mu\right)^2(v+\mathrm{H})^2 \,. \tag{3}$$

With the definition of the $\mathrm{W}^\pm$ bosons,

$$\mathrm{W}^\pm_\mu = \tfrac{1}{\sqrt{2}}\left(\mathrm{W}^1_\mu \mp i\mathrm{W}^2_\mu\right) \,,$$

and with the defintion of the Z boson as a superposition of W^3 and B (Weinberg rotation), show then that Eq. (3) can be written in terms of the physical gauge bosons as

$$D^\mu\phi^\dagger D_\mu\phi = \tfrac{1}{2}\partial^\mu \mathrm{H}\partial_\mu \mathrm{H} + \tfrac{1}{2}\tfrac{g^2}{4}(v+\mathrm{H})^2\left(\mathrm{W}^+_\mu\mathrm{W}^{+\mu} + \mathrm{W}^-_\mu\mathrm{W}^{-\mu}\right) + \tfrac{1}{2}\tfrac{g^2+g'^2}{4}(v+\mathrm{H})^2\,\mathrm{Z}_\mu\mathrm{Z}^\mu \,.$$

What are the resulting gauge boson masses?

This approach results in addition into coupling terms between the gauge bosons and the Higgs boson H. Express the terms by the gauge boson masses and the vacuum expectation value v of the Higgs field. How does the coupling depend on the gauge boson masses?

Exercise 2: Masses for the Fermions

In the Standard Model, the Higgs doublet can also be used to generate mass terms for the fermions. They emerge dynamically from additionally introduced Yukawa coupling terms

$$\mathcal{L}_{\mathrm{Yukawa}} = -y_f\left(\overline{\psi}_L\phi\psi_R + \overline{\psi}_R\phi^\dagger\psi_L\right) \tag{4}$$

between the Higgs field ϕ and the fermion fields ψ. Here, ψ_L denotes a weak isospin doublet of left-handed fermions, and ψ_R denotes the corresponding singlet of right-handed fermions, e. g. in case of the first generation leptons

$$\psi_L = \begin{pmatrix}\nu_e\\ e\end{pmatrix}_L \,, \quad \psi_R = e_R \,.$$

Show that $\mathcal{L}_{\mathrm{Yukawa}}$ Eq. (4) is invariant under both $\mathrm{U}(1)_Y$ transformations $\mathcal{A}_Y$ and $\mathrm{SU}(2)_L$ transformations $\mathcal{B}_L$, where

$$\begin{aligned}
\mathcal{A}_Y &: \mathrm{F}_{L/R} \to \exp[i\tfrac{g'}{2}Y_\mathrm{F}\alpha(x)]\mathrm{F}_{L/R}\\
\mathcal{B}_L &: \;\mathrm{F}_L \;\to \exp[i\tfrac{g}{2}\tau^a\alpha_a(x)]\mathrm{F}_L\\
\mathcal{B}_L &: \;\mathrm{F}_R \;\to \mathrm{F}_R \,.
\end{aligned}$$

and F_L represents the isospin doublets spinor ψ_L and Higgs field ϕ_L, and F_R the isospin singlet spinor ψ_R. Note that $\mathcal{A}_Y$ depends on the weak hypercharge Y_F of the field F it acts on, and that the weak hypercharge of the Higgs field is $Y_\phi = +1$.

Now, work out the fermion mass terms resulting from $\mathcal{L}_{\text{Yukawa}}$ Eq. (4). Demonstrate this for the case of the first generation leptons and assume neutrinos to be massless. Start with expanding the Higgs field around its ground state ϕ_0 Eq. (1) by a small perturbation H, identified with the Higgs boson, as in Eq. (2). Show that this leads to

$$\mathcal{L}_{\text{Yukawa}} = -\tfrac{y_e}{\sqrt{2}} \left[\overline{e}_L (v + \mathrm{H}) e_R + \overline{e}_R (v + \mathrm{H}) e_L \right] ,$$

and derive the electron mass term from this. The approach results in addition into coupling terms between the electron and the Higgs boson. Show explicitly the proportionality of the coupling to the fermion mass.

As part of the calculation, you will need to show that

$$\overline{e}e = \overline{e}_L e_R + \overline{e}_R e_L \, .$$

Consider decays of the Higgs boson into pairs of $\tau^+\tau^-$ and $\mu^+\mu^-$ leptons. What is the relative frequency of the decays?

Solutions

Useful definitions:

$$A^\dagger \equiv (A^*)^T\,, \qquad \overline{A} \equiv A^\dagger \gamma^0\,.$$

Useful identities of γ^μ matrices:

$$\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu}\,, \qquad \gamma^0 = (\gamma^0)^\dagger\,, \qquad \gamma^a = -(\gamma^a)^\dagger\,,$$

and of γ^5:

$$\gamma^5 = (\gamma^5)^\dagger\,, \qquad (\gamma^5)^2 = 1\,, \qquad \{\gamma^5, \gamma^\mu\} = 0\,.$$

It is further

$$(A \cdot B)^\dagger = B^\dagger \cdot A^\dagger\,.$$

Solution to Exercise 1

Using Eq. (2) and $Y_\phi = +1$, it is

$$\begin{aligned} D_\mu \phi &= \left[\partial_\mu + i\tfrac{g}{2}\tau_a \mathrm{W}^a_\mu + i\tfrac{g'}{2} Y_\phi \mathrm{B}_\mu\right]\phi \\ &= \tfrac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ \partial_\mu(v+\mathrm{H})\end{pmatrix} + \tfrac{i}{\sqrt{8}}\left[g\tau_a \mathrm{W}^a_\mu + g'\mathrm{B}_\mu\right]\begin{pmatrix} 0 \\ v+\mathrm{H}\end{pmatrix}\,. \end{aligned}$$

With the Pauli matrices

$$\tau_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0\end{pmatrix}\,, \quad \tau_2 = \begin{pmatrix} 0 & -i \\ i & 0\end{pmatrix}\,, \quad \tau_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1\end{pmatrix}\,,$$

the sum $\tau_a \mathrm{W}^a_\mu$ becomes

$$\tau_a \mathrm{W}^a_\mu = \begin{pmatrix} 0 & \mathrm{W}^1_\mu \\ \mathrm{W}^1_\mu & 0\end{pmatrix} + \begin{pmatrix} 0 & -i\mathrm{W}^2_\mu \\ i\mathrm{W}^2_\mu & 0\end{pmatrix} + \begin{pmatrix} \mathrm{W}^3_\mu & 0 \\ 0 & -\mathrm{W}^3_\mu\end{pmatrix}\,.$$

With this and since $\partial_\mu v = 0$, it is

$$\begin{aligned} D_\mu \phi &= \tfrac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ \partial_\mu \mathrm{H}\end{pmatrix} + \tfrac{i}{\sqrt{8}} g \begin{pmatrix} \mathrm{W}^3_\mu & \mathrm{W}^1_\mu - i\mathrm{W}^2_\mu \\ \mathrm{W}^1_\mu + i\mathrm{W}^2_\mu & -\mathrm{W}^3_\mu\end{pmatrix}\begin{pmatrix} 0 \\ v+\mathrm{H}\end{pmatrix} + \tfrac{i}{\sqrt{8}} g'\mathrm{B}_\mu \begin{pmatrix} 0 \\ v+\mathrm{H}\end{pmatrix} \\ &= \tfrac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ \partial_\mu \mathrm{H}\end{pmatrix} + \tfrac{i}{\sqrt{8}}\begin{pmatrix} g(\mathrm{W}^1_\mu - i\mathrm{W}^2_\mu)(v+\mathrm{H}) \\ -g\mathrm{W}^3_\mu(v+\mathrm{H})\end{pmatrix} + \tfrac{i}{\sqrt{8}}\begin{pmatrix} 0 \\ g'\mathrm{B}_\mu(v+\mathrm{H})\end{pmatrix} \\ &= \tfrac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ \partial_\mu \mathrm{H}\end{pmatrix} + \tfrac{i}{\sqrt{8}}\begin{pmatrix} g(\mathrm{W}^1_\mu - i\mathrm{W}^2_\mu) \\ -g\mathrm{W}^3_\mu + g'\mathrm{B}_\mu\end{pmatrix}(v+\mathrm{H})\,. \end{aligned}$$

For convenience, this can be rearranged into real and imaginary parts as

$$D_\mu\phi = \underbrace{\begin{pmatrix} \frac{1}{\sqrt{8}} g \mathrm{W}_\mu^2 (v+\mathrm{H}) \\ \frac{1}{\sqrt{2}} \partial_\mu \mathrm{H} \end{pmatrix}}_{\mathcal{A}} + i \underbrace{\frac{1}{\sqrt{8}} \begin{pmatrix} g\mathrm{W}_\mu^1 \\ -g\mathrm{W}_\mu^3 + g'\mathrm{B}_\mu \end{pmatrix} (v+\mathrm{H})}_{\mathcal{B}} ,$$

from which one can see that

$$D^\mu\phi^\dagger D_\mu\phi = (\mathcal{A}^T - i\mathcal{B}^T)(\mathcal{A} + i\mathcal{B}) = |\mathcal{A}|^2 + |\mathcal{B}|^2$$

and thus

$$D^\mu\phi^\dagger D_\mu\phi = \tfrac{1}{2}\partial^\mu \mathrm{H}\partial_\mu \mathrm{H} + \tfrac{1}{8} g^2 \left(|\mathrm{W}^1|^2 + |\mathrm{W}^2|^2\right)(v+\mathrm{H})^2 + \tfrac{1}{8}\left(-g\mathrm{W}_\mu^3 + g'\mathrm{B}_\mu\right)^2 (v+\mathrm{H})^2 .$$

With the defintion $\mathrm{W}_\mu^\pm = \frac{1}{\sqrt{2}}\left(\mathrm{W}_\mu^1 \mp i\mathrm{W}_\mu^2\right)$, it is

$$\begin{aligned} \mathrm{W}_\mu^1 &= \tfrac{1}{\sqrt{2}}\left(\mathrm{W}_\mu^+ + \mathrm{W}_\mu^-\right) \\ \mathrm{W}_\mu^2 &= \tfrac{i}{\sqrt{2}}\left(\mathrm{W}_\mu^+ - \mathrm{W}_\mu^-\right) \ , \end{aligned}$$

and thus

$$\begin{aligned} |\mathrm{W}^1|^2 + |\mathrm{W}^2|^2 = \tfrac{1}{2}\big(& |\mathrm{W}^+|^2 + |\mathrm{W}^-|^2 + 2\mathrm{W}_\mu^+\mathrm{W}^{-\mu} \\ & + |\mathrm{W}^+|^2 + |\mathrm{W}^-|^2 - 2\mathrm{W}_\mu^+\mathrm{W}^{-\mu}\big) \\ = \quad & |\mathrm{W}^+|^2 + |\mathrm{W}^-|^2 . \end{aligned}$$

The Z boson is defined by the Weinberg rotation as

$$\mathrm{Z}_\mu = \cos\theta_W \mathrm{W}_\mu^3 - \sin\theta_W \mathrm{B}_\mu$$

with

$$\sin\theta_W \equiv \tfrac{g'}{\sqrt{g^2+g'^2}} , \quad \cos\theta_W \equiv \tfrac{g}{\sqrt{g^2+g'^2}} ,$$

and thus it is

$$-\sqrt{g^2+g'^2}\,\mathrm{Z}_\mu = -g\mathrm{W}_\mu^3 + g'\mathrm{B}_\mu ,$$

such that $D^\mu\phi^\dagger D_\mu\phi$ can be expressed in terms of the physical gauge bosons as

$$D^\mu\phi^\dagger D_\mu\phi = \tfrac{1}{2}\partial^\mu \mathrm{H}\partial_\mu \mathrm{H} + \tfrac{1}{8} g^2 \left(|\mathrm{W}^1|^2 + |\mathrm{W}^2|^2\right)(v+\mathrm{H})^2 + \tfrac{1}{8}\left(-g\mathrm{W}_\mu^3 + g'\mathrm{B}_\mu\right)^2 (v+\mathrm{H})^2 .$$

Expanding the $(v+\mathrm{H})^2$ terms yields, looking only at the terms involving W^+ bosons,

$$\begin{aligned} & \tfrac{1}{2}\tfrac{g^2}{4}(v+\mathrm{H})^2\,\mathrm{W}_\mu^+\mathrm{W}^{+\mu} \\ = & \tfrac{1}{2}\underbrace{\tfrac{g^2}{4}v^2}_{\equiv m_\mathrm{W}^2}\mathrm{W}_\mu^+\mathrm{W}^{+\mu} + \tfrac{g^2}{4}v\mathrm{H}\mathrm{W}_\mu^+\mathrm{W}^{+\mu} + \tfrac{1}{2}\tfrac{g^2}{4}\mathrm{H}^2\mathrm{W}_\mu^+\mathrm{W}^{+\mu} . \end{aligned}$$

The identification of the W-boson mass as

$$m_{\mathrm{W}} = \tfrac{g}{2} v$$

is motivated by the fact that in the Lagrangian that yields the Proca equation for massive vector bosons A^μ, the mass enters as a term

$$\tfrac{1}{2} m_{\mathrm{A}}^2 \mathrm{A}_\mu \mathrm{A}^\mu \,.$$

Rewriting the above in terms of m_{W} yields

$$\begin{aligned} &\tfrac{1}{2}\tfrac{g^2}{4}\left(v+\mathrm{H}\right)^2 \mathrm{W}_\mu^+ \mathrm{W}^{+\mu} \\ =&\tfrac{1}{2} m_{\mathrm{W}}^2 \mathrm{W}_\mu^+ \mathrm{W}^{+\mu} + \tfrac{m_{\mathrm{W}}^2}{v} \mathrm{H} \mathrm{W}_\mu^+ \mathrm{W}^{+\mu} + \tfrac{1}{2}\tfrac{m_{\mathrm{W}}^2}{v^2} \mathrm{H}^2 \mathrm{W}_\mu^+ \mathrm{W}^{+\mu} \,. \end{aligned}$$

The last two terms can be interpreted as three-point and four-point interaction terms between a Higgs boson and two W bosons or two Higgs bosons and two W bosons, respectively.

Interpretation of the terms with W^- and Z works in complete analogy.

Solution to Exercise 2

For the $\mathrm{U}(1)_Y$ transformations, it is

$$\begin{aligned} \left(\overline{\psi}_L \phi \psi_R\right)' &= \left(\overline{(\mathcal{A}_{Y_L}\psi_L)}\right)\left(\mathcal{A}_{Y_\phi}\phi\right)\left(\mathcal{A}_{Y_R}\psi_R\right) \\ &= \left(\overline{\psi}_L \mathcal{A}_{Y_L}^\dagger\right)\left(\mathcal{A}_{Y_\phi}\phi\right)\left(\mathcal{A}_{Y_R}\psi_R\right) \\ &= \mathcal{A}_{Y_L}^\dagger \mathcal{A}_{Y_\phi} \mathcal{A}_{Y_R}\left(\overline{\psi}_L \phi \psi_R\right) \\ &= \exp\left[i\tfrac{g'}{2}(-Y_L + Y_\phi + Y_R)\alpha(x)\right]\left(\overline{\psi}_L \phi \psi_R\right) \\ &= \exp\left[i\tfrac{g'}{2}(-(-1) + (+1) + (-2))\alpha(x)\right]\left(\overline{\psi}_L \phi \psi_R\right) \\ &= \left(\overline{\psi}_L \phi \psi_R\right) , \end{aligned}$$

where we have used that

$$\begin{aligned} \overline{(\mathcal{A}\psi)} &= (\mathcal{A}\psi)^\dagger \gamma^0 \\ &= \psi^\dagger \mathcal{A}^\dagger \gamma^0 \\ &= \psi^\dagger \gamma^0 \mathcal{A}^\dagger \\ &= \overline{\psi} \mathcal{A}^\dagger \end{aligned}$$

and that the weak hypercharges for the first-generation leptons are $Y_L = -1$ and $Y_R = -2$ and for the Higgs field $Y_\phi = +1$.

For the $\mathrm{SU}(2)_L$ transformations, it is

$$\begin{aligned}
\left(\overline{\psi}_L\phi\psi_R\right)' &= \left(\overline{(\mathcal{B}_L\psi_L)}\right)\left(\mathcal{B}_L\phi\right)\left(\mathcal{B}_L\psi_R\right)\\
&= \left(\overline{\psi}_L\mathcal{B}_L^\dagger\right)\left(\mathcal{B}_L\phi\right)\left(\psi_R\right)\\
&= \left(\overline{\psi}_L\underbrace{\mathcal{B}_L^\dagger\mathcal{B}_L}_{=1}\phi\psi_R\right)\\
&= \left(\overline{\psi}_L\phi\psi_R\right)
\end{aligned}$$

Analogously, one shows invariance for the $\left(\overline{\psi}_R\phi^\dagger\psi_L\right)$ terms.

Expanding around ϕ_0 leads to

$$\begin{aligned}
\mathcal{L}_{\text{Yukawa}} &= -y_e\left(\overline{\psi}_L\phi\psi_R + \overline{\psi}_R\phi^\dagger\psi_L\right)\\
&= -y_e\left[(\overline{\nu}\quad\overline{e})_L\tfrac{1}{\sqrt{2}}\begin{pmatrix}0\\ v+\mathrm{H}\end{pmatrix}e_R + \overline{e}_R\tfrac{1}{\sqrt{2}}(0\quad v+\mathrm{H})\begin{pmatrix}\nu\\ e\end{pmatrix}_L\right]\\
&= -\tfrac{y_e}{\sqrt{2}}\left[\overline{e}_L(v+\mathrm{H})e_R + \overline{e}_R(v+\mathrm{H})e_L\right]\\
&= -\tfrac{y_e}{\sqrt{2}}\left[v\overline{e}_Le_R + v\overline{e}_Re_L + \mathrm{H}\overline{e}_Le_R + \mathrm{H}\overline{e}_Re_L\right]\\
&= -\tfrac{y_e}{\sqrt{2}}v\underbrace{\left(\overline{e}_Le_R + \overline{e}_Re_L\right)}_{\overline{e}e} - \tfrac{y_e}{\sqrt{2}}\mathrm{H}\underbrace{\left(\overline{e}_Le_R + \overline{e}_Re_L\right)}_{\overline{e}e},
\end{aligned}$$

where the identity in the last step is proven further below. Comparison to the *Dirac* mass term

$$-m\overline{\psi}\psi\,,$$

i. e. the mass term in the Lagrangian that leads to the Dirac equation for massive fermions, reveals

$$m_e = \tfrac{y_e}{\sqrt{2}}v\,.$$

Rewriting in terms of m_e leads to

$$\mathcal{L}_{\text{Yukawa}} = -m_e\overline{e}e - \tfrac{m_e}{v}\mathrm{H}\overline{e}e\,.$$

The last term can be interpreted as interaction between the Higgs boson and two electrons with a coupling strength $g_{\mathrm{H}ee} \propto m_e/v$. Compare this to the coupling strength $g_{\mathrm{HVV}} \propto m_{\mathrm{V}}^2/v$ of the respective three-point interaction HVV between the Higgs boson and the gauge bosons V.

With the coupling strength $g_{\mathrm{H}ff} \propto m_f/v$ of the Higgs boson to fermions f, the decay width follows from Fermi's Golden rule as

$$\Gamma(\mathrm{H}\to\bar{f}f) \propto \sigma \propto |\mathcal{M}|^2 \propto g_{\mathrm{H}ff}^2 \propto \left(\frac{m_f}{v}\right)^2\,.$$

Thus, assuming $m_{\mathrm{H}} \gg m_\tau, m_\mu$, one can assume the factors of proportionality (phase-space) to be the same for τ and μ, and hence

$$\frac{\Gamma(\mathrm{H} \to \tau^+\tau^-)}{\Gamma(\mathrm{H} \to \mu^+\mu^-)} = \left(\frac{m_\tau}{m_\mu}\right)^2 = \left(\frac{1.8\,\mathrm{GeV}}{0.1\,\mathrm{GeV}}\right)^2 = 18^2 = 324\,.$$

It remains to be shown that

$$\overline{e}e = \overline{e}_L e_R + \overline{e}_R e_L\,.$$

Each spinor can be decomposed into its left- and right-handed component as

$$e = e_L + e_R$$

with the projection operators

$$\mathcal{P}_{L/R} : e_{L/R} = \mathcal{P}_{L/R} e = \tfrac{1}{2}(1 \mp \gamma^5)e\,.$$

Thus, it is

$$\overline{e}e = \overline{(e_L + e_R)}(e_L + e_R) = \overline{e}_L e_L + \overline{e}_R e_R + \overline{e}_L e_R + \overline{e}_R e_L\,.$$

We now first show that

$$\begin{aligned}
\overline{e}_{L/R} &= \overline{\tfrac{1}{2}(1 \mp \gamma^5)e} \\
&= \left(\tfrac{1}{2}(1 \mp \gamma^5)e\right)^\dagger \gamma^0 \\
&= \tfrac{1}{2}e^\dagger(1 \mp \gamma^5)^\dagger\gamma^0 \\
&= \tfrac{1}{2}e^\dagger(1 \mp \gamma^5)\gamma^0 \qquad \text{since } (\gamma^5)^\dagger = \gamma^5 \\
&= \tfrac{1}{2}e^\dagger\gamma^0(1 \pm \gamma^5) \qquad \text{since } \{\gamma^\mu, \gamma^5\} = 0 \\
&= \tfrac{1}{2}\overline{e}(1 \pm \gamma^5)\,.
\end{aligned}$$

Thus, it is

$$\begin{aligned}
\overline{e}_{L/R} e_{L/R} &= \tfrac{1}{2}\overline{e}(1 \pm \gamma^5)\,\tfrac{1}{2}e(1 \mp \gamma^5) \\
&= \tfrac{1}{4}\overline{e}(1 \pm \gamma^5)(1 \mp \gamma^5)e \\
&= \tfrac{1}{4}\overline{e}(1 - (\gamma^5)^2)e \\
&= 0 \qquad\qquad \text{since } (\gamma^5)^2 = 1\,.
\end{aligned}$$