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Teilchenphysik 2 — W/Z/Higgs an Collidern

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Exercises No. 2 Discussion on May 15, 2019

Exercise 1: Masses for the Gauge Bosons

In the Standard Model, the mass terms for the gauge bosons $W^\pm$ and Z emerge dynamically from their coupling to the Higgs field via the covariant derivative. We want to study this in the following.

The Higgs field ϕ of the Standard Model is a weak-isospin doublet, and its covariant derivative is

$$D_\mu \phi = \left[\partial_\mu + i\tfrac{g}{2}\tau_a \mathrm{W}^a_\mu + i\tfrac{g'}{2}Y_\phi \mathrm{B}_\mu\right]\phi$$

with the three $\mathrm{SU}(2)_\mathrm{L}$ gauge bosons W^a, the $\mathrm{U}(1)_\mathrm{Y}$ gauge boson B, the three Pauli matrices τ^a, and the weak hypercharge $Y_\phi = +1$ of the Higgs field. After electroweak symmetry breaking, the ground state ϕ_0 of the Higgs field can be chosen as

$$\phi_0 = \tfrac{1}{\sqrt{2}}\begin{pmatrix}0\\ v\end{pmatrix}, \qquad v = \sqrt{-\tfrac{\mu^2}{\lambda}}\,. \tag{1}$$

As a first step, the Higgs field is expanded around its ground state by a small perturbation $\mathrm{H}(x) \equiv \mathrm{H}$, identified with the Higgs boson, such that ϕ becomes

$$\phi = \tfrac{1}{\sqrt{2}}\begin{pmatrix}0\\ v+\mathrm{H}\end{pmatrix}. \tag{2}$$

(Note that ϕ has two components because it is an isospin doublet.)

Show that, with Eq. (2), the covariant derivative and its conjugate of the Higgs field become

$$\begin{aligned}
D_\mu\phi &= \tfrac{1}{\sqrt{2}}\begin{pmatrix}0\\ \partial_\mu \mathrm{H}\end{pmatrix} + \tfrac{i}{\sqrt{8}}\begin{pmatrix} g(\mathrm{W}^1_\mu - i\mathrm{W}^2_\mu)\\ -g\mathrm{W}^3_\mu + g'\mathrm{B}_\mu\end{pmatrix}(v+\mathrm{H})\\
D^\mu\phi^\dagger &= \tfrac{1}{\sqrt{2}}\begin{pmatrix}0 & \partial^\mu \mathrm{H}\end{pmatrix} - \tfrac{i}{\sqrt{8}}\begin{pmatrix} g(\mathrm{W}^{1,\mu} + i\mathrm{W}^{2,\mu}) & -g\mathrm{W}^{3,\mu} + g'\mathrm{B}^\mu\end{pmatrix}(v+\mathrm{H})
\end{aligned}$$

and that the dynamic term in the Higgs Lagrangian becomes

$$D^\mu\phi^\dagger D_\mu\phi = \tfrac{1}{2}\partial^\mu \mathrm{H}\partial_\mu \mathrm{H} + \tfrac{1}{8}g^2\left(|\mathrm{W}^1|^2 + |\mathrm{W}^2|^2\right)(v+\mathrm{H})^2 + \tfrac{1}{8}\left(-g\mathrm{W}^3_\mu + g'\mathrm{B}_\mu\right)^2(v+\mathrm{H})^2 . \tag{3}$$

With the definition of the $\mathrm{W}^\pm$ bosons,

$$\mathrm{W}^\pm_\mu = \tfrac{1}{\sqrt{2}}\left(\mathrm{W}^1_\mu \mp i\mathrm{W}^2_\mu\right) ,$$

and with the defintion of the Z boson as a superposition of W^3 and B (Weinberg rotation), show then that Eq. (3) can be written in terms of the physical gauge bosons as

$$D^\mu\phi^\dagger D_\mu\phi = \tfrac{1}{2}\partial^\mu \mathrm{H}\partial_\mu \mathrm{H} + \tfrac{1}{2}\tfrac{g^2}{4}(v+\mathrm{H})^2\left(\mathrm{W}^+_\mu \mathrm{W}^{+\mu} + \mathrm{W}^-_\mu \mathrm{W}^{-\mu}\right) + \tfrac{1}{2}\tfrac{g^2+g'^2}{4}(v+\mathrm{H})^2\,\mathrm{Z}_\mu \mathrm{Z}^\mu .$$

What are the resulting gauge boson masses?

This approach results in addition into coupling terms between the gauge bosons and the Higgs boson H. Express the terms by the gauge boson masses and the vacuum expectation value v of the Higgs field. How does the coupling depend on the gauge boson masses?

<u>Exercise 2</u>: Masses for the Fermions

In the Standard Model, the Higgs doublet can also be used to generate mass terms for the fermions. They emerge dynamically from additionally introduced Yukawa coupling terms

$$\mathcal{L}_{\text{Yukawa}} = -y_f\left(\overline{\psi}_L\phi\psi_R + \overline{\psi}_R\phi^\dagger\psi_L\right) \tag{4}$$

between the Higgs field ϕ and the fermion fields ψ. Here, ψ_L denotes a weak isospin doublet of left-handed fermions, and ψ_R denotes the corresponding singlet of right-handed fermions, e. g. in case of the first generation leptons

$$\psi_L = \begin{pmatrix}\nu_e\\ e\end{pmatrix}_L , \quad \psi_R = e_R .$$

Show that $\mathcal{L}_{\text{Yukawa}}$ Eq. (4) is invariant under both $\mathrm{U}(1)_Y$ transformations $\mathcal{A}_Y$ and $\mathrm{SU}(2)_L$ transformations $\mathcal{B}_L$, where

$$\begin{aligned}
\mathcal{A}_Y &: \mathrm{F}_{L/R} \rightarrow \exp[i\tfrac{g'}{2}Y_\mathrm{F}\alpha(x)]\mathrm{F}_{L/R}\\
\mathcal{B}_L &: \ \mathrm{F}_L \ \rightarrow \exp[i\tfrac{g}{2}\tau^a\alpha_a(x)]\mathrm{F}_L\\
\mathcal{B}_L &: \ \mathrm{F}_R \ \rightarrow \mathrm{F}_R .
\end{aligned}$$

and F_L represents the isospin doublets spinor ψ_L and Higgs field ϕ_L, and F_R the isospin singlet spinor ψ_R. Note that $\mathcal{A}_Y$ depends on the weak hypercharge Y_F of the field F it acts on, and that the weak hypercharge of the Higgs field is $Y_\phi = +1$.

Now, work out the fermion mass terms resulting from $\mathcal{L}_{\text{Yukawa}}$ Eq. (4). Demonstrate this for the case of the first generation leptons and assume neutrinos to be massless. Start with expanding the Higgs field around its ground state ϕ_0 Eq. (1) by a small perturbation H, identified with the Higgs boson, as in Eq. (2). Show that this leads to

$$\mathcal{L}_{\text{Yukawa}} = -\tfrac{y_e}{\sqrt{2}}\left[\overline{e}_L(v + \mathrm{H})e_R + \overline{e}_R(v + \mathrm{H})e_L\right] ,$$

and derive the electron mass term from this. The approach results in addition into coupling terms between the electron and the Higgs boson. Show explicitly the proportionality of the coupling to the fermion mass.

As part of the calculation, you will need to show that

$$\overline{e}e = \overline{e}_L e_R + \overline{e}_R e_L \, .$$

Consider decays of the Higgs boson into pairs of $\tau^+\tau^-$ and $\mu^+\mu^-$ leptons. What is the relative frequency of the decays?