Karlsruher Institut für Technologie

Teilchenphysik 2 — W/Z/Higgs an Collidern

Sommersemester 2019

Matthias Schröder und Roger Wolf | Vorlesung 2

INSTITUT FÜR EXPERIMENTELLE TEILCHENPHYSIK (ETP)

Programme

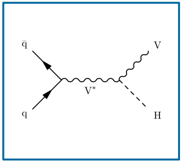
Date	Room	Type	Topic
Wed Apr 24.	Kl. HS B	LE 01	1. Organisation and introduction: particle physics at colliders + W/Z/H history
Tue Apr 30.	30.23 11/12	—	*no class*
Wed May 01.	Kl. HS B	—	*no class*
Tue May 07.	30.23 11/12	LE 02	2.1 Gauge theory & 2.2 The electroweak sector of the SM I
Wed May 08.	Kl. HS B	LE 03, EX 01	2.3 Discovery of the W and Z bosons & EX gauge theories
Tue May 14.	30.23 11/12	LE 04	2.4 The Higgs mechanism
Wed May 15.	Kl. HS B	EX 02	Exercise “SM Higgs mechanism”
Tue May 21.	30.23 11/12	—	*no class*
Wed May 22.	Kl. HS B	LE 05	2.5 The electroweak sector of the SM II (Higgs mechanism + Yukawa couplings)
Tue May 28.	30.23 11/12	SP 01	Specialisation of 2.4 and 2.5
Wed May 29.	Kl. HS B	LE 06	3.1 From theory to observables & 3.2 Reconstruction + analysis of exp. data
Tue Jun 04.	30.23 11/12	EX 03	Exercise “Trigger efficiency measurement”
Wed Jun 05.	Kl. HS B	LE 07	3.3 Measurements in particle physics (part 1)
Tue Jun 11.	30.23 11/12	EX 04	Exercise on statistical methods
Wed Jun 12.	Kl. HS B	LE 08	3.3 Measurements in particle physics (part 2)
Tue Jun 18.	30.23 11/12	SP 02	Specialisation “Limit setting”
Wed Jun 19.	Kl. HS B	SP 03	Specialisation “Unfolding”
Tue Jun 25.	30.23 11/12	LE 09	4.1 Determination of SM parameters
Wed Jun 26.	Kl. HS B	LE 10	4.2 Measurement and role of W/Z bosons at the LHC
Tue Jul 02.	30.23 11/12	EX 05	Paper seminar “Z pole measurements”
Wed Jul 03.	Kl. HS B	LE 11	4.3 Processes with several W/Z bosons
Tue Jul 09.	30.23 11/12	EX 06	Paper seminar Higgs
Wed Jul 10.	Kl. HS B	LE 12	5.1 Discovery and first measurements of the Higgs boson
Tue Jul 16.	30.23 11/12	EX 07	Exercise “Machine learning in physics analysis”
Wed Jul 17.	Kl. HS B	LE 13	5.2 Measurement of couplings and kinematic properties
Tue Jul 23.	30.23 11/12	EX 08	Presentations: results of ML challenge
Wed Jul 24.	Kl. HS B	LE 14	5.3 Search for Higgs physics beyond the SM & 5.4 Future Higgs physics

Programme

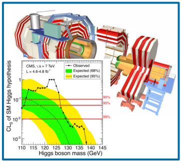
Basics of electroweak theory

$$
\begin{aligned}
\mathcal{L} = &-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} \\
&+ i\bar{\Psi}\not{D}\psi \\
&+ D_\mu\Phi^\dagger D^\mu\Phi - V(\Phi) \\
&+ \bar{\Psi}_L\hat{Y}\Phi\Psi_R + h.c.
\end{aligned}
$$

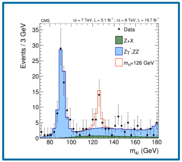
From theory to observables



Experimental techniques



Results and open questions



2. The Electroweak Sector of the Standard Model

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2.1 Gauge theory
- Global and local phase transformations
- Example: QED
- Abelian and non-Abelian gauge theories

2.2 The electroweak sector of the Standard Model – I
- Properties of the weak interaction, weak isospin
- Formulation of the Standard Model (without masses)

2.3 Discovery of W and Z bosons
- History towards discovery
- Experimental methods

2.4 The Higgs mechanism
- Problem of massive gauge bosons and massive fermions
- Idea of the Higgs mechanism: examples of spontaneous symmetry breaking

2.5 The electroweak sector of the Standard Model – II
- The Standard Model Higgs mechanism
- Yukawa couplings and fermion masses
- The Higgs boson

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2.1 Gauge theory

Symmetries

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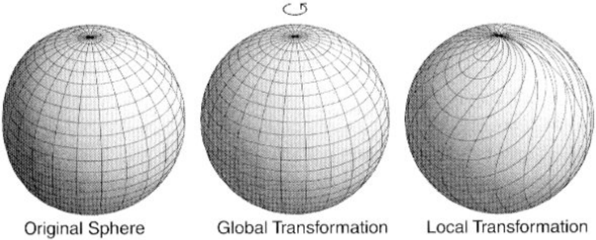
Fundamental physical theories are based on symmetry principles

- Symmetries of a system
  - **Discrete**: e. g. under reflection, particle-antiparticle exchange
  - **Continuous**: e. g. under space-time translations or rotations
- Noether's theorem (E. Noether 1918): *To each continuous symmetry of a system, there is a conserved quantity.*
  - For example, quantum-mechanical phase of a charged particle cannot be observed, i. e. system is symmetric under rotations of the phase (phase transformation) $\longrightarrow$ charge conservation

Symmetries

- Standard Model: interactions as consequence of symmetries



universe-review.ca

Postulation: equations of motion stay invariant under local phase transformations

→ **Consequence**: existence of fundamental interactions

Global Phase Transformations

- The Lagrangians for fermions and bosons are **invariant under global phase transformations**
  - The **phase is the same at each space-time point** x: $\alpha = \text{const}$
- For example, Lagrangian of free fermions

$$\boxed{\begin{aligned}\psi(x) &\to \psi'(x) = e^{i\alpha}\psi(x)\\ \overline{\psi}(x) &\to \overline{\psi'}(x) = \overline{\psi}(x)e^{-i\alpha}\end{aligned}}$$

Proof:
$$\begin{aligned}\mathcal{L}' &= \overline{\psi'}(i\gamma^\mu\partial_\mu - m)\psi'\\ &= \overline{\psi}e^{-i\alpha}\,(i\gamma^\mu\partial_\mu - m)\,e^{i\alpha}\psi\\ &= \overline{\psi}(i\gamma^\mu\partial_\mu - m)\psi = \mathcal{L} \qquad \checkmark\end{aligned}$$

Local Phase Transformations

- But: Lagrangian is **not invariant under local phase transformations**
  - Different phases at each space-time point $\alpha = \alpha(x)$?

$$\psi(x) \rightarrow \psi'(x) = e^{i\alpha(x)}\psi(x)$$
$$\overline{\psi}(x) \rightarrow \overline{\psi'}(x) = \overline{\psi}(x)e^{-i\alpha(x)}$$

Proof:
$$\begin{aligned}\mathcal{L}' &= \overline{\psi'}(i\gamma^\mu\partial_\mu - m)\psi' \\ &= \overline{\psi}e^{-i\alpha(x)}\,(i\gamma^\mu\partial_\mu - m)\,e^{i\alpha(x)}\psi \\ &= \overline{\psi}(i\gamma^\mu(\partial_\mu + i\partial_\mu\alpha(x)) - m)\psi \neq \mathcal{L}\end{aligned}$$

breaks invariance due to $\partial_\mu = \lim_{\Delta x \rightarrow 0} \frac{\psi(x+\Delta x)-\psi(x)}{\Delta x}$ in $\mathcal{L}$
(connects neighbouring space-time points)

Covariant Derivative

- Invariance can be achieved by introducing the **covariant derivative** $\partial_\mu \to D_\mu = \partial_\mu + iqA_\mu$ with arbitrary **gauge field** A_μ and transformation behaviour

$$\begin{aligned}
\psi(x) &\to \psi'(x) &&= e^{i\alpha(x)}\psi(x) \\
\overline{\psi}(x) &\to \overline{\psi'}(x) &&= \overline{\psi}(x)e^{-i\alpha(x)} \\
A(x)_\mu &\to A'(x)_\mu &&= A(x)_\mu - \frac{1}{q}\partial_\mu\alpha(x)
\end{aligned}$$

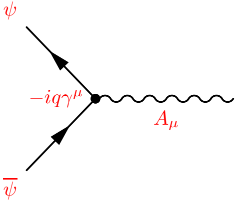
$$\begin{aligned}
\text{Proof: } \mathcal{L}' &= \overline{\psi'}(i\gamma^\mu D'_\mu - m)\psi' \\
&= \overline{\psi'}(i\gamma^\mu(\partial_\mu + iqA'_\mu) - m)\psi' \\
&= \overline{\psi}e^{-i\alpha(x)}\,(i\gamma^\mu(\partial_\mu + iqA_\mu - i\partial_\mu\alpha(x)) - m)\,e^{i\alpha(x)}\psi \\
&= \overline{\psi}\,(i\gamma^\mu(\partial_\mu + i\partial_\mu\alpha(x) + iqA_\mu - i\partial_\mu\alpha(x)) - m)\,\psi \\
&= \overline{\psi}(i\gamma^\mu D_\mu - m)\psi = \mathcal{L} \qquad \checkmark
\end{aligned}$$

The Gauge Field

- **Covariant derivative introduces gauge field A_μ**
- Allows arbitrary phase $\alpha(x)$ of $\psi(x)$ at each space-time point x
  - A_μ 'transports' this information from point to point (physical: no instantaneous information exchange)
- A_μ couples to property q of spinor field $\psi(x)$
  - q can be identified with electric charge

$$\mathcal{L} = \overline{\psi}\,(i\gamma^\mu D_\mu - m)\,\psi$$

$$= \underbrace{\overline{\psi}\,(i\gamma^\mu \partial_\mu - m)\,\psi}_{\text{free fermion}} \underbrace{-q(\overline{\psi}\gamma^\mu\psi)A_\mu}_{\text{interaction}} - \frac{1}{4}F^{\mu\nu}F_{\mu\nu}$$



- **A_μ can be identified with photon field**
- Dynamics of A_μ given by $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu = \frac{i}{q}[D_\mu, D_\nu]$

$$\mathcal{L}_{\text{kin}} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu}$$ (→Proca equation for massless vector boson)

Lagrange Density of QED

- Postulation of local U(1) gauge symmetry leads to **Lagrangian of QED**

$$\begin{aligned}\mathcal{L}_{\text{QED}} &= \overline{\psi}\,(i\gamma^{\mu} D_{\mu} - m)\,\psi - \tfrac{1}{4} F^{\mu\nu} F_{\mu\nu} \\ &= \underbrace{\overline{\psi}\,(i\gamma^{\mu}\partial_{\mu} - m)\,\psi}_{\text{free fermion}} - \underbrace{q(\overline{\psi}\gamma^{\mu}\psi)A_{\mu}}_{\text{interaction}} - \underbrace{\tfrac{1}{4} F^{\mu\nu} F_{\mu\nu}}_{\text{gauge field}}\end{aligned}$$

Lagrange Density of QED

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$$\mathcal{L}_{\mathrm{QED}} = \overline{\psi}\,(i\gamma^{\mu}D_{\mu} - m)\,\psi - \frac{1}{4}F^{\mu\nu}F_{\mu\nu}$$
$$= \underbrace{\overline{\psi}\,(i\gamma^{\mu}\partial_{\mu} - m)\,\psi}_{\text{free fermion}} - \underbrace{q(\overline{\psi}\gamma^{\mu}\psi)A_{\mu}}_{\text{interaction}} - \underbrace{\frac{1}{4}F^{\mu\nu}F_{\mu\nu}}_{\text{gauge field}}$$

Electromagnetic interaction consequence of local gauge invariance

- Continuous transformations $U = e^{i\alpha(x)}$ form **Abelian group U**(1) under multiplication and thus commute, i. e. $[U_i, U_j] = 0$

Yang–Mills Theories

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- Extension of the gauge principle to **non-Abelian groups**
  - Standard Model: in particular SU(2) and SU(3)
- SU(n) transformations $\psi \to \exp[i\frac{1}{2}g\alpha^a(x)\tau^a]\psi$
  - **$n^2 - 1$ generators τ^a**
  - Non-Abelian algebra $[\tau^a, \tau^b] = if^{abc}\tau^c$ with **structure constants f^{abc}**
- Analogue to QED: invariance under local SU(n) transformations by introducing covariant derivative and field-strength tensor

$$D_\mu = \partial_\mu + ig\tau^a A^a_\mu$$

with

$$A^a_\mu \to A^a_\mu + \frac{1}{g}\partial_\mu\alpha^a(x) + f^{abc}\alpha^b(x)A^c_\mu$$

$$[D_\mu, D_\nu]^a = igF^a_{\mu\nu}, \quad F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + gf^{abc}A^b_\mu A^c_\nu$$

- **Non-zero structure constants lead to gauge boson self-interaction**
- NB: above relations also hold for U(1)

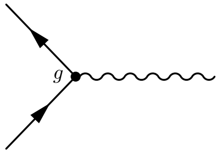
Example: Invariance Under Local SU(2)

- **Generators: 3 Pauli matrices τ^a** with $f^{abc} = \epsilon^{abc}$
  - Act on isospin doublets, e. g. $\psi = \begin{pmatrix} \nu \\ e \end{pmatrix}$
- 3 vector fields F^a_μ: **3 vector bosons**
- Additional terms in field-strength tensor (from non-zero commutator):

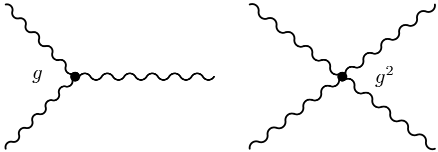
$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + \boxed{g\epsilon^{abc} A^b_\mu A^c_\nu} \rightarrow \text{vector boson } \textbf{self-interaction}$$

- Lagrangian

$$\mathcal{L}_{\mathrm{SU(2)}} = \overline{\psi}(i\not\partial - m)\psi - \underbrace{g\left(\overline{\psi}\gamma^\mu\tau^a\psi\right)A^a_\mu}_{\text{fermion-boson interaction}} - \underbrace{\frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu}}_{\text{boson propagation and self-interaction}}$$



fermion-boson int.



boson self-interaction

Summary

- **Symmetry** as basic principle of physical theories
- Concept of **local gauge theories**: invariance of Lagrangian under local gauge transformations
  - Requires introduction of vector fields (gauge bosons) with specific coupling structure
- QED: symmetry under $\mathrm{U}(1)$ gauge group $\rightarrow$ introduction of photon
- Yang–Mills theories: non-Abelian gauge groups $\rightarrow$ more complex structure, e. g. self-interactions of gauge bosons

2.2 The electroweak sector of the SM — I

The Standard Model

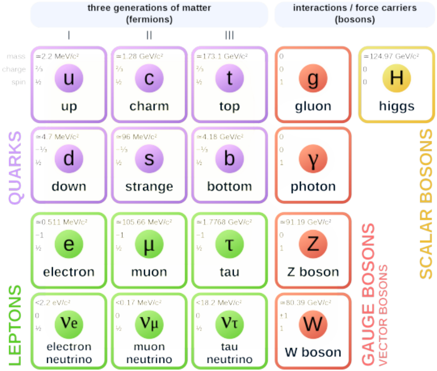
Constituents and Interactions

3 (r,g,b) x 6 (L,R)

3 (r,g,b) x 6 (L,R)

6 (L,R)

3 (L)



8 SU(3)

3+1 $SU(2)_L \times U(1)_Y$

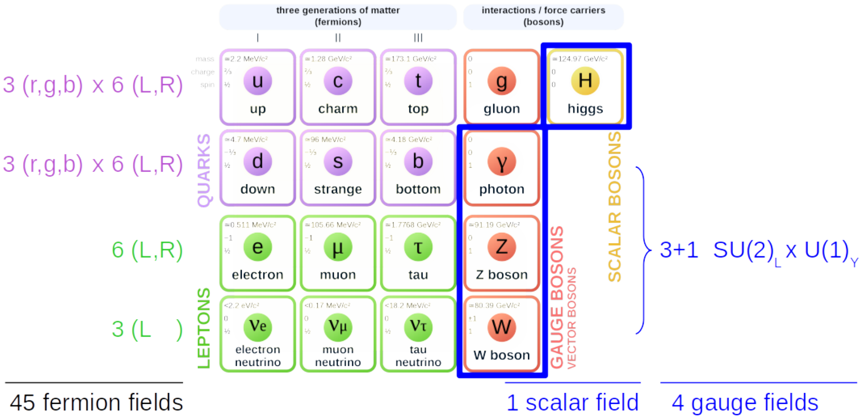
45 fermion fields

1 scalar field

12 gauge fields

The Standard Model

Constituents and Interactions: Electroweak Sector



2.2.1. Properties of the weak interaction

Weak Interaction: Change of Flavour

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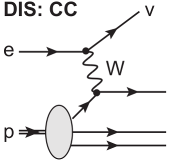
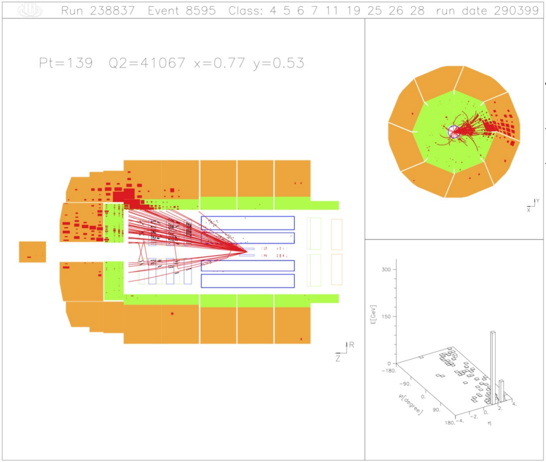
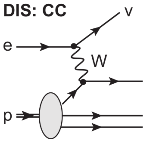
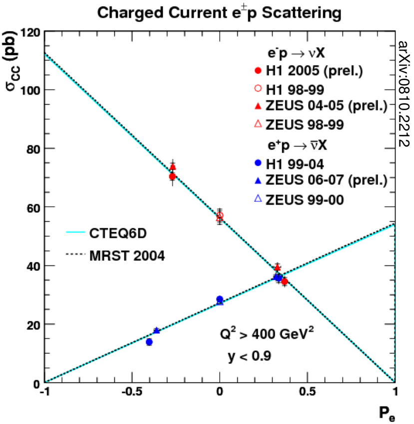




Rev.Mod.Phys. 86 (2014) no.3, 1037

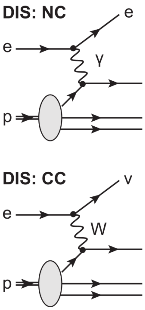
Weak Interaction: Parity Violation

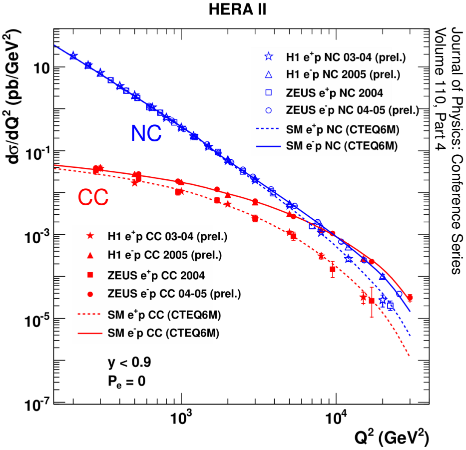




- W bosons couple only to **left-handed particles** (and right-handed antiparticles): weak interaction is **maximally parity violating**
- Also CP violating, e. g. K^0 system

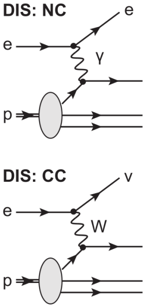
Weak Interaction: Heavy Mediators

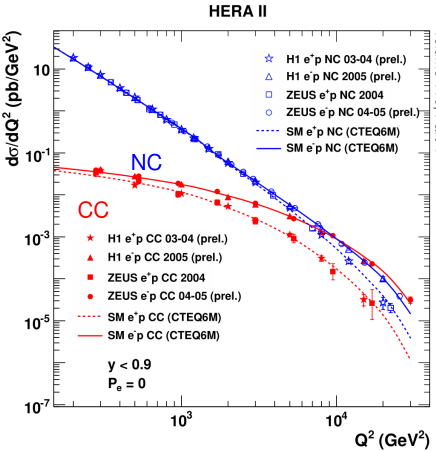




- **Heavy mediators**: short range/weakness of interaction
  - Propagator suppressed by large mass in denominator
  - Resolves divergencies in 4-point contact-interaction model (Fermi theory)

Electroweak Unification

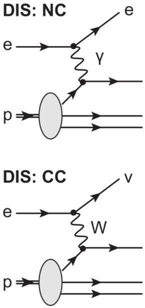


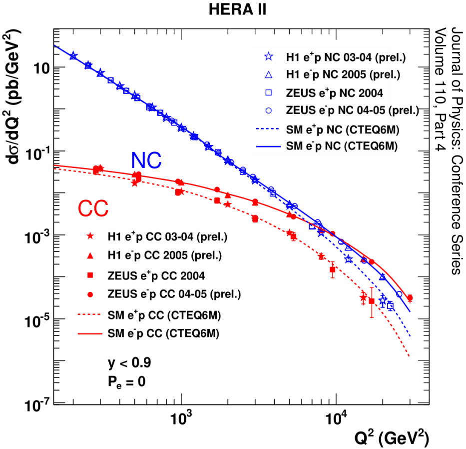


Journal of Physics: Conference Series Volume 110, Part 4

- **Electroweak unification**: same coupling at high energies
  - Also: resolves divergencies in $e^+e^- \rightarrow WW$ by contributions from triple-gauge couplings γWW, ZWW (prediction of Z boson!)

Electroweak Gauge Group





- Simplest combination of gauge-symmetry groups for unified electroweak interaction: $SU(2)_L \times U(1)_Y$
  - $SU(2)_L$: **weak isospin** acts on left-handed particles only
  - $U(1)_Y$: **hypercharge** acts on all particles ($\neq$ U(1) gauge group of QED!)

Electroweak Gauge Group

- **Particle content**: distinguish left-handed and right-handed particles
  - **Left-handed particles**: weak isospin **doublets** ($I = 1/2$, $I_3 = \pm 1/2$)

$$\boxed{\psi_L = \begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_\mathrm{L}, \ldots, \begin{pmatrix} u \\ d \end{pmatrix}_\mathrm{L}, \ldots}$$

  - **Right-handed particles**: weak isospin **singlets** ($I = I_3 = 0$)

$$\boxed{\psi_R = e^-_\mathrm{R}, \ldots, u_\mathrm{R}, d_\mathrm{R}, \ldots}$$

- Left- and right-handed (chirality!) components of fermions can be projected with

$$\boxed{\psi_{L/R} \equiv \tfrac{1}{2}\left(1 \mp \gamma^5\right)\psi \quad \Rightarrow \quad \psi = \psi_L + \psi_R} \quad \gamma^5 \equiv i\gamma^0\gamma^1\gamma^2\gamma^3$$

  - Important equality: scalar bilinear form $\overline{\psi}\psi = \overline{\psi}_L\psi_R + \overline{\psi}_R\psi_L$

Electroweak Gauge Group

- Gauge transformation of $\mathrm{SU}(2)_\mathrm{L}$: $U(x) = \exp[i\frac{g}{2}\alpha^a(x)\tau^a]$
  - Coupling constant g
  - Acts on **isospin doublets**
  - 3 generators: Pauli matrices $\tau^a = \sigma_a \to$ **3 gauge bosons $\mathbf{W}^i_\mu$**
- Gauge transformation of $\mathrm{U}(1)_\mathrm{Y}$: $U(x) = \exp[i\frac{g'}{2}Y\alpha(x)]$
  - Coupling constant g'
  - Weak hypercharge Y (additive quantum number)
  - Acts on **isospin doublets and singlets**
  - **Single gauge boson $\mathbf{B}_\mu$**
- Require that $\mathrm{SU}(2)_\mathrm{L}$ doublets are $\mathrm{U}(1)_\mathrm{Y}$ singlets

  $\to$ **Gell-Mann–Nishijima formula** $I_3 = Q - \frac{1}{2}Y$

Particles and Quantum Numbers

Fermion	Chirality	Isospin (I, I_3)	Hypercharge Y	Charge Q (e)
Neutrinos: ν_e, ν_μ, ν_τ	*L*	(1/2, +1/2)	–1	0
	R	Not part of the standard model		
Charged leptons: *e*, μ, τ	*L*	(1/2, –1/2)	–1	–1
	R	(0, 0)	–2	–1
up-type quarks: *u*, *c*, *t*	*L*	(1/2, +1/2)	+1/3	+2/3
	R	(0, 0)	+4/3	+2/3
down-type quarks: *d*, *s*, *b*	*L*	(1/2, –1/2)	+1/3	–1/3
	R	(0, 0)	–2/3	–1/3

Electroweak Lagrangian

(without gauge-boson mass terms)

$$\mathcal{L}_{\text{EWK}} = \boxed{i\overline{\psi}_L\gamma^\mu D_\mu\psi_L + i\overline{\psi}_R\gamma^\mu D_\mu\psi_R} \; \boxed{-\frac{1}{4}B_{\mu\nu}B^{\mu\nu} - \frac{1}{4}W^a_{\mu\nu}W^{a,\mu\nu}}$$

1. Covariant derivatives

$$\boxed{\begin{aligned} D_\mu\psi_L &= \left(\partial_\mu - i\frac{g}{2}\tau^a W^a_\mu - i\frac{g'}{2}Y_L\mathbb{1}_2 B_\mu\right)\psi_L \\ D_\mu\psi_R &= \left(\partial_\mu - i\frac{g'}{2}Y_R B_\mu\right)\psi_R \end{aligned}}$$

Covariant Derivative of $SU(2)_L \times U(1)_Y$

Covariant derivative corresponding to SU(2) acts on isospin doublet only

$$\mathcal{L} = i\bar{\psi}_L\gamma^\mu \left[\partial_\mu + i\frac{g}{2}\tau^a W^a_\mu + i\frac{g'}{2}Y_L B_\mu\right]\psi_L + i\bar{\psi}_R\gamma^\mu\left[\partial_\mu + i\frac{g'}{2}Y_R B_\mu\right]\psi_R$$

$$\tau^a W^a_\mu = \sqrt{2}\left(\tau^+ W^+_\mu + \tau^- W^-_\mu\right) + \tau^3 W^3_\mu$$

$$W^\pm_\mu = \frac{1}{\sqrt{2}}(W^1_\mu \mp iW^2_\mu), \qquad \tau^+ \equiv \frac{1}{2}(\tau^1 + i\tau^2) = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \quad \text{(ascending operator)}$$

$$\tau^- \equiv \frac{1}{2}(\tau^1 - i\tau^2) = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} \quad \text{(descending operator)}$$

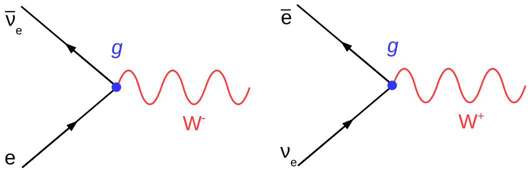
$$\tau^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Charged Currents [Example: 1. Generation Leptons]

$$\mathcal{L}_{CC} = -\frac{g}{\sqrt{2}}\Big[\underbrace{(\overline{\nu}_e \gamma^\mu e_L)}_{J^{\mu,+}_{CC}} W^+_\mu + \underbrace{(\overline{e}_L \gamma^\mu \nu_e)}_{J^{\mu,-}_{CC}} W^-_\mu \Big]$$

$$= -\frac{g}{\sqrt{2}}\Big[(\overline{\nu}_e \gamma^\mu \underbrace{\tfrac{1}{2}(1-\gamma_5)}_{\text{V-A}} e) W^+_\mu + (\overline{e} \gamma^\mu \underbrace{\tfrac{1}{2}(1-\gamma_5)}_{\text{V-A}} \nu_e) W^-_\mu \Big]$$

Operator W^+_μ:
annihilates W^+ or creates W^-



- Transitions within isospin doublets
  - Simultaneous **change of charge** (by $\pm e$) and **flavour** ($e \leftrightarrow \nu_e$)
- **Parity violation**: W boson couples only to left-handed particles
  - Only left-handed particles carry “weak isospin charge” under I_3
- **V-A interaction** (“vector minus axial vector current”)

Covariant Derivative of $SU(2)_L \times U(1)_Y$

Covariant derivative corresponding to SU(2) acts on isospin doublet only

$$\mathcal{L} = i\bar{\psi}_L\gamma^\mu \left[\partial_\mu + i\frac{g}{2}\tau^a W^a_\mu + i\frac{g'}{2}Y_L B_\mu\right]\psi_L + i\bar{\psi}_R\gamma^\mu\left[\partial_\mu + i\frac{g'}{2}Y_R B_\mu\right]\psi_R$$

$$\tau^a W^a_\mu = \sqrt{2}\left(\tau^+ W^+_\mu + \tau^- W^-_\mu\right) + \tau^3 W^3_\mu$$

$$W^\pm_\mu = \frac{1}{\sqrt{2}}(W^1_\mu \mp iW^2_\mu), \quad \tau^+ \equiv \frac{1}{2}(\tau^1 + i\tau^2) = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \quad \text{(ascending operator)}$$

$$\tau^- \equiv \frac{1}{2}(\tau^1 - i\tau^2) = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} \quad \text{(descending operator)}$$

$$\tau^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Covariant Derivative of $\mathbf{SU}(2)_L \times \mathbf{U}(1)_Y$

Covariant derivative corresponding to SU(2) acts on isospin doublet only

$$\mathcal{L} = i\overline{\psi}_L\gamma^\mu \left[\partial_\mu + i\frac{g}{2}\tau^a W^a_\mu + i\frac{g'}{2} Y_L B_\mu\right]\psi_L + i\overline{\psi}_R\gamma^\mu \left[\partial_\mu + i\frac{g'}{2} Y_R B_\mu\right]\psi_R$$

	$Y_{L/R}$	I_3	Q
ν	-1	$+1/2$	0
e_L	-1	$-1/2$	-1
e_R	-2	0	-1

$Y_{L/R}$: hypercharge
I_3: isospin
Q: electric charge

$Q = I_3 + \frac{Y}{2}$ (Gell-Mann–Nishijima)

Covariant derivative corresponding to U(1) acts on isospin doublet and on isospin singlet

Neutral Currents [Example: 1. Generation Leptons]

$$\mathcal{L}_{\text{NC}} = -\Big[\underbrace{\tfrac{g}{2}W^3_\mu - \tfrac{g'}{2}B_\mu}_{-c_1 Z_\mu}\Big](\overline{\nu}\gamma^\mu\nu) + \Big[\underbrace{\tfrac{g}{2}W^3_\mu + \tfrac{g'}{2}B_\mu}_{[c_2 Z_\mu + c_4 A_\mu]}\Big](\overline{e}_L\gamma^\mu e_L) + \underbrace{g' B_\mu}_{[c_3 Z_\mu + c_4 A_\mu]}(\overline{e}_R\gamma^\mu e_R)$$

Weinberg rotation:

$$\begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta_W & -\sin\theta_W \\ \sin\theta_W & \cos\theta_W \end{pmatrix} \begin{pmatrix} W^3_\mu \\ B_\mu \end{pmatrix}$$

$$\sin\theta_W \equiv \frac{g'}{\sqrt{g^2+g'^2}}\,, \qquad \cos\theta_W \equiv \frac{g}{\sqrt{g^2+g'^2}}$$

$$\begin{aligned}
\mathcal{L}_{\text{NC}} = & -\frac{\sqrt{g^2+g'^2}}{2} \quad Z_\mu \quad (\overline{\nu}\gamma^\mu\nu) \\
& + \frac{\sqrt{g^2+g'^2}}{2}\Big[(\cos^2\theta_W - \sin^2\theta_W)Z_\mu + 2\sin\theta_W\cos\theta_W A_\mu\Big](\overline{e}_L\gamma^\mu e_L) \\
& + \frac{\sqrt{g^2+g'^2}}{2}\Big[\qquad (-2\sin^2\theta_W)Z_\mu + 2\sin\theta_W\cos\theta_W A_\mu\Big](\overline{e}_R\gamma^\mu e_R)
\end{aligned}$$

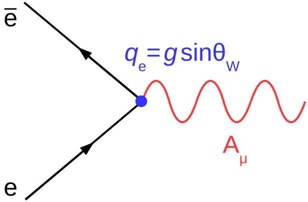
Electromagnetic Interaction [Example: Electrons]

$$
\begin{aligned}
\mathcal{L}_{\mathrm{em}} &= \frac{\sqrt{g^2+g'^2}}{2} 2 \sin\theta_W \cos\theta_W \cdot A_\mu \cdot \left[(\bar{e}_L \gamma^\mu e_L) + (\bar{e}_R \gamma^\mu e_R) \right] \\
&= \frac{gg'}{\sqrt{g^2+g'^2}} \cdot A_\mu \cdot (\bar{e}\gamma^\mu e) \\
&= q_e \cdot A_\mu \cdot j^\mu_{\mathrm{em}}
\end{aligned}
$$

→ QED **vector current** j^μ_{em} recovered

- Electron charge related to electroweak **coupling constants** g and g'

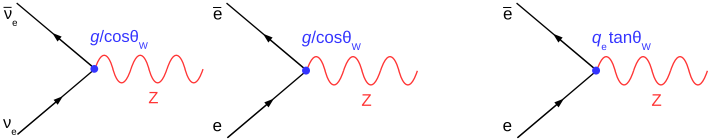
$$q_e = \frac{gg'}{\sqrt{g^2+g'^2}} = g \sin\theta_W = g' \cos\theta_W$$



- Photon field A_μ couples "as desired"
  - Photon couples to **all charged particles**
  - **Symmetric** coupling for left-handed and right-handed components

Z-Boson Exchange [Example: 1. Generation Leptons]

$$
\begin{aligned}
\mathcal{L}_Z &= -\frac{\sqrt{g^2+g'^2}}{2} J^\mu_{NC} Z_\mu \\
&= -\frac{g}{2\cos\theta_W}\left[\overline{\nu}_e\gamma^\mu\nu_e - \left(\cos^2\theta_W - \sin^2\theta_W\right)\overline{e}_L\gamma^\mu e_L + 2\sin^2\theta_W \overline{e}_R\gamma^\mu e_R\right] Z_\mu \\
&= -\frac{g}{2\cos\theta_W}\left[\overline{\nu}_e\gamma^\mu \tfrac{1}{2}(1-\gamma_5)\nu_e - \overline{e}\gamma^\mu\tfrac{1}{2}(1-\gamma_5)e + 2\sin^2\theta_W\left(\overline{e}\gamma^\mu e\right)\right] Z_\mu \\
&= -\frac{g}{\cos\theta_W}\left[I_3^\nu\,\overline{\nu}_e\tfrac{1}{2}\gamma^\mu(1-\gamma_5)\nu_e + I_3^e\,\overline{e}\gamma^\mu\tfrac{1}{2}(1-\gamma_5)e\right] Z_\mu - q_e\tan\theta_W \overline{e}\gamma^\mu e\, Z_\mu
\end{aligned}
$$

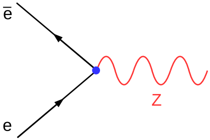


Left-handed (V-A) couplings

Vector couplings

Example: Z-Boson Coupling to Electrons

$$\mathcal{L}_{Ze} = -\frac{g}{2\cos\theta_W}\left[-\bar{e}\gamma^\mu \tfrac{1}{2}(1-\gamma_5)e + 2\sin^2\theta_W\,(\bar{e}\gamma^\mu e)\right]Z_\mu$$
$$= -\frac{g}{4\cos\theta_W}\left[\bar{e}\gamma^\mu(4\sin^2\theta_W - 1 + \gamma_5)\right]e\,Z_\mu$$
$$= -\frac{g}{4\cos\theta_W}\,\bar{e}\gamma^\mu\left(C_V + C_A\gamma_5\right)e\,Z_\mu$$

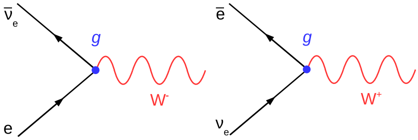


$$C_V = 4\sin^2\theta_W - 1 \text{ (vector coupling)}$$
$$C_A = 1 \text{ (axial-vector coupling)}$$

With $\sin^2\theta_W = 0.22$: C_V is small
$\rightarrow$ **electron couples mostly via axial-vector couplings to the Z boson**

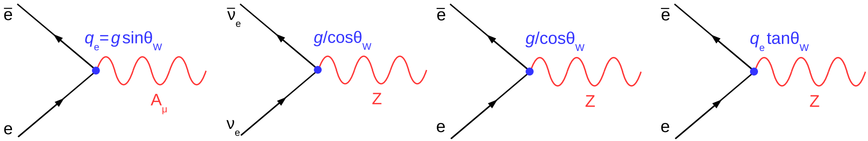
Covariant Derivative of $SU(2)_L \times U(1)_Y$

Charged-current interactions (W boson exchange)



Left-handed (V-A) couplings

Neutral-current interactions (photon and Z boson exchange)



Vector couplings | Left-handed (V-A) couplings | Vector couplings

Electroweak Lagrangian

(without gauge-boson mass terms)

$$\mathcal{L}_{\text{EWK}} = \boxed{i\overline{\psi}_L\gamma^\mu D_\mu\psi_L + i\overline{\psi}_R\gamma^\mu D_\mu\psi_R} \; \boxed{-\frac{1}{4}B_{\mu\nu}B^{\mu\nu} - \frac{1}{4}W^a_{\mu\nu}W^{a,\mu\nu}}$$

1. Covariant derivatives

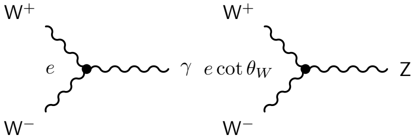
$$\begin{aligned} D_\mu\psi_L &= \left(\partial_\mu - i\frac{g}{2}\tau^a W^a_\mu - i\frac{g'}{2}Y_L \mathbb{1}_2 B_\mu\right)\psi_L \\ D_\mu\psi_R &= \left(\partial_\mu - i\frac{g'}{2}Y_R B_\mu\right)\psi_R \end{aligned}$$

2. Field strength tensors

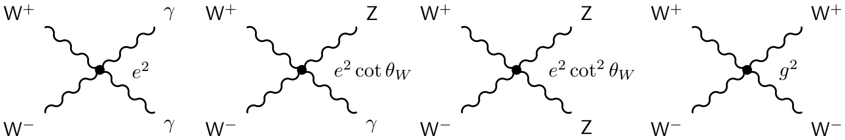
$$\begin{aligned} B_{\mu\nu} &= \partial_\mu B_\nu - \partial_\nu B_\mu \\ W^i_{\mu\nu} &= \partial_\mu W^i_\nu - \partial_\nu W^i_\mu - \underbrace{g\epsilon^{ijk}W^j_\mu W^k_\nu}_{\text{gauge boson self-interaction}} \end{aligned}$$

Gauge-Boson Self-Interaction

Triple gauge couplings



Quartic gauge couplings



Summary

- Standard Model: **all fundamental interactions as consequence of local gauge invariance**
  - Invariance requires introduction of gauge fields
  - Geometrical interpretation: gauge bosons transport phase information between space-time points
- Gauge groups of the Standard Model: $SU(3)_C \times SU(2)_L \times U(1)_Y$
- Electroweak gauge group $SU(2)_L \times U(1)_Y$ has peculiar structure
  - Physical gauge boson ($W^\pm$, Z, γ) superposition of underlying gauge fields W^a (from $SU(2)_L$) and B (from $U(1)_Y$)
  - Interaction different for left- and right-handed states, leading to e. g. parity violation

The Standard Model — Part 1