Karlsruher Institut für Technologie

Teilchenphysik 2 — W/Z/Higgs an Collidern

Sommersemester 2019

Matthias Schröder und Roger Wolf | Vorlesung 4

INSTITUT FÜR EXPERIMENTELLE TEILCHENPHYSIK (ETP)

2. The Electroweak Sector of the Standard Model

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2.1 Gauge theory
- Global and local phase transformations
- Example: QED
- Abelian and non-Abelian gauge theories

2.2 The electroweak sector of the Standard Model – I
- Properties of the weak interaction, weak isospin
- Formulation of the Standard Model (without masses)

2.3 Discovery of W and Z bosons
- History towards discovery
- Experimental methods

2.4 The Higgs mechanism
- Problem of massive gauge bosons and massive fermions
- Idea of the Higgs mechanism: examples of spontaneous symmetry breaking

2.5 The electroweak sector of the Standard Model – II
- The Standard Model Higgs mechanism
- Yukawa couplings and fermion masses
- The Higgs boson

Status Standard Model

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- **All fundamental interactions as consequence of local gauge invariance**
  - Invariance requires introduction of gauge fields
  - Geometrical interpretation: gauge bosons transport phase information between space-time points
- Gauge groups of the Standard Model: $SU(3)_C \times SU(2)_L \times U(1)_Y$
- Electroweak gauge group $SU(2)_L \times U(1)_Y$ has peculiar structure
  - Physical gauge boson ($W^{\pm}$, Z, γ) superposition of underlying gauge fields W^a (from $SU(2)_L$) and B (from $U(1)_Y$)
  - Chiral theory: interaction different for left- and right-handed states, e. g. leading to parity violation

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2.4 The Higgs mechanism

2.4.1. Problem of massive gauge bosons and massive fermions

Problem of Massive Gauge Bosons

- **Invariance of $\mathcal{L}$ under local gauge transformation** achieved by introduction of vector field(s) with specific transformation behaviour $\rightarrow$ **cause of interactions**
- Example QED: from invariance under local U(1) transformations
  - Vector field (photon) transforms as $A_\mu \rightarrow A'_\mu = A_\mu - \frac{1}{q}\partial_\mu \alpha$
  - Transformation of mass terms

$$
\begin{aligned}
\frac{1}{2} m_A^2 A_\mu A^\mu \rightarrow & \frac{1}{2} m_A^2 A'_\mu A'^\mu \\
& = \frac{1}{2} m_A^2 A_\mu A^\mu - \frac{1}{q} m_A^2 \left(A_\mu \partial^\mu \alpha + A^\mu \partial_\mu \alpha \right) + \frac{1}{2q^2} m_A^2 \partial_\mu \alpha \partial^\mu \alpha \\
& \neq \frac{1}{2} m_A^2 A_\mu A^\mu
\end{aligned}
$$

✗ Gauge-boson **mass terms break local gauge invariance**
  - Property of all gauge-field theories

✗ **Fundamental problem**: W and Z bosons have masses!

Problem of Massive Fermions?

- **Invariance of $\mathcal{L}$ under local gauge transformation** achieved by introduction of vector field(s) with specific transformation behaviour $\rightarrow$ **cause of interactions**
- Example QED: from invariance under local U(1) transformations
  - Spinor transforms as $\psi \rightarrow \psi' = \mathrm{e}^{i\alpha}\psi$ and $\overline{\psi} \rightarrow \overline{\psi}' = \overline{\psi}\mathrm{e}^{-i\alpha}$
  - Transformation of mass terms ("Dirac mass" term)

$$m_f\overline{\psi}\psi \longrightarrow m_f\overline{\psi}'\psi' = m_f\overline{\psi}\psi$$

✓ No problem with fermion masses for U(1) transformations

✓ Similarly, no problem in SU(3) (non-Abelian gauge group)

Problem of Massive Fermions!

- $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ transformations act differently on chiral components
- Decomposition of mass term

$$m_f \overline{\psi}\psi = m_f \left(\overline{\psi}_R \psi_L + \overline{\psi}_L \psi_R\right)$$

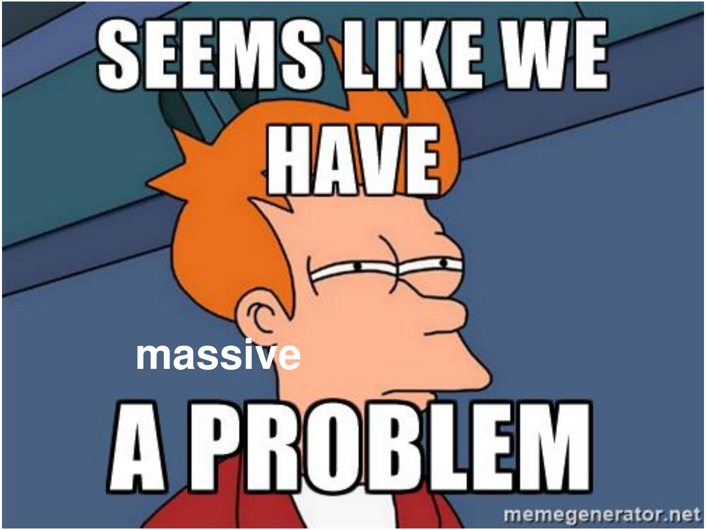
- **Left- and right-handed components transform differently!**

$$\psi_L \to \psi_L' = e^{i\alpha^a \tau^a + i\alpha Y} \psi_L \quad \text{(component of isospin doublet, } I = \tfrac{1}{2}\text{)}$$

$$\psi_R \to \psi_R' = e^{i\alpha Y} \psi_R \qquad \text{(isospin singlet, } I = 0\text{)}$$

✗ Left- and right-handed fermions transform differently under $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$

✗ **Fermion mass terms in chiral theory are not gauge invariant**


SEEMS LIKE WE
HAVE
massive
A PROBLEM
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Status Standard Model

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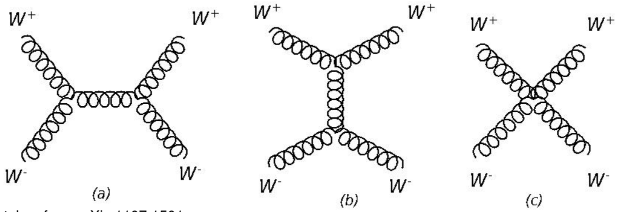
  - Geometrical interpretation: gauge bosons transport phase information between space-time points
- Gauge groups of the Standard Model: $SU(3)_C \times SU(2)_L \times U(1)_Y$
- Electroweak gauge group $SU(2)_L \times U(1)_Y$
  - Chiral theory: interaction different for left- and right-handed states
- **Fundamental problem of the Standard Model**
  - **Gauge-boson mass terms violate gauge invariance** (gauge theories in general)
  - **Fermion mass terms violate invariance under electroweak ($\mathbf{SU}(2)_L \times \mathbf{U}(1)_Y$) symmetry** (because of the chiral structure)

Unitarity Violation

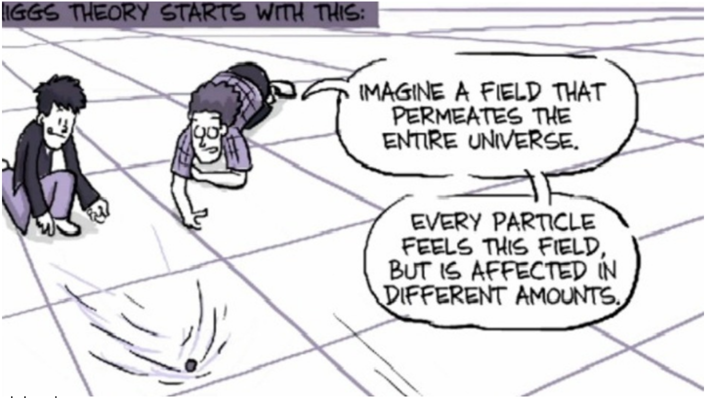
- Several Standard Model scattering cross-sections violate unitarity, i. e. become divergent at large $\sqrt{s}$, for example
  - $e^+e^- \to$ WW (for $m_e \neq 0$)
  - WW $\to$ WW scattering



taken from arXiv:1107.1501

$\rightarrow$ **theory becomes non-renormalisable**

How to Solve the Problems?



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2.4.2. Idea of the Higgs mechanism

How to Solve the Problems?

- Concept of **spontaneous symmetry breaking** (SSB)
  - Applied to the Standard Model:
    the Higgs mechanism (1960s)

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F. Englert and **R. Brout**,
Broken symmetry and the mass of gauge vector mesons,
Phys. Rev. Lett. 13 (1964) 321-323.

P. W. Higgs,
Broken symmetries, massless particles and gauge fields,
Phys. Lett. 12 (1964) 132-133.

P. W. Higgs,
Broken symmetries and the masses of gauge bosons,
Phys. Rev. Lett. 13 (1964) 508-509.

G. Guralnik, **C. Hagen**, and **T. Kibble**,
Global conservation laws and massless particles,
Phys. Rev. Lett. 13 (1964) 585-587.

P. W. Higgs,
Spontaneous symmetry breakdown without massless bosons,
Phys. Rev. 145 (1966) 1156-1163.

T. Kibble,
Symmetry breaking in non-Abelian gauge theories,
Phys. Rev. 155 (1967) 1554-1561.

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  - Applied to the Standard Model:
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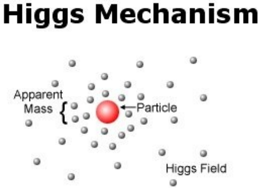
- New **background field that has non-zero amplitude** v in ground state everywhere
  - **Particles interact with the field and get 'slowed down'**: movement as if they have mass
  - Mass explained as restoring force



$$m \propto v \qquad (v = \text{field amplitude})$$

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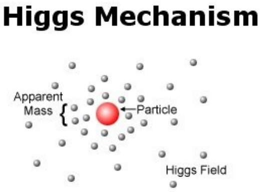
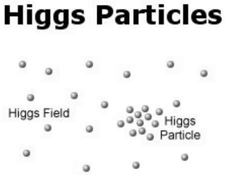
$$m \propto v \qquad (v = \text{field amplitude})$$

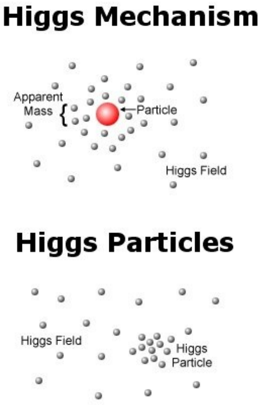
  - Detection: **excitation of background field** $\rightarrow$ **new particle**





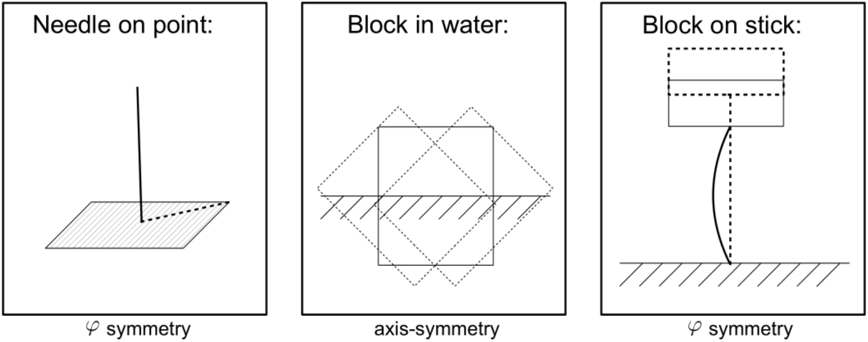
How to Solve the Problems?

- Concept of **spontaneous symmetry breaking** (SSB)
  - Applied to the Standard Model:
    the Englert–Brout–Higgs–Guralnik–Hagen–Kibble mechanism (1960s)
- In the Standard Model
  - Weak interactions themselves have infinite range and are described by gauge-invariant theory
  - **Interactions are screened by background field**: effective masses for the gauge bosons
  - SSB: field spontaneously takes ground-state which does not have symmetry
  - But mechanism would be better called ‘hidden gauge symmetry’ (background field hiding the gauge invariance)



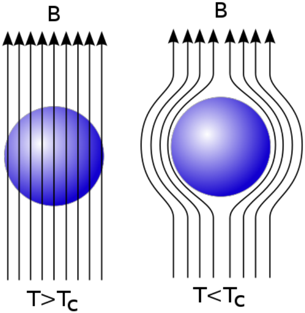
SSB in Classical Mechanics

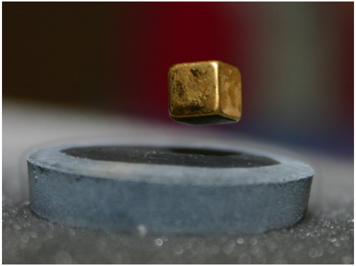
- Symmetry is present in the system (i. e. the Lagrangian)
- But it is broken in the energy ground-state



Analogy: Meißner–Ochsenfeld Effect

- Below critical temperature: magn. field expulsed from superconductor
  - Only small penetration depth λ of the magnetic field
- Expulsion occurs due to interaction of photons of magnetic field with Cooper pairs in the superconductor
- If one ignores 'background field' of Cooper pairs: appears as if photons have acquired a mass $M \propto \frac{1}{\lambda}$





How to Solve the Problems?

- Concept of **spontaneous symmetry breaking** (SSB)
  - Applied to the Standard Model:
    the Englert–Brout–Higgs–Guralnik–Hagen–Kibble mechanism (1960s)
- **Introduce a background field with a specific potential** that
  - keeps the **full Lagrangian invariant** under $SU(2)_L \times U(1)_Y$,
  - but will make the energy **ground-state *not* invariant** under this symmetry

→ **Higgs mechanism**
  - Solves all the discussed problems
  - Introduces a **fundamental scalar particle: the Higgs boson**

Examples: Spontaneous Symmetry Breaking

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1) Introducing the Higgs potential & spontaneous symmetry breaking
2) Breaking global gauge symmetry
3) Breaking local gauge invariance: the Higgs mechanism

1) Introducing the Higgs potential & spontaneous symmetry breaking

Simple Example of SSB

- **Illustrate idea** of Higgs field and **spontaneous symmetry breaking**
- Real scalar field $\phi(x)$ in specific potential $V(\phi)$

$$\mathcal{L} = \underbrace{\frac{1}{2}\left(\partial_\mu \phi(x)\right)\left(\partial^\mu \phi(x)\right)}_{T(\phi)} - \underbrace{\left[\frac{1}{2}\mu^2\phi^2(x) + \frac{1}{4}\lambda\phi^4(x)\right]}_{V(\phi)}$$

  - $\mathcal{L}$ symmetric under global phase transformation $\phi(x) \rightarrow -\phi(x)$
  - $\lambda > 0$: V has absolute minimum
  - Two possibilities for sign of μ^2
- Investigate particle spectrum: **investigate $\mathcal{L}$ around energy ground-state** (*vacuum expectation value* or short *vacuum*)

Energy ground-state at minimum of Hamiltonian density

$$\mathcal{H} = \frac{\partial\mathcal{L}}{\partial(\partial_0\phi)}(\partial_0\phi) - \mathcal{L} = \frac{1}{2}[(\partial_0\phi)^2 + (\nabla\phi)^2] + V(\phi)$$

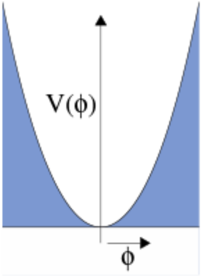
Lowest energy if $\phi(x) = \phi_0 = \text{const}$ and $V(\phi_0)$ minimal

Simple Example of SSB

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)(\partial^\mu \phi) - \left[\frac{1}{2}\mu^2\phi^2 + \frac{1}{4}\lambda\phi^4\right]$$

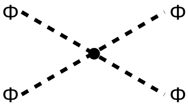
- Case $\mu^2 > 0$:
  - Minimum of $V(\phi)$ at $\phi(x) = \phi_0 = 0$: ground state
  - Ground state retains symmetry in $\phi \to -\phi$

$$\mathcal{L} = \underbrace{\left[\frac{1}{2}(\partial_\mu \phi)(\partial^\mu \phi) - \frac{1}{2}\mu^2\phi^2\right]}_{\text{free particle, mass } \mu} - \underbrace{\frac{1}{4}\lambda\phi^4}_{\text{interaction}}$$



→ **free scalar particle with mass μ and four-point self-interaction**

- Mass = excitation against “restoring force”

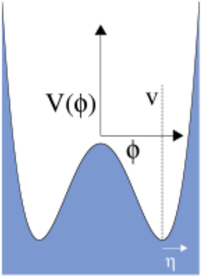


Simple Example of SSB

$$\mathcal{L} = \frac{1}{2}\left(\partial_\mu\phi\right)\left(\partial^\mu\phi\right) - \left[\frac{1}{2}\mu^2\phi^2 + \frac{1}{4}\lambda\phi^4\right]$$

- Case $\mu^2 < 0$: particle with imaginary mass?
  - No stable minimum of $V(\phi)$ at $\phi(x) = 0$ (perturbation theory will not converge)
  - Ground state(s) located at $\boxed{\phi_0 = \sqrt{-\frac{\mu^2}{\lambda}} \equiv v}$
  - Study **states close to minimum**:

$\boxed{\phi(x) \equiv v + \eta(x)}$ (perturbations $\eta(x)$ around v)



Kinetic term: $T = \frac{1}{2}\left[\partial_\mu(v+\eta)\partial^\mu(v+\eta)\right]$

$= \frac{1}{2}\left(\partial_\mu\eta\right)\left(\partial^\mu\eta\right)$, since $\partial_\mu v = 0$

Potential term: $V = \frac{1}{2}\mu^2(v+\eta)^2 + \frac{1}{4}\lambda(v+\eta)^4$

$= \lambda v^2\eta^2 + \lambda v\eta^3 + \frac{1}{4}\lambda\eta^4 - \underbrace{\frac{1}{4}\lambda v^4}_{\text{const}}$, since $\mu^2 = -\lambda v^2$

Simple Example of SSB

$$\mathcal{L} = \frac{1}{2}\left(\partial_\mu \phi\right)\left(\partial^\mu \phi\right) - \left[\frac{1}{2}\mu^2\phi^2 + \frac{1}{4}\lambda\phi^4\right]$$

- Case $\mu^2 < 0$: particle with imaginary mass?~~particle with imaginary mass?~~
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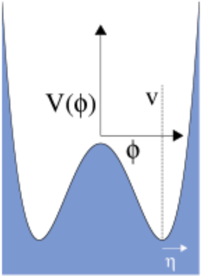
  - Ground state(s) located at $\boxed{\phi_0 = \sqrt{-\frac{\mu^2}{\lambda}} \equiv v}$
  - Study **states close to minimum**:

$$\mathcal{L} = \left[\frac{1}{2}\left(\partial_\mu \eta\right)\left(\partial^\mu \eta\right) - \lambda v^2\eta^2\right] - \lambda v\eta^3 - \frac{1}{4}\lambda\eta^4$$

  - Scalar particle η with mass $\boxed{\frac{1}{2}m_\eta^2 \equiv \lambda v^2 = -\mu^2 \Rightarrow m_\eta = \sqrt{2\lambda v^2}}$
  - Additional 3- and 4-point self-interactions



Symmetry in ϕ retained but ground state not symmetric in η: $\mathcal{L}(\eta) \neq \mathcal{L}(-\eta)$

$\rightarrow$ **spontaneous symmetry breaking (SSB)**

Summary

- Lagrangian for scalar field ϕ without mass terms + potential $V(\phi)$ with minimum (= ground-state of system) at $\phi \equiv v \neq 0$
- Particle spectrum obtained by investigating $\mathcal{L}$ close to the minimum: expansion of ϕ around the minimum v
- Adding V leads to massive scalar particle (consequence of ‘restoring force’ in potential) with self-interaction
  - Keeps the full Lagrangian invariant under the original symmetry
  - But makes the energy ground-state *not* invariant under this symmetry

$\rightarrow$ tools needed for the Higgs mechanism

2) Breaking global gauge symmetry

Breaking Global Gauge Symmetry

- Example: complex scalar field $\phi = \frac{1}{\sqrt{2}}(\phi_1 + i\phi_2)$
  (NB: field for charged particles, see Exercise No. 1.2)

- **Higgs potential** $$\boxed{V(\phi) = \mu^2|\phi|^2 + \lambda|\phi|^4}$$

- Lagrangian $\mathcal{L} = (\partial_\mu\phi^*)(\partial^\mu\phi) - V(\phi)$

- $V = V(|\phi|^2) \rightarrow$ **invariant under global U(1) transformations**

$$\begin{aligned} \phi &\rightarrow e^{i\alpha}\phi \\ \phi^* &\rightarrow e^{-i\alpha}\phi^* \end{aligned} \qquad \alpha = \text{const}$$

- $\mu^2 > 0$: ground state at $|\phi_0| = 0$
  $\rightarrow$ 2 massive scalar particles with additional self-interaction

Breaking Global Gauge Symmetry

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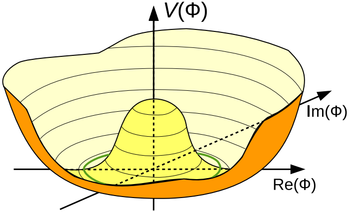


- $\mu^2 < 0$: infinitely many ground states on circle with

$$\boxed{|\phi| = \sqrt{\tfrac{1}{2}(\phi_1^2 + \phi_2^2)} = \sqrt{\tfrac{-\mu^2}{2\lambda}} \equiv \tfrac{v}{\sqrt{2}}}$$

Breaking Global Gauge Symmetry

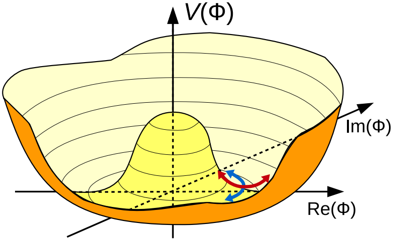
- Œ choose real ground state (U(1) symmetry!)

$$\boxed{\phi_0 = \frac{v}{\sqrt{2}} = \sqrt{\frac{-\mu^2}{2\lambda}}}$$

- Study perturbation around ϕ_0:

$$\phi(x) = \frac{1}{\sqrt{2}}\left(v + \eta(x) + i\zeta(x)\right)$$

$\eta(x)$, $\zeta(x)$: infinitesimal field amplitudes



Breaking Global Gauge Symmetry

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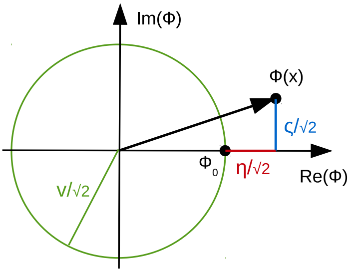

$$T = \tfrac{1}{2}\partial_\mu(v + \eta - i\zeta)\partial^\mu(v + \eta + i\zeta)$$
$$= \tfrac{1}{2}\left(\partial_\mu\eta\right)\left(\partial^\mu\eta\right) + \tfrac{1}{2}\left(\partial_\mu\zeta\right)\left(\partial^\mu\zeta\right), \quad \partial_\mu v = 0$$

$$V = \mu^2|\phi|^2 + \lambda|\phi|^4$$
$$= -\tfrac{1}{2}\lambda v^2\left[(v+\eta)^2 + \zeta^2\right] + \tfrac{1}{4}\lambda\left[(v+\eta)^2 + \zeta^2\right]^2, \quad \mu^2 = -\lambda v^2$$
$$= +\lambda v^2\eta^2 + \mathcal{O}(\eta^3, \eta^4, \zeta^4, \eta\zeta^2, \eta^2\zeta^2, \ldots)$$

Breaking Global Gauge Symmetry

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- Full Lagrangian after symmetry breaking

$$\mathcal{L} = \underbrace{\tfrac{1}{2}(\partial_\mu \eta)(\partial^\mu \eta) - \lambda v^2 \eta^2}_{\text{massive scalar particle}} + \underbrace{\tfrac{1}{2}(\partial_\mu \zeta)(\partial^\mu \zeta)}_{\text{massless scalar particle}} + \underbrace{\text{higher-order terms}}_{\text{self interaction}}$$

- η: massive scalar particle with $m_\eta = \sqrt{2\lambda v^2}$
  - Consequence of 'restoring force' in radial direction
- ζ: massless scalar particle "**Goldstone Boson**"
  - No restoring force in azimuth, consequence of the global U(1) symmetry

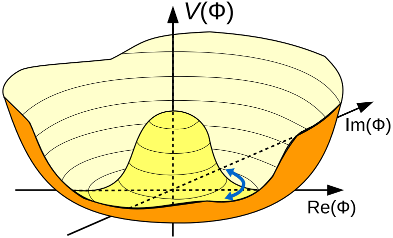
Goldstone Theorem *For each generator of a spontaneously broken*[1] *continuous symmetry*[2]*, a massless spin-zero particle will appear*

[1] a symmetry of $\mathcal{L}$ that is not present in the ground state
[2] that 'connects' the ground states

$\approx$ example: chiral symmetry breaking in QCD (pions = pseudo Goldstone bosons)

Summary

Spontaneously breaking a continuous global symmetry leads to the appearance of a massless Goldstone boson



3) Breaking local gauge invariance: the Higgs mechanism

Higgs Mechanism: Breaking Local Symmetry

Example QED: local U(1) symmetry

- Invariance under local U(1) gauge transformations

$$\psi(x) \to \psi'(x) = e^{i\alpha(x)}\psi(x)$$

achieved by introduction of covariant derivative

$$\partial_\mu \to D_\mu = \partial_\mu + iqA_\mu \quad \text{with} \quad A_\mu \to A'_\mu = A_\mu - \frac{1}{q}\partial_\mu\alpha(x)$$

- Adding a **complex scalar Higgs field** $\phi = \frac{1}{\sqrt{2}}(\phi_1 + i\phi_2)$
(also transforms under U(1)!)
- Local-U(1) **gauge-invariant Lagrangian** for Higgs and photon field
(omitting fermion terms)

$$\boxed{\mathcal{L} = (D_\mu\phi)^\dagger (D^\mu\phi) - V(\phi) - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}}$$

with Higgs potential $V(\phi) = \mu^2|\phi|^2 + \lambda|\phi|^4$ with $\mu^2 < 0$

Higgs Mechanism: Breaking Local Symmetry

- Higgs field $\phi = \frac{1}{\sqrt{2}} (v + \eta + i\zeta)$ close to ground state $v = \sqrt{-\mu^2/\lambda}$
- Kinetic and potential term of Lagrangian

$$
\begin{aligned}
T &= (D_\mu \phi)^\dagger (D^\mu \phi) \\
&= \tfrac{1}{2} \left[(\partial_\mu - iqA_\mu)(v + \eta - i\zeta) \right] \left[(\partial^\mu + iqA^\mu)(v + \eta + i\zeta) \right] \\
&= \tfrac{1}{2} \Big[(\partial_\mu \eta)(\partial^\mu \eta) + (\partial_\mu \zeta)(\partial^\mu \zeta) \\
&\quad + q^2 \left((v+\eta)^2 + \zeta^2 \right) A_\mu A^\mu + 2qvA_\mu (\partial^\mu \zeta) \Big] + \text{higher orders}
\end{aligned}
$$

$$V = +\lambda v^2 \eta^2 + \text{higher orders} \qquad \text{(see previous example)}$$

$$\to \mathcal{L} = \underbrace{\tfrac{1}{2} (\partial_\mu \eta)^2 - \lambda v^2 \eta^2}_{\text{massive scalar boson}} + \underbrace{\tfrac{1}{2} (\partial_\mu \zeta)^2}_{\text{Goldstone boson}} \underbrace{- \tfrac{1}{4} F_{\mu\nu} F^{\mu\nu} + \tfrac{1}{2} q^2 v^2 A_\mu A^\mu}_{\text{photon with mass term}}$$

$$+ \underbrace{qvA_\mu (\partial^\mu \zeta)}_{?} + \text{interaction } \eta/\zeta A_\mu + \text{ self-interaction } \eta/\zeta$$

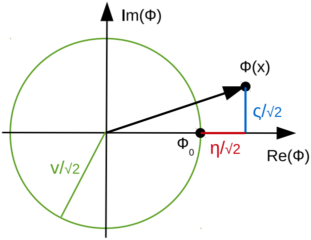
Rewriting Lagrangian in Unitary Gauge

- Terms involving ζ and A_μ

$$\frac{1}{2}\left(\partial_\mu\zeta\right)^2+\frac{1}{2}q^2v^2A_\mu A^\mu+qvA_\mu\left(\partial^\mu\zeta\right)=\frac{1}{2}q^2v^2\left[A_\mu+\frac{1}{qv}\left(\partial^\mu\zeta\right)\right]^2=\frac{1}{2}q^2v^2\left(A'_\mu\right)^2$$

- **Exploiting local gauge invariance**
  - A_μ fixed up to a term $\frac{1}{q}\partial_\mu\alpha(x)$ (because $A_\mu \to A'_\mu = A_\mu - \frac{1}{q}\partial_\mu\alpha(x)$)
  - Gauge transformation with $\boxed{\alpha(x) = -\frac{1}{v}\zeta(x)}$ (*unitary gauge*)

$$\begin{aligned}
A'_\mu &= A_\mu + \frac{1}{qv}\left(\partial_\mu\zeta\right)\\
\phi' &= e^{i\alpha}\phi = e^{-i\frac{1}{v}\zeta}\phi\\
&= e^{-i\frac{1}{v}\zeta}\frac{1}{\sqrt{2}}\left(v+\eta+i\zeta\right)\\
&\approx \left(1-i\frac{1}{v}\zeta\right)\frac{1}{\sqrt{2}}\left(v+\eta+i\zeta\right)\\
&\approx \frac{1}{\sqrt{2}}\left(v+\eta\right)
\end{aligned}$$



Rewriting Lagrangian in Unitary Gauge

(for simplicity, from now on writing: $\phi' = \phi$, $A'_\mu = A_\mu$)

$$\mathcal{L} = \left(D_\mu \phi'\right)^\dagger \left(D^\mu \phi'\right) - V(\phi')$$
$$= \frac{1}{2}\left[\left(\partial_\mu - iqA'_\mu\right)\left(v + \eta\right)\right]\left[\left(\partial^\mu + iqA'^\mu\right)\left(v + \eta\right)\right] - V(\phi')$$

Rewriting Lagrangian in Unitary Gauge

(for simplicity, from now on writing: $\phi' = \phi$, $A'_\mu = A_\mu$)

$$\begin{aligned}
\mathcal{L} &= (D_\mu \phi)^\dagger (D^\mu \phi) - V(\phi) \\
&= \tfrac{1}{2} \left[\Big(\partial_\mu - iqA_\mu \Big) \Big(v + \eta \Big) \right] \left[\Big(\partial^\mu + iqA^\mu \Big) \Big(v + \eta \Big) \right] - V(\phi) \\
&= \tfrac{1}{2} (\partial_\mu \eta)^2 + \tfrac{1}{2} q^2 (v + \eta)^2 A_\mu^2 - \underbrace{\left(\lambda v^2 \eta^2 + \lambda v \eta^3 + \tfrac{1}{4} \lambda \eta^4 - \tfrac{1}{4} \lambda v^4 \right)}_{= V(\phi) \text{ (see example 1))}} \\
&= \underbrace{\tfrac{1}{2} (\partial_\mu \eta)^2 - \lambda v^2 \eta^2}_{\text{massive Higgs boson}} + \underbrace{\tfrac{1}{2} q^2 v^2 A_\mu^2}_{\text{photon mass}} + \underbrace{q^2 v A_\mu^2 \eta + \tfrac{1}{2} q^2 A_\mu^2 \eta^2}_{\text{Higgs-photon interaction}} - \underbrace{\lambda v \eta^3 - \tfrac{1}{4} \lambda \eta^4}_{\text{Higgs self-interaction}}
\end{aligned}$$

Summary: The Higgs Mechanism

$$\mathcal{L} = \underbrace{\tfrac{1}{2}(\partial_\mu \eta)^2 - \lambda v^2 \eta^2}_{\text{massive Higgs boson}} + \underbrace{\tfrac{1}{2} q^2 v^2 A_\mu^2}_{\text{photon mass}} + \underbrace{q^2 v A_\mu^2 \eta + \tfrac{1}{2} q^2 A_\mu^2 \eta^2}_{\text{Higgs-photon interaction}} - \underbrace{\lambda v \eta^3 - \tfrac{1}{4}\lambda \eta^4}_{\text{Higgs self-interaction}}$$

- Expansion of $\phi \to \phi(\eta, \zeta)$ around energy ground-state of Higgs potential generates **mass term $m_A = qv$** for gauge field A_μ from coupling $q^2|\phi|^2 A_\mu^2$ **by covariant derivative**
- **Requires non-vanishing** v: particular shape of potential ($\mu^2 < 0$)
- From point-of-view of the gauge field, two interpretations
  1. Photon field interacts with external background (Higgs) field: *dynamic* mass term
  2. Background field unknown: interpretation as massive photon field

Summary: The Higgs Mechanism

$$\mathcal{L} = \underbrace{\tfrac{1}{2}(\partial_\mu \eta)^2 - \lambda v^2 \eta^2}_{\text{massive Higgs boson}} + \underbrace{\tfrac{1}{2} q^2 v^2 A_\mu^2}_{\text{photon mass}} + \underbrace{q^2 v A_\mu^2 \eta + \tfrac{1}{2} q^2 A_\mu^2 \eta^2}_{\text{Higgs-photon interaction}} - \underbrace{\lambda v \eta^3 - \tfrac{1}{4}\lambda \eta^4}_{\text{Higgs self-interaction}}$$

- What about the massless ζ field (Goldstone boson)?
  - Removed by gauge transformation (absorbed into A'_μ)
  - Responsible for longitudinal component of massive vector field

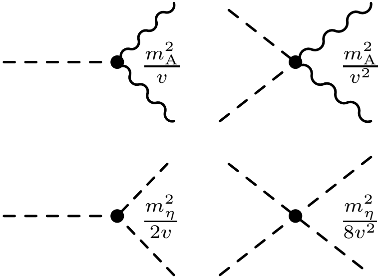
w/o ϕ-A_μ interaction	with ϕ-A_μ interaction
1 η field, $m_\eta = \sqrt{2\lambda v^2}$	1 η field, $m_\eta = \sqrt{2\lambda v^2}$
1 ζ field, $m_\zeta = 0$	
2 states of A_μ (helicity ± 1)	3 states of A_μ (helicity ± 1, 0)

“The gauge boson has eaten up the Goldstone boson and has become fat on it.”

Summary: The Higgs Boson

$$\mathcal{L} = \underbrace{\tfrac{1}{2}\left(\partial_\mu \eta\right)^2 - \lambda v^2 \eta^2}_{\text{massive Higgs boson}} + \underbrace{\tfrac{1}{2} q^2 v^2 A_\mu^2}_{\text{photon mass}} + \underbrace{q^2 v A_\mu^2 \eta + \tfrac{1}{2} q^2 A_\mu^2 \eta^2}_{\text{Higgs-photon interaction}} - \underbrace{\lambda v \eta^3 - \tfrac{1}{4}\lambda \eta^4}_{\text{Higgs self-interaction}}$$

- **Higgs mechanism predicts massive scalar particle η** (Higgs boson) with self interaction
  - **NB: Gauge boson mass acquired by interaction with Higgs background field, not with Higgs boson!**
- Interaction of photon and Higgs boson
  - Photon-Higgs three-point interaction
  - Photon-Higgs four-point interaction
- Higgs self-interaction
  - Three-point self-coupling
  - Four-point self-coupling



This was just an Example

- Previous discussion was just an example to illustrate the Higgs mechanism: Apparently, **there is no charged Higgs field** with $v > 0$ because the **photon is massless**!
- But principle can be applied to $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ symmetry of the Standard Model

Examples: Spontaneous Symmetry Breaking

1) Lagrangian for scalar field ϕ without mass terms
   + Higgs potential $V(\phi)$ with ground state at $\phi_0 \equiv v \neq 0$
   - Lagrangian invariant under global phase transformation but ground state ϕ_0 is not $\rightarrow$ spontaneous symmetry breaking
   - Massless $\phi \rightarrow$ massive scalar particle (consequence of 'restoring force' in potential) with self-interaction

2) Breaking global gauge symmetry: complex scalar field ϕ with $V(\phi)$
   - Massive scalar particle ('restoring force' in radial direction)
   - Massless scalar particle (along circle of ground states): "Goldstone boson"

3) Breaking local gauge symmetry: complex scalar field ϕ with $V(\phi)$
   - Mass terms of gauge boson (via covariant derivative of ϕ)
   - Massless scalar particle (Goldstone boson) removed by gauge transformation (d.o.f. appears as mass of vector boson)
   - Massive scalar particle (Higgs boson) with self-interaction and interaction with gauge boson

Programme

Date	Room	Type	Topic
Wed Apr 24.	Kl. HS B	LE 01	1. Organisation and introduction: particle physics at colliders + W/Z/H history
Tue Apr 30.	30.23 11/12	—	*no class*
Wed May 01.	Kl. HS B	—	*no class*
Tue May 07.	30.23 11/12	LE 02	2.1 Gauge theory & 2.2 The electroweak sector of the SM I
Wed May 08.	Kl. HS B	LE 03, EX 01	2.3 Discovery of the W and Z bosons & EX gauge theories
→ Tue May 14.	30.23 11/12	LE 04	2.4 The Higgs mechanism
Wed May 15.	Kl. HS B	EX 02	Exercise “SM Higgs mechanism”
Tue May 21.	30.23 11/12	—	*no class*
Wed May 22.	Kl. HS B	LE 05	2.5 The electroweak sector of the SM II (Higgs mechanism + Yukawa couplings)
Tue May 28.	30.23 11/12	SP 01	Specialisation of 2.4 and 2.5
Wed May 29.	Kl. HS B	LE 06	3.1 From theory to observables & 3.2 Reconstruction + analysis of exp. data
Tue Jun 04.	30.23 11/12	EX 03	Exercise “Trigger efficiency measurement”
Wed Jun 05.	Kl. HS B	LE 07	3.3 Measurements in particle physics (part 1)
Tue Jun 11.	30.23 11/12	EX 04	Exercise on statistical methods
Wed Jun 12.	Kl. HS B	LE 08	3.3 Measurements in particle physics (part 2)
Tue Jun 18.	30.23 11/12	SP 02	Specialisation “Limit setting”
Wed Jun 19.	Kl. HS B	SP 03	Specialisation “Unfolding”
Tue Jun 25.	30.23 11/12	LE 09	4.1 Determination of SM parameters
Wed Jun 26.	Kl. HS B	LE 10	4.2 Measurement and role of W/Z bosons at the LHC
Tue Jul 02.	30.23 11/12	EX 05	Paper seminar “Z pole measurements”
Wed Jul 03.	Kl. HS B	LE 11	4.3 Processes with several W/Z bosons
Tue Jul 09.	30.23 11/12	EX 06	Paper seminar Higgs
Wed Jul 10.	Kl. HS B	LE 12	5.1 Discovery and first measurements of the Higgs boson
Tue Jul 16.	30.23 11/12	EX 07	Exercise “Machine learning in physics analysis”
Wed Jul 17.	Kl. HS B	LE 13	5.2 Measurement of couplings and kinematic properties
Tue Jul 23.	30.23 11/12	EX 08	Presentations: results of ML challenge
Wed Jul 24.	Kl. HS B	LE 14	5.3 Search for Higgs physics beyond the SM & 5.4 Future Higgs physics