Teilchenphysik 2 — W/Z/Higgs an Collidern

Sommersemester 2019

Matthias Schröder und Roger Wolf | Vorlesung 5

INSTITUT FÜR EXPERIMENTELLE TEILCHENPHYSIK (ETP)

Laptops for Exercises/Specialisations

The exercise will be a computer exercise, and it will be done during class time ("Präsenzübung"). The exercise runs standalone on a ROOT input file. *Please bring a laptop and make sure* ***beforehand*** *that there is a working installation of a recent ROOT6 and Python 2 version* (the exercise has been tested with ROOT version 6.06/06 and Python version 2.7.6). It is encouraged that you work in small groups of up to three persons, and it is sufficient to have one laptop per group.

Laptops for Exercises/Specialisations

	Date	Room	Type	Topic
	Wed Apr 24.	Kl. HS B	LE 01	1. Organisation and introduction: particle physics at colliders + W/Z/H history
	Tue Apr 30.	30.23 11/12	—	*no class*
	Wed May 01.	Kl. HS B	—	*no class*
	Tue May 07.	30.23 11/12	LE 02	2.1 Gauge theory & 2.2 The electroweak sector of the SM I
	Wed May 08.	Kl. HS B	LE 03, EX 01	2.3 Discovery of the W and Z bosons & EX gauge theories
	Tue May 14.	30.23 11/12	LE 04	2.4 The Higgs mechanism
	Wed May 15.	Kl. HS B	EX 02	Exercise “SM Higgs mechanism”
	Tue May 21.	30.23 11/12	—	*no class*
	Wed May 22.	Kl. HS B	LE 05	2.5 The electroweak sector of the SM II (Higgs mechanism + Yukawa couplings)
	Tue May 28.	30.23 11/12	SP 01	Specialisation of 2.4 and 2.5
	Wed May 29.	Kl. HS B	LE 06	3.1 From theory to observables & 3.2 Reconstruction + analysis of exp. data
→	Tue Jun 04.	30.23 11/12	EX 03	Exercise “Trigger efficiency measurement”
	Wed Jun 05.	Kl. HS B	LE 07	3.3 Measurements in particle physics (part 1)
	Tue Jun 11.	30.23 11/12	EX 04	Exercise on statistical methods
	Wed Jun 12.	Kl. HS B	LE 08	3.3 Measurements in particle physics (part 2)
→	Tue Jun 18.	30.23 11/12	SP 02	Specialisation “Limit setting”
→	Wed Jun 19.	Kl. HS B	SP 03	Specialisation “Unfolding”
	Tue Jun 25.	30.23 11/12	LE 09	4.1 Determination of SM parameters
	Wed Jun 26.	Kl. HS B	LE 10	4.2 Measurement and role of W/Z bosons at the LHC
	Tue Jul 02.	30.23 11/12	EX 05	Paper seminar “Z pole measurements”
	Wed Jul 03.	Kl. HS B	LE 11	4.3 Processes with several W/Z bosons
	Tue Jul 09.	30.23 11/12	EX 06	Paper seminar Higgs
	Wed Jul 10.	Kl. HS B	LE 12	5.1 Discovery and first measurements of the Higgs boson
→	Tue Jul 16.	30.23 11/12	EX 07	Exercise “Machine learning in physics analysis”
	Wed Jul 17.	Kl. HS B	LE 13	5.2 Measurement of couplings and kinematic properties
	Tue Jul 23.	30.23 11/12	EX 08	Presentations: results of ML challenge
	Wed Jul 24.	Kl. HS B	LE 14	5.3 Search for Higgs physics beyond the SM & 5.4 Future Higgs physics

2. The Electroweak Sector of the Standard Model

2. Electroweak Sector of the Standard Model

2.1 Gauge theory

- Global and local phase transformations
- Example: QED
- Abelian and non-Abelian gauge theories

2.2 The electroweak sector of the Standard Model – I

- Properties of the weak interaction, weak isospin
- Formulation of the Standard Model (without masses)

2.3 Discovery of W and Z bosons

- History towards discovery
- Experimental methods

2.4 The Higgs mechanism

- Problem of massive gauge bosons and massive fermions
- Idea of the Higgs mechanism: examples of spontaneous symmetry breaking

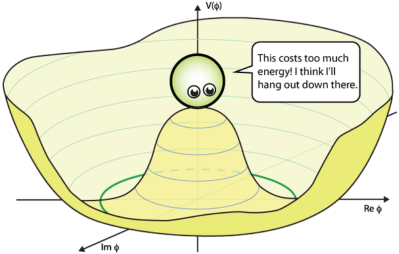
2.5 The electroweak sector of the Standard Model – II

- The Standard Model Higgs mechanism
- Yukawa couplings and fermion masses
- The Higgs boson

Let's Complete the Standard Model!

- Standard Model does not allow for naive mass terms
- But can **create mass terms dynamically** via Higgs mechanism
  - Requires the gauge symmetries in energy ground-state to be spontaneously broken
  - All fields introduced so far obey all symmetries, also in their energy ground-state

→ **Need new field with self-interaction that leads to spontaneously symmetry-breaking (Higgs) potential**



Reminder: SSB Examples

1) Lagrangian for scalar field ϕ without mass terms
   + Higgs potential $V(\phi)$ with ground state at $\phi_0 \equiv v \neq 0$
   - Lagrangian invariant under global phase transformation but ground state ϕ_0 is not $\rightarrow$ spontaneous symmetry breaking
   - Massless $\phi \rightarrow$ massive scalar particle (consequence of 'restoring force' in potential) with self-interaction

2) Breaking global gauge symmetry: complex scalar field ϕ with $V(\phi)$
   - Massive scalar particle ('restoring force' in radial direction)
   - Massless scalar particle (along circle of ground states): "Goldstone boson"

3) Breaking local gauge symmetry: complex scalar field ϕ with $V(\phi)$
   - Mass terms of gauge boson (via covariant derivative of ϕ)
   - Massless scalar particle (Goldstone boson) removed by gauge transformation (d.o.f. appears as mass of vector boson)
   - Massive scalar particle (Higgs boson) with self-interaction and interaction with gauge boson

Let's Complete the Standard Model!

2. Electroweak Sector of the Standard Model

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2.1 Gauge theory
- Global and local phase transformations
- Example: QED
- Abelian and non-Abelian gauge theories

2.2 The electroweak sector of the Standard Model – I
- Properties of the weak interaction, weak isospin
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- Idea of the Higgs mechanism: examples of spontaneous symmetry breaking

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- The Standard Model Higgs mechanism
- Yukawa couplings and fermion masses
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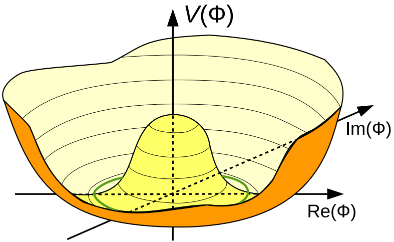
2.5 The electroweak sector of the Standard Model – II

2.5.1. The Standard Model Higgs mechanism

The Standard-Model Higgs Field ϕ

- $\mathcal{L}_{\text{SM}}$ should retain all gauge symmetries: add Higgs field ϕ as **left-chiral weak-isospin doublet of two complex fields**

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_1 + i\phi_2 \\ \phi_3 + i\phi_4 \end{pmatrix}$$



- Lagrangian for the Higgs field

$$\mathcal{L}_{\text{Higgs}} = (\partial_\mu \phi^\dagger)(\partial^\mu \phi) - V(\phi)$$

$$V(\phi) = \mu^2|\phi|^2 + \lambda|\phi|^4$$

with $\mu^2 < 0$ (SSB!)

- The Standard-Model Lagrangian becomes

$$\mathcal{L}_{\text{SM}} = \mathcal{L}_{\text{kin}} + \mathcal{L}_{\text{CC}} + \mathcal{L}_{\text{NC}} + \mathcal{L}_{\text{gauge}} + \mathcal{L}_{\text{Higgs}}$$

The Standard-Model Higgs Field ϕ

- $\mathcal{L}_{\text{SM}}$ should retain all gauge symmetries: add Higgs field ϕ as **left-chiral weak-isospin doublet of two complex fields**

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_1 + i\phi_2 \\ \phi_3 + i\phi_4 \end{pmatrix}$$

- Invariance of $\mathcal{L}_{\text{Higgs}}$ under local $\text{SU}(2)_\text{L} \times \text{U}(1)_\text{Y}$ transformations

$$\phi(x) \rightarrow e^{[i\frac{g}{2}\alpha^a(x)\tau^a]} e^{[i\frac{g'}{2}\alpha(x)Y_\phi]}\phi(x)$$

  enforced by **covariant derivative**

$$\begin{aligned}
\partial_\mu &\rightarrow \partial_\mu + i\tfrac{g}{2}\tau_a W^a_\mu + i\tfrac{g'}{2} Y_\phi B_\mu \\
W^a_\mu &\rightarrow W^a_\mu - \partial_\mu \alpha^a(x) - g\epsilon^{abc}\alpha_b(x) W_{c,\mu} \\
B_\mu &\rightarrow B_\mu - \partial_\mu \alpha(x)
\end{aligned}$$

The Standard-Model Higgs Field ϕ

- $\mathcal{L}_{\mathrm{SM}}$ should retain all gauge symmetries: add Higgs field ϕ as **left-chiral weak-isospin doublet of two complex fields**

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- Invariance of $\mathcal{L}_{\mathrm{Higgs}}$ under local $\mathrm{SU(2)_L} \times \mathrm{U(1)_Y}$ transformations

$$\phi(x) \to e^{[i\frac{g}{2}\alpha^a(x)\tau^a]} e^{[i\frac{g'}{2}\alpha(x)Y_\phi]} \phi(x)$$

enforced by **covariant derivative**

$$\partial_\mu \to \partial_\mu + i\frac{g}{2}\tau_a W^a_\mu + i\frac{g'}{2} Y_\phi B_\mu$$

$$W^a_\mu \to W^a_\mu - \partial_\mu \alpha^a(x) - g\epsilon^{abc}\alpha_b(x) W_{c,\mu}$$

$$B_\mu \to B_\mu - \partial_\mu \alpha(x)$$

SU(2) × U(1) hypercharges of ϕ

field	Y_ϕ	I_3	Q
ϕ^+	+1	$+1/2$	$+1$
ϕ^0	+1	$-1/2$	0

$Q = I_3 + \frac{Y}{2}$ (Gell-Mann–Nishijima)

Choice of Vacuum

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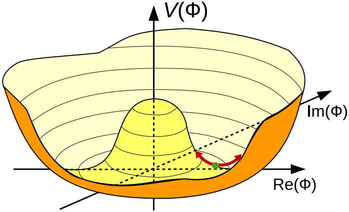
- Ground state ϕ_0 with non-zero amplitude $\phi_0 \equiv v/\sqrt{2}$ ($\rightarrow$ SSB)
- Choose **ground state** with $I_3 = -\frac{1}{2}$, $Q = 0$ (i. e. $\phi^+ = 0$):

$$\phi_0 = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}_0 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}, \quad v = \sqrt{-\frac{\mu^2}{\lambda}}$$

Any state with $(\phi^+)^2 + (\phi^0)^2 = v^2$ possible, but $|\phi^+| \neq 0$ together with $Y_\phi = +1$ leads to massive photon

- Expansion of ϕ around ground state in **unitarity gauge**

$$\phi(x) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + H \end{pmatrix}$$



Vacuum expectation value $v \neq 0$: gauge-boson masses

Radial excitation: the Higgs boson

Goldstone boson (term $i\zeta$) eliminated by gauge transformation

Dynamic Term in $\mathcal{L}_{\text{Higgs}}$

(see Exercises No. 2)

- **Covariant derivative** will give rise to
  - Masses for gauge bosons ($\propto v$)
  - Interactions between gauge bosons and Higgs boson ($\propto v\mathrm{H}$, $\propto \mathrm{H}^2$)

$$D_\mu\phi = \frac{1}{\sqrt{2}}\partial_\mu \begin{pmatrix} 0 \\ v+\mathrm{H} \end{pmatrix} + \frac{i}{\sqrt{2}}\left[\frac{g}{2}\tau_a \mathrm{W}^a_\mu + \frac{g'}{2} Y_\phi B_\mu\right] \begin{pmatrix} 0 \\ v+\mathrm{H} \end{pmatrix}$$

$$= \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ \partial_\mu \mathrm{H} \end{pmatrix} + \frac{i}{\sqrt{8}} \begin{pmatrix} g(\mathrm{W}^1_\mu - i\mathrm{W}^2_\mu) \\ -g\mathrm{W}^3_\mu + g' Y_\phi B_\mu \end{pmatrix} (v+\mathrm{H})$$

$$D^\mu\phi^\dagger = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & \partial^\mu \mathrm{H} \end{pmatrix} - \frac{i}{\sqrt{8}}\begin{pmatrix} g(\mathrm{W}^{1,\mu} + i\mathrm{W}^{2,\mu}) & -g\mathrm{W}^{3,\mu} + g' Y_\phi B^\mu \end{pmatrix}\left(v+\mathrm{H}\right)$$

- With Pauli matrices τ_a

$$\tau_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \tau_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \tau_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Dynamic Term in $\mathcal{L}_{\text{Higgs}}$

(see Exercises No. 2)

- **Covariant derivative** will give rise to
  - Masses for gauge bosons ($\propto v$)
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$$\begin{aligned} D_\mu\phi &= \frac{1}{\sqrt{2}}\partial_\mu \begin{pmatrix} 0 \\ v+\mathrm{H} \end{pmatrix} + \frac{i}{\sqrt{2}}\left[\frac{g}{2}\tau_a \mathrm{W}^a_\mu + \frac{g'}{2}Y_\phi B_\mu\right]\begin{pmatrix} 0 \\ v+\mathrm{H} \end{pmatrix} \\ &= \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ \partial_\mu \mathrm{H} \end{pmatrix} + \frac{i}{\sqrt{8}}\begin{pmatrix} g(\mathrm{W}^1_\mu - i\mathrm{W}^2_\mu) \\ -g\mathrm{W}^3_\mu + g' Y_\phi B_\mu \end{pmatrix}(v+\mathrm{H}) \end{aligned}$$

$$D^\mu\phi^\dagger = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & \partial^\mu \mathrm{H} \end{pmatrix} - \frac{i}{\sqrt{8}}\begin{pmatrix} g(\mathrm{W}^{1,\mu} + i\mathrm{W}^{2,\mu}) & -g\mathrm{W}^{3,\mu} + g' Y_\phi B^\mu \end{pmatrix}\begin{pmatrix} v+\mathrm{H} \end{pmatrix}$$

- Full dynamic term in $\mathcal{L}_{\text{Higgs}}$

$$D^\mu\phi^\dagger D_\mu\phi = \frac{1}{2}\left(\partial_\mu \mathrm{H}\right)^2 + \frac{g^2}{8}(v+\mathrm{H})^2\left(|\mathrm{W}^1|^2 + |\mathrm{W}^2|^2\right) + \frac{1}{8}(v+\mathrm{H})^2\left(-g\mathrm{W}^3_\mu + g' Y_\phi B_\mu\right)^2$$

Dynamic Term in $\mathcal{L}_{\text{Higgs}}$

(see Exercises No. 2)

- Re-writing $D^\mu \phi^\dagger D_\mu \phi$ in terms of **physical bosons**:

$$\frac{1}{2}\partial^\mu \mathrm{H}\partial_\mu \mathrm{H} + \frac{g^2}{8}(v+\mathrm{H})^2\left(|\mathrm{W}^1|^2 + |\mathrm{W}^2|^2\right) \quad + \frac{1}{8}(v+\mathrm{H})^2\left(-g\mathrm{W}^3_\mu + g' Y_\phi B_\mu\right)^2$$

- $\mathrm{W}^\pm$ bosons

$$\mathrm{W}^\pm_\mu = \frac{1}{\sqrt{2}}\left(\mathrm{W}^1_\mu \mp i\mathrm{W}^2_\mu\right) \quad \Rightarrow \quad |\mathrm{W}^1|^2 + |\mathrm{W}^2|^2 = |\mathrm{W}^+|^2 + |\mathrm{W}^-|^2$$

Dynamic Term in $\mathcal{L}_{\text{Higgs}}$

(see Exercises No. 2)

- Re-writing $D^{\mu}\phi^{\dagger}D_{\mu}\phi$ in terms of **physical bosons**

$$\tfrac{1}{2}\partial^{\mu}\text{H}\partial_{\mu}\text{H} + \tfrac{g^2}{8}(v+\text{H})^2\left(|\text{W}^+|^2 + |\text{W}^-|^2\right) + \tfrac{1}{8}(v+\text{H})^2\left(-g\text{W}^3_{\mu} + g'Y_{\phi}B_{\mu}\right)^2$$

- W$^{\pm}$ bosons

$$\text{W}^{\pm}_{\mu} = \tfrac{1}{\sqrt{2}}\left(\text{W}^1_{\mu} \mp i\text{W}^2_{\mu}\right) \quad \Rightarrow \quad |\text{W}^1|^2 + |\text{W}^2|^2 = |\text{W}^+|^2 + |\text{W}^-|^2$$

Dynamic Term in $\mathcal{L}_{\text{Higgs}}$

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- Re-writing $D^\mu \phi^\dagger D_\mu \phi$ in terms of **physical bosons**

$$\frac{1}{2}\partial^\mu \mathrm{H}\partial_\mu \mathrm{H} + \frac{g^2}{8}(v+\mathrm{H})^2\left(|\mathrm{W}^+|^2 + |\mathrm{W}^-|^2\right) + \frac{1}{8}(v+\mathrm{H})^2\left(-g\mathrm{W}^3_\mu + g' Y_\phi B_\mu\right)^2$$

- Weinberg rotation: photon and Z boson

$$\begin{pmatrix} \mathrm{Z}_\mu \\ A_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta_W & -\sin\theta_W \\ \sin\theta_W & \cos\theta_W \end{pmatrix} \begin{pmatrix} \mathrm{W}^3_\mu \\ B_\mu \end{pmatrix}$$

$$\underbrace{\sin\theta_W \equiv \frac{g'}{\sqrt{g^2+g'^2}}, \qquad \cos\theta_W \equiv \frac{g}{\sqrt{g^2+g'^2}}}$$

$$\left(-g\mathrm{W}^3_\mu + g' B_\mu\right) = -\sqrt{g^2+g'^2}\,\mathrm{Z}_\mu + 0\cdot A_\mu$$

Dynamic Term in $\mathcal{L}_{\text{Higgs}}$

(see Exercises No. 2)

- Re-writing $D^\mu \phi^\dagger D_\mu \phi$ in terms of **physical bosons**

$$\frac{1}{2}\partial^\mu \mathrm{H}\partial_\mu \mathrm{H} + \frac{g^2}{8}(v+\mathrm{H})^2\left(|\mathrm{W}^+|^2 + |\mathrm{W}^-|^2\right) + \frac{g^2+g'^2}{8}(v+\mathrm{H})^2|\mathrm{Z}|^2$$

- Weinberg rotation: photon and Z boson

$$\begin{pmatrix} \mathrm{Z}_\mu \\ A_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta_W & -\sin\theta_W \\ \sin\theta_W & \cos\theta_W \end{pmatrix} \begin{pmatrix} \mathrm{W}^3_\mu \\ B_\mu \end{pmatrix}$$

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$$\left(-g\mathrm{W}^3_\mu + g' B_\mu\right) = -\sqrt{g^2+g'^2}\,\mathrm{Z}_\mu + 0\cdot A_\mu$$

Dynamic Term in $\mathcal{L}_{\text{Higgs}}$

(see Exercises No. 2)

- Higgs doublet and **choice of specific ground state** leads to

$$D^\mu\phi^\dagger D_\mu\phi = \tfrac{1}{2}\partial_\mu \mathrm{H}\partial_\mu \mathrm{H}$$

$$+\tfrac{1}{2}\underbrace{\tfrac{g^2}{4}(v}_{}+\mathrm{H})^2\left(|\mathrm{W}^+|^2+|\mathrm{W}^-|^2\right)+\tfrac{1}{2}\underbrace{\tfrac{g^2+g'^2}{4}(v}_{}+\mathrm{H})^2|\mathrm{Z}|^2$$

$$m_\mathrm{W} \equiv \tfrac{1}{2}gv \qquad\qquad m_\mathrm{Z} \equiv \tfrac{1}{2}\sqrt{g^2+g'^2}\,v$$

Mass terms for the $\mathrm{W}^\pm$ and Z bosons
No mass term for the photon

- **Results depend on choice of Higgs-sector structure** ($v = 0$ for ϕ^+)
- Absolute masses of gauge bosons not predicted but their relation

$$\rho = \frac{m_\mathrm{W}}{m_\mathrm{Z}\cos\theta_W} = 1 \quad\Rightarrow\quad m_\mathrm{Z} > m_\mathrm{W}$$

Vacuum Expectation Value *v*

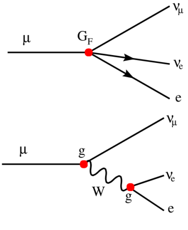
- **Higgs mechanism does not predict value of $v = \sqrt{-\mu^2/\lambda}$**
- But **estimate from relation to W-boson mass** possible

$$m_W^2 = \left(\tfrac{1}{2} g v\right)^2 \quad \text{(from Higgs mechanism)}$$

$$m_W^2 = \frac{\sqrt{2} g^2}{8 G_F} \quad \text{(from Fermi theory)}$$

Fermi: $\propto \frac{G_F}{\sqrt{2}}$

EW: $\propto \frac{g^2}{8 M_W^2}$



- $G_F = (1.16639 \pm 0.00002) \cdot 10^{-5}\,\mathrm{GeV}^{-2}$
  from muon-lifetime measurement

> $v = 246.22\,\mathrm{GeV}$ sets the scale of electroweak symmetry breaking

The Standard Model Higgs Mechanism

- Adding ϕ as $\mathrm{SU}(2)_L$ doublet with specific non-zero ground-state

$$
\begin{aligned}
\mathcal{L}_{\mathrm{Higgs}} = & \frac{1}{2}\left(\partial_\mu \mathrm{H}\right)\left(\partial^\mu \mathrm{H}\right) - \lambda v^2 \mathrm{H}^2 + \lambda v \mathrm{H}^3 - \frac{1}{4}\lambda \mathrm{H}^4 \\
& + \frac{1}{2} m_\mathrm{Z}^2 \mathrm{Z}_\mu \mathrm{Z}^\mu + \frac{m_\mathrm{Z}^2}{v} \mathrm{H} \mathrm{Z}_\mu \mathrm{Z}^\mu + \frac{1}{2}\frac{m_\mathrm{Z}^2}{v^2} \mathrm{H}^2 \mathrm{Z}_\mu \mathrm{Z}^\mu \\
& + m_\mathrm{W}^2 \mathrm{W}_\mu^+ \mathrm{W}^{-,\mu} + 2\frac{m_\mathrm{W}^2}{v} \mathrm{H} \mathrm{W}_\mu^+ \mathrm{W}^{-,\mu} + \frac{m_\mathrm{W}^2}{v^2} \mathrm{H}^2 \mathrm{W}_\mu^+ \mathrm{W}^{-,\mu}
\end{aligned}
$$

(Here, the equality $|\mathrm{W}^+|^2 + |\mathrm{W}^-|^2 = 2\mathrm{W}^+\mathrm{W}^-$ was used)

The Standard Model Higgs Mechanism

- Adding ϕ as $\mathrm{SU}(2)_L$ doublet with specific non-zero ground-state

$$\begin{aligned}\mathcal{L}_{\text{Higgs}} = \tfrac{1}{2}\left(\partial_\mu \mathrm{H}\right)\left(\partial^\mu \mathrm{H}\right) - \lambda v^2 \mathrm{H}^2 + \lambda v \mathrm{H}^3 - \tfrac{1}{4}\lambda \mathrm{H}^4 \\ + \tfrac{1}{2} m_\mathrm{Z}^2 \mathrm{Z}_\mu \mathrm{Z}^\mu + \frac{m_\mathrm{Z}^2}{v} \mathrm{H} \mathrm{Z}_\mu \mathrm{Z}^\mu + \frac{1}{2}\frac{m_\mathrm{Z}^2}{v^2} \mathrm{H}^2 \mathrm{Z}_\mu \mathrm{Z}^\mu \\ + m_\mathrm{W}^2 \mathrm{W}_\mu^+ \mathrm{W}^{-,\mu} + 2\frac{m_\mathrm{W}^2}{v} \mathrm{H} \mathrm{W}_\mu^+ \mathrm{W}^{-,\mu} + \frac{m_\mathrm{W}^2}{v^2} \mathrm{H}^2 \mathrm{W}_\mu^+ \mathrm{W}^{-,\mu}\end{aligned}$$

- Masses (mass terms) for the gauge bosons $\mathrm{W}^\pm$ and Z

The Standard Model Higgs Mechanism

- Adding ϕ as $\mathrm{SU}(2)_L$ doublet with specific non-zero ground-state

$$
\begin{aligned}
\mathcal{L}_{\text{Higgs}} = & \tfrac{1}{2}\left(\partial_\mu \mathrm{H}\right)\left(\partial^\mu \mathrm{H}\right) - \lambda v^2 \mathrm{H}^2 + \lambda v \mathrm{H}^3 - \tfrac{1}{4}\lambda \mathrm{H}^4 \\
& + \tfrac{1}{2} m_\mathrm{Z}^2 \mathrm{Z}_\mu \mathrm{Z}^\mu + \frac{m_\mathrm{Z}^2}{v} \mathrm{H} \mathrm{Z}_\mu \mathrm{Z}^\mu + \frac{1}{2}\frac{m_\mathrm{Z}^2}{v^2} \mathrm{H}^2 \mathrm{Z}_\mu \mathrm{Z}^\mu \\
& + m_\mathrm{W}^2 \mathrm{W}_\mu^+ \mathrm{W}^{-,\mu} + 2\frac{m_\mathrm{W}^2}{v} \mathrm{H} \mathrm{W}_\mu^+ \mathrm{W}^{-,\mu} + \frac{m_\mathrm{W}^2}{v^2} \mathrm{H}^2 \mathrm{W}_\mu^+ \mathrm{W}^{-,\mu}
\end{aligned}
$$

- Masses (mass terms) for the gauge bosons $\mathrm{W}^\pm$ and Z

w/o ϕ-W/Z interaction: d.o.f.		with ϕ-W/Z interaction: d.o.f.	
4 massless vector fields W^a, B:	8	3 massive vector fields $\mathrm{W}^\pm$, Z:	9
2 complex Higgs fields:	4	1 massless vector field A:	2
		1 massive scalar:	1
total number d.o.f.:	12	total number d.o.f.:	12

The Standard Model Higgs Mechanism

- Adding ϕ as $\mathrm{SU}(2)_L$ doublet with specific non-zero ground-state

$$
\begin{aligned}
\mathcal{L}_{\mathrm{Higgs}} = & \frac{1}{2}\left(\partial_\mu \mathrm{H}\right)\left(\partial^\mu \mathrm{H}\right) - \lambda v^2 \mathrm{H}^2 + \lambda v \mathrm{H}^3 - \frac{1}{4}\lambda \mathrm{H}^4 \\
& + \frac{1}{2} m_\mathrm{Z}^2 \mathrm{Z}_\mu \mathrm{Z}^\mu + \frac{m_\mathrm{Z}^2}{v} \mathrm{H} \mathrm{Z}_\mu \mathrm{Z}^\mu + \frac{1}{2}\frac{m_\mathrm{Z}^2}{v^2} \mathrm{H}^2 \mathrm{Z}_\mu \mathrm{Z}^\mu \\
& + m_\mathrm{W}^2 \mathrm{W}_\mu^+ \mathrm{W}^{-,\mu} + 2\frac{m_\mathrm{W}^2}{v} \mathrm{H} \mathrm{W}_\mu^+ \mathrm{W}^{-,\mu} + \frac{m_\mathrm{W}^2}{v^2} \mathrm{H}^2 \mathrm{W}_\mu^+ \mathrm{W}^{-,\mu}
\end{aligned}
$$

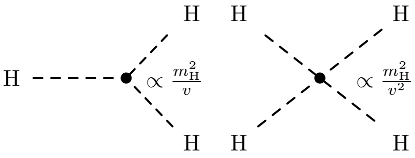
- Masses (mass terms) for the gauge bosons $\mathrm{W}^\pm$ and Z
- A massive scalar particle H (Higgs boson) with self-interaction

The Standard Model Higgs Mechanism

- Adding ϕ as $\mathrm{SU}(2)_L$ doublet with specific non-zero ground-state

$$
\begin{aligned}
\mathcal{L}_{\mathrm{Higgs}} = & \tfrac{1}{2}\left(\partial_\mu \mathrm{H}\right)\left(\partial^\mu \mathrm{H}\right) - \tfrac{1}{2} m_{\mathrm{H}} \mathrm{H}^2 + \frac{m_{\mathrm{H}}^2}{2v}\mathrm{H}^3 - \frac{m_{\mathrm{H}}^2}{8v^2}\mathrm{H}^4 \\
& + \tfrac{1}{2} m_{\mathrm{Z}}^2 \mathrm{Z}_\mu \mathrm{Z}^\mu + \frac{m_{\mathrm{Z}}^2}{v}\mathrm{H}\mathrm{Z}_\mu \mathrm{Z}^\mu + \frac{1}{2}\frac{m_{\mathrm{Z}}^2}{v^2}\mathrm{H}^2 \mathrm{Z}_\mu \mathrm{Z}^\mu \\
& + m_{\mathrm{W}}^2 \mathrm{W}_\mu^+ \mathrm{W}^{-,\mu} + 2\frac{m_{\mathrm{W}}^2}{v}\mathrm{H}\mathrm{W}_\mu^+ \mathrm{W}^{-,\mu} + \frac{m_{\mathrm{W}}^2}{v^2}\mathrm{H}^2 \mathrm{W}_\mu^+ \mathrm{W}^{-,\mu}
\end{aligned}
$$

- Masses (mass terms) for the gauge bosons $\mathrm{W}^\pm$ and Z
- A massive scalar particle H (Higgs boson) with self-interaction
  - Higgs-boson mass $m_{\mathrm{H}} = \sqrt{2\lambda v^2}$
  - Three-point Higgs-boson self-coupling
  - Four-point Higgs-boson self-coupling

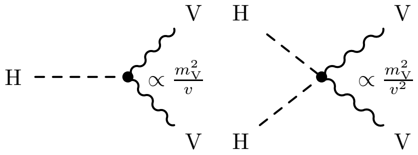


The Standard Model Higgs Mechanism

- Adding ϕ as $SU(2)_L$ doublet with specific non-zero ground-state

$$\begin{aligned}\mathcal{L}_{\text{Higgs}} = & \frac{1}{2}(\partial_\mu H)(\partial^\mu H) - \frac{1}{2} m_H H^2 + \frac{m_H^2}{2v} H^3 - \frac{m_H^2}{8v^2} H^4 \\ & + \frac{1}{2} m_Z^2 Z_\mu Z^\mu + \frac{m_Z^2}{v} H Z_\mu Z^\mu + \frac{1}{2}\frac{m_Z^2}{v^2} H^2 Z_\mu Z^\mu \\ & + m_W^2 W_\mu^+ W^{-,\mu} + 2\frac{m_W^2}{v} H W_\mu^+ W^{-,\mu} + \frac{m_W^2}{v^2} H^2 W_\mu^+ W^{-,\mu}\end{aligned}$$

- Masses (mass terms) for the gauge bosons $W^\pm$ and Z
- A massive scalar particle H (Higgs boson) with self-interaction
- Interactions of the Higgs boson with the $W^\pm$ and Z bosons
  - V-Higgs three-point interaction
  - V-Higgs four-point interaction



The Standard Model Higgs Mechanism

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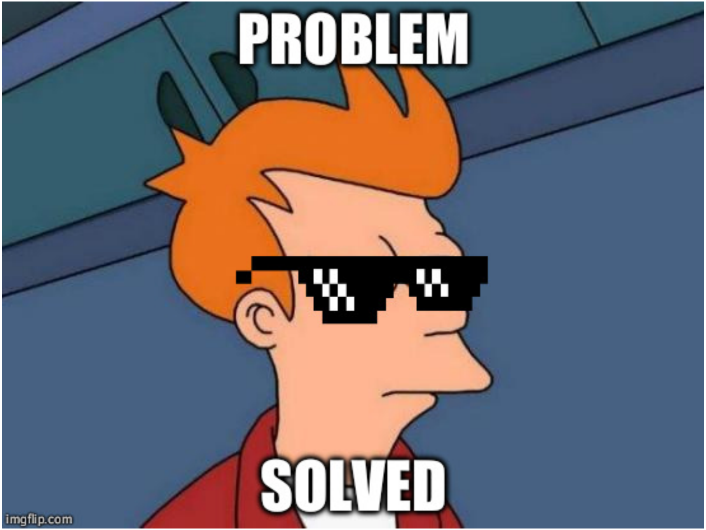
- Adding ϕ as $\mathrm{SU}(2)_L$ doublet with specific non-zero ground-state

$$\begin{aligned}\mathcal{L}_{\text{Higgs}} = & \tfrac{1}{2}\left(\partial_\mu \mathrm{H}\right)\left(\partial^\mu \mathrm{H}\right) - \tfrac{1}{2} m_{\mathrm{H}} \mathrm{H}^2 + \frac{m_{\mathrm{H}}^2}{2v}\mathrm{H}^3 - \frac{m_{\mathrm{H}}^2}{8v^2}\mathrm{H}^4 \\ & + \tfrac{1}{2} m_{\mathrm{Z}}^2 \mathrm{Z}_\mu \mathrm{Z}^\mu + \frac{m_{\mathrm{Z}}^2}{v}\mathrm{H}\mathrm{Z}_\mu\mathrm{Z}^\mu + \frac{1}{2}\frac{m_{\mathrm{Z}}^2}{v^2}\mathrm{H}^2\mathrm{Z}_\mu\mathrm{Z}^\mu \\ & + m_{\mathrm{W}}^2 \mathrm{W}_\mu^+ \mathrm{W}^{-,\mu} + 2\frac{m_{\mathrm{W}}^2}{v}\mathrm{H}\mathrm{W}_\mu^+\mathrm{W}^{-,\mu} + \frac{m_{\mathrm{W}}^2}{v^2}\mathrm{H}^2\mathrm{W}_\mu^+\mathrm{W}^{-,\mu}\end{aligned}$$

- **Masses (mass terms) for the gauge bosons $\mathrm{W}^\pm$ and Z**
- A massive scalar particle H (Higgs boson) with self-interaction
- Interactions of the Higgs boson with the $\mathrm{W}^\pm$ and Z bosons

Intermediate Summary

- Concept of **spontaneous symmetry breaking** (SSB)
  - Scalar field with specific potential: full Lagrangian has symmetry but energy ground-state is not
  - Applied to the Standard Model: the Englert–Brout–Higgs–Guralnik–Hagen–Kibble mechanism (1960s)
- **Allows masses of the elementary particles** without breaking local gauge invariance (but no prediction of the masses!)
  - Background (Higgs) field ϕ with SSB potential: non-zero vacuum expectation value v
  - Coupling of gauge bosons to ϕ (by covariant derivative) generates boson mass-terms $\propto v$ ('eat up' Goldstone bosons to gain mass)
- Predicts a **massive scalar particle (the Higgs boson)**
  - Coupling to gauge bosons depending on their masses
  - Additional self-interaction


PROBLEM
SOLVED
imgflip.com


WAIT A SECOND...
What about the fermion masses?
memegenerator.net

2.5.2. Yukawa couplings and fermion masses

Reminder: Problem of Massive Fermions

- $SU(2)_L \times U(1)_Y$ transformations act differently on chiral components
- Decomposition of mass term

$$m_f \overline{\psi}\psi = m_f \left(\overline{\psi}_R \psi_L + \overline{\psi}_L \psi_R \right)$$

- **Left- and right-handed components transform differently!**

$$\psi_L \to \psi'_L = e^{i\alpha^a \tau^a + i\alpha Y} \psi_L \quad \text{(component of isospin doublet, } I = \tfrac{1}{2}\text{)}$$

$$\psi_R \to \psi'_R = e^{i\alpha Y} \psi_R \qquad \text{(isospin singlet, } I = 0\text{)}$$

✗ Left- and right-handed fermions transform differently under $SU(2)_L \times U(1)_Y$

✗ **Fermion mass terms in chiral theory are not gauge invariant**

$\mathbf{SU(2)_L \times U(1)_Y}$ Invariant Fermion-Mass Term

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- Higgs field can also be used to generate mass terms for fermions!
- Terms as the following are gauge invariant under $SU(2)_L \times U(1)_Y$

$$\boxed{\mathcal{L}_{\text{Yukawa}} = -y_f \overline{\psi}_L \phi \psi_R - y_f \overline{\psi}_R \phi^\dagger \psi_L}$$

y_f: "**Yukawa coupling**"

Proof (see Exercises No. 2):

$$\begin{aligned}
\overline{\psi}_L \phi \psi_R \to & \left(\overline{\psi}_L A^\dagger_{Y_L} B^\dagger\right)\left(A_{Y_\phi} B\phi\right)\left(A_{Y_R}\psi_R\right) \\
&= A^\dagger_{Y_L} A_{Y_\phi} A_{Y_R} \overline{\psi}_L B^\dagger B \phi \psi_R \\
&= e^{i\frac{g'}{s}(-Y_L+Y_\phi+Y_R)\alpha(x)} \overline{\psi}_L \underbrace{B^\dagger B}_{=1} \phi\psi_R \\
&= e^{i\frac{g'}{s}(-(-1)+(+1)+(-2))\alpha(x)} \overline{\psi}_L \phi\psi_R \\
&= \overline{\psi}_L \phi \psi_R
\end{aligned}$$

... and analogously for $\overline{\psi}_R \phi^\dagger \psi_L$

Transformations:

$$U(1)_Y \;: A_Y \equiv e^{i\frac{g'}{2} Y\alpha(x)}$$

$$SU(2)_L : B \;\equiv e^{i\frac{g}{2}\tau^a \alpha_a(x)}$$

Hypercharges Y, e.g. for e:

e_L	-1
e_R	-2
ϕ	+1

$\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ Invariant Fermion-Mass Term

- Higgs field can also be used to generate mass terms for fermions!
- Terms as the following are gauge invariant under $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$

$$\boxed{\mathcal{L}_{\text{Yukawa}} = -y_f \overline{\psi}_L \phi \psi_R - y_f \overline{\psi}_R \phi^\dagger \psi_L}$$

y_f: "**Yukawa coupling**"

- Summary: under $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ transformations

Dirac mass terms	$m_f(\overline{\psi}_L \psi_R + \overline{\psi}_R \psi_L)$	break invariance
Yukawa mass terms	$y_f(\overline{\psi}_L \phi \psi_R + \overline{\psi}_R \phi^\dagger \psi_L)$	are invariant

Coupling to Higgs field restores gauge invariance!

. . . and how does this help?

Example: Electron Mass

- Expand ϕ around vacuum $|\phi_0| = \frac{v}{\sqrt{2}}$: $\phi \to \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + \mathrm{H} \end{pmatrix}$

$$\begin{aligned}
\mathcal{L}^{\text{electron}}_{\text{Yukawa}} &= -y_e \overline{\psi}_L \phi e_R - y_e \overline{e}_R \phi^\dagger \psi_L \\
&= -y_e \frac{1}{\sqrt{2}} \left[(\overline{\nu} \quad \overline{e})_L \begin{pmatrix} 0 \\ v + \mathrm{H} \end{pmatrix} e_R + \overline{e}_R (0 \quad v + \mathrm{H}) \begin{pmatrix} \nu \\ e \end{pmatrix}_L \right] \\
&= -\frac{y_e}{\sqrt{2}} (v + \mathrm{H}) [\underbrace{\overline{e}_L e_R + \overline{e}_R e_L}_{= \overline{e} e}] \\
&= -\underbrace{\frac{y_e}{\sqrt{2}}}_{e \text{ mass}} v \, \overline{e} e - \underbrace{\frac{y_e}{\sqrt{2}}}_{\mathrm{H}\overline{e}e \text{ interaction}} \mathrm{H} \overline{e} e \quad \equiv -m_e \overline{e} e - \frac{m_e}{v} \mathrm{H} \overline{e} e
\end{aligned}$$

- **Yukawa coupling of electron with Higgs field**
  $\to$ **electron-mass term** (cf. Dirac equation) in gauge-invariant way!
  - Electron mass: $m_e = \frac{y_e}{\sqrt{2}} v$
- In addition: **interaction of electron with Higgs boson** $\propto m_e$
- **No prediction of electron mass**: free parameter y_e

Fermion Masses

- $\overline{\psi}_L \phi \psi_R + \overline{\psi}_R \phi^\dagger \psi_L$: masses only for 'down'-type fermions
- Additional **term for 'up'-type fermions**:

$$\boxed{\overline{\psi}_L \phi^c \psi_R\,, \qquad \phi^c \equiv i\tau_2 \phi^* = \begin{pmatrix} \phi^{0*} \\ -\phi^{-*} \end{pmatrix}}$$

ϕ^c: charge conjugate of ϕ
$Y_{\phi^c} = -1$

- Conjugate ϕ^c transforms in same way as ϕ under $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$: above terms are gauge invariant

Fermion Masses

- $\overline{\psi}_L \phi \psi_R + \overline{\psi}_R \phi^\dagger \psi_L$: masses only for 'down'-type fermions
- Additional **term for 'up'-type fermions**:

$$\overline{\psi}_L \phi^c \psi_R \,, \qquad \phi^c \equiv i\tau_2 \phi^* \overset{\text{SSB}}{=} \frac{1}{\sqrt{2}} \begin{pmatrix} v \\ 0 \end{pmatrix}$$

ϕ^c: charge conjugate of ϕ

$Y_{\phi^c} = -1$

- **Fermion-mass terms** (without *h.c.* terms):

$$\text{d-type:} \quad -y_d (\overline{u}_L \quad \overline{d}_L) \phi d_R = -\frac{y_d}{\sqrt{2}} (\overline{u}_L \quad \overline{d}_L) \begin{pmatrix} 0 \\ v \end{pmatrix} d_R = -\frac{y_d}{\sqrt{2}} v \overline{d}_L d_R$$

$$\text{u-type:} \quad -y_d (\overline{u}_L \quad \overline{d}_L) \phi^c u_R = -\frac{y_d}{\sqrt{2}} (\overline{u}_L \quad \overline{d}_L) \begin{pmatrix} v \\ 0 \end{pmatrix} u_R = -\frac{y_d}{\sqrt{2}} v \overline{u}_L u_R$$

- $\mathcal{L}_{\text{Yukawa}}$ for generation i (massless neutrinos)

$$\mathcal{L}_{\text{Yukawa}} = -y_i^d \overline{Q}_{Li} \phi d_{Ri} - y_i^u \overline{Q}_{Li} \phi^c u_{Ri} - y_i^l \overline{L}_{Li} \phi l_{Ri} - h.c.$$

Quark Masses

- Most general case: **$y_i \rightarrow G_{ij}$ complex matrices**

$$\mathcal{L}^{\text{quarks}}_{\text{Yukawa}} = G_{ij}\overline{\psi}_{Li}\phi\psi_{Rj} = -G^d_{ij}\overline{Q}'_{Li}\phi d'_{Rj} - G^u_{ij}\overline{Q}'_{Li}\phi^c u'_{Rj} - h.c.$$

  - $d', \ldots$: states in **flavour (= SU(2)-interaction) base**

- For example, first term (after electroweak symmetry breaking)

$$G^d_{ij}\overline{Q}'_{Li}\phi d'_{Rj} = \frac{1}{\sqrt{2}}\left[\begin{pmatrix} G^d_{dd}\cdot\overline{(u \quad d)}'_L & G^d_{ds}\cdot\overline{(u \quad d)}'_L & G^d_{db}\cdot\overline{(u \quad d)}'_L \\ G^d_{sd}\cdot\overline{(c \quad s)}'_L & G^d_{ss}\cdot\overline{(c \quad s)}'_L & G^d_{sb}\cdot\overline{(c \quad s)}'_L \\ G^d_{bd}\cdot\overline{(t \quad b)}'_L & G^d_{bs}\cdot\overline{(t \quad b)}'_L & G^d_{bb}\cdot\overline{(t \quad b)}'_L \end{pmatrix}\begin{pmatrix} 0 \\ v \end{pmatrix}\right]\cdot\begin{pmatrix} d'_R \\ s'_R \\ b'_R \end{pmatrix}$$

- **Lagrangian** becomes

$$\begin{aligned}\mathcal{L}^{\text{quarks}}_{\text{Yukawa}} &= -\ G^d_{dd}\frac{v}{\sqrt{2}}\cdot\overline{d}'_L d'_R - G^d_{ds}\frac{v}{\sqrt{2}}\cdot\overline{d}'_L s'_R - \ldots - h.c.\\ &= -\ M^{d'}_{dd}\cdot\overline{d}'_L d'_R - M^{d'}_{ds}\cdot\overline{d}'_L s'_R - \ldots - h.c.\\ &= -\ \underbrace{M'_{dd}\cdot\overline{d}'d'}_{d\text{-quark mass}} - \underbrace{M'_{ds}\cdot\overline{d}'s'}_{?} - \ldots\end{aligned}$$

Quark Masses

- States with proper mass terms by **diagonalizing the mass matrices**

$$M^d = V_L^d M^{d'} V_R^{d\dagger} = \begin{pmatrix} m_d & 0 & 0 \\ 0 & m_s & 0 \\ 0 & 0 & m_b \end{pmatrix}, \quad M^u = V_L^u M^{u'} V_R^{u\dagger} = \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_c & 0 \\ 0 & 0 & m_t \end{pmatrix}$$

with unitary matrices V (i. e. $V^\dagger V = \mathbb{1}$)

$$\begin{aligned} \mathcal{L}_{\text{Yukawa}}^{\text{quarks}} &= - \quad \overline{d}'_{Li} \quad M_{ij}^{d'} \quad d'_{Rj} - \quad \overline{u}'_{Li} \quad M_{ij}^{u'} \quad u'_{Rj} \\ &= - \overline{d}'_{Li} V_L^{d\dagger} V_L^d M_{ij}^{d'} V_R^{d\dagger} \; V_R^d d'_{Rj} \; - \overline{u}'_{Li} V_L^{u\dagger} V_L^u M_{ij}^{u'} V_R^{u\dagger} \; V_R^u u'_{Rj} \end{aligned}$$

Quark Masses

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- States with proper mass terms by **diagonalizing the mass matrices**

$$M^d = V_L^d M^{d'} V_R^{d\dagger} = \begin{pmatrix} m_d & 0 & 0 \\ 0 & m_s & 0 \\ 0 & 0 & m_b \end{pmatrix} , \quad M^u = V_L^u M^{u'} V_R^{u\dagger} = \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_c & 0 \\ 0 & 0 & m_t \end{pmatrix}$$

with unitary matrices V (i. e. $V^\dagger V = \mathbb{1}$)

$$\begin{aligned} \mathcal{L}^{\text{quarks}}_{\text{Yukawa}} &= - \overline{d}'_{Li} M^{d'}_{ij} d'_{Rj} - \overline{u}'_{Li} M^{u'}_{ij} u'_{Rj} \\ &= - \underbrace{\overline{d}'_{Li} V_L^{d\dagger}} \underbrace{V_L^d M^{d'}_{ij} V_R^{d\dagger}} \underbrace{V_R^d d'_{Rj}} - \underbrace{\overline{u}'_{Li} V_L^{u\dagger}} \underbrace{V_L^u M^{u'}_{ij} V_R^{u\dagger}} \underbrace{V_R^u u'_{Rj}} \\ &= - \overline{d}_{Li} M^d_{ij} d_{Rj} - \overline{u}_{Li} M^u_{ij} u_{Rj} \end{aligned}$$

with **quark mass-eigenstates**

$$\boxed{\begin{aligned} d_{Li} &= (V_L^d)_{ij} d'_{Lj} & d_{Ri} &= (V_R^d)_{ij} d'_{Rj} \\ u_{Li} &= (V_L^u)_{ij} u'_{Lj} & u_{Ri} &= (V_R^u)_{ij} u'_{Rj} \end{aligned}}$$

SM Interactions in Quark Mass-Eigenstates

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- Electroweak interaction terms rewritten in SU(2)-interaction base

$$\mathcal{L}_{\text{EWK}} = i\overline{Q}'_L\gamma^\mu\left[\partial_\mu + i\frac{g}{2}\mathrm{W}^a_\mu\tau^a + i\frac{g'}{2}Y_L B_\mu\right]Q'_L + i\overline{q'}_R\gamma^\mu\left[\partial_\mu + i\frac{g'}{2}Y_L B_\mu\right]q'_R$$

$$= \quad \ldots \text{ see lecture 2}$$

$$= \quad i\overline{Q}'_L\gamma^\mu\partial_\mu Q'_L + i\overline{q}'_R\gamma^\mu\partial_\mu q'_R \qquad \longrightarrow \mathcal{L}_{\text{kin}}$$

$$+\ \overline{Q}'_L\gamma^\mu\mathrm{W}^\pm_\mu\tau^\pm Q'_L \qquad \longrightarrow \mathcal{L}_{\text{CC}}$$

$$+\ \overline{Q}'_L\gamma^\mu(c^Z_L Z_\mu, c^A A_\mu)Q'_L + \overline{q}'_R\gamma^\mu(c^Z_R Z_\mu, c^A A_\mu)q'_R \qquad \longrightarrow \mathcal{L}_{\text{NC}}$$

SM Interactions in Quark Mass-Eigenstates

(see Specialisations No. 1)

- Electroweak interaction terms rewritten in SU(2)-interaction base
- For $\mathcal{L}_{\text{NC}}$ (and similarly $\mathcal{L}_{\text{kin}}$), terms of the form

$$\overline{Q}'_L \gamma^\mu (Z_\mu, A_\mu) Q'_L = \overline{(u \;\; d)}'_{Li} \gamma^\mu (Z_\mu, A_\mu) \begin{pmatrix} u \\ d \end{pmatrix}'_{Li}$$

$$= \overline{u}'_{Li} \gamma^\mu (Z_\mu, A_\mu) u'_{Li} + \ldots$$

$$= \gamma^\mu (Z_\mu, A_\mu) \overline{u}'_{Li} u'_{Li} + \ldots$$

$$= \gamma^\mu (Z_\mu, A_\mu) \underbrace{\overline{u}_{Li} (V_L^u)_{ij}}_{\overline{u}'_{Li}} \underbrace{(V_L^{u\dagger})_{ij} u_{Lj}}_{u'_{Li}} + \ldots$$

SM Interactions in Quark Mass-Eigenstates

(see Specialisations No. 1)

- Electroweak interaction terms rewritten in SU(2)-interaction base
- For $\mathcal{L}_{\mathrm{NC}}$ (and similarly $\mathcal{L}_{\mathrm{kin}}$), terms of the form

$$\overline{Q}'_L \gamma^\mu (Z_\mu, A_\mu) Q'_L = \overline{(u \;\; d)}'_{Li} \gamma^\mu (Z_\mu, A_\mu) \begin{pmatrix} u \\ d \end{pmatrix}'_{Li}$$

$$= \overline{u}'_{Li} \gamma^\mu (Z_\mu, A_\mu) u'_{Li} + \ldots$$

$$= \gamma^\mu (Z_\mu, A_\mu) \overline{u}'_{Li} u'_{Li} + \ldots$$

$$= \gamma^\mu (Z_\mu, A_\mu) \overline{u}_{Li} \underbrace{(V^u_L V^{u\dagger}_L)_{ij}}_{\delta_{ij}} u_{Lj} + \ldots$$

Kinetic and NC interaction terms act on quark mass-eigentstates

SM Interactions in Quark Mass-Eigenstates

(see Specialisations No. 1)

- Electroweak interaction terms rewritten in SU(2)-interaction base
- For $\mathcal{L}_{\mathrm{CC}}$, e. g. W^+, terms of the form

$$\overline{Q}'_L \gamma^\mu \mathrm{W}^+_\mu \tau^+ Q'_L = \overline{(u \;\; d)}'_{Li} \gamma^\mu \mathrm{W}^+_\mu \tau^+ \begin{pmatrix} u \\ d \end{pmatrix}'_{Li}$$

SM Interactions in Quark Mass-Eigenstates

(see Specialisations No. 1)

- Electroweak interaction terms rewritten in SU(2)-interaction base
- For $\mathcal{L}_{\text{CC}}$, e. g. W^+, terms of the form

$$
\begin{aligned}
\overline{Q}'_L \gamma^\mu \text{W}^+_\mu \tau^+ Q'_L &= \gamma^\mu \text{W}^+_\mu \, \overline{u}'_{Li} d'_{Li} + \dots \\
&= \gamma^\mu \text{W}^+_\mu \, \underbrace{\overline{u}_{Li} (V^u_L)_{ij}}_{\overline{u}'_{Li}} \underbrace{(V^{d\dagger}_L)_{ij} d_{Lj}}_{d'_{Li}} + \dots \\
&= \gamma^\mu \text{W}^+_\mu \, \overline{u}_{Li} \underbrace{(V^u_L V^{d\dagger}_L)_{ij}}_{V^{ij}_{\text{CKM}}} d_{Lj} + \dots
\end{aligned}
$$

CC act on superposition of mass-eigentstates (**quark mixing**)

$V^u_L V^{d\dagger}_L = V_{\text{CKM}}$: Cabibbo-Kobayashi-Maskawa (CKM) matrix

Convention: V_{CKM} elements such that no mixing for u-type quarks: $u'_i = u_i$

Summary

- Higgs field can also help to obtain **mass terms for fermions**
- Gauge invariant **Yukawa coupling** terms between fermions and Higgs field
  - Couples left- and right-handed fermion components
  - **Allows fermion mass terms** (but does not predict their values)
- Leads in addition to **interaction between Higgs boson and fermions**
  - Coupling strength $\propto m_f$
- **Mixing of mass- and interaction eigentstates**: CKM matrix
  - CP violation, transitions between generations in charged-current interactions
  - Allows mixing but does not predict value of matrix elements

Note on Neutrino Masses

- Here, we have assumed massless neutrinos
  (as in original Standard Model formulation)
- **Mass terms for neutrinos can be added analogously** to the up-type quark case, adding also right-handed neutrinos to the fermion sector
  (this is assuming neutrinos are Dirac particles)
- In that case, additional matrix that mixes lepton flavour- and mass-eigenstates: **Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix**
  - By convention, interaction and mass eigenstates chosen to be the same for down-type states (charged leptons)

Illustration: © Johan Jarnestad/The Royal Swedish Academy of Sciences

- Both for CKM and PMNS matrices
  - **Origin of matrix** in **Yukawa coupling terms**
  - **Origin of observed values** of matrix elements (structure) **unknown!**

Summary on Masses

- Three mechanisms of mass generation in the Standard Model via coupling to the Higgs field

1. via **covariant derivative**	$\frac{1}{2}\frac{g^2}{4}v^2 W^{\pm}_{\mu} W^{\mu\pm}, \ \frac{1}{2}\frac{g^2+g'^2}{4}v^2 Z_{\mu} Z^{\mu}$
2. via **Yukawa coupling**	$\frac{y_f}{\sqrt{2}} v f \bar{f}$
3. via **Goldstone potential**	$\lambda v^2 H^2 = -\mu^2 H^2$

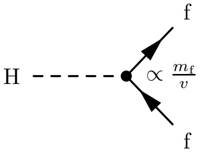
  - 1. and 2.: require non-vanishing vacuum expectation value v
  - 3.: would also yield mass for $v = 0$: in that case, $m_H = \mu^2 > 0$
- Full Lagrangian retains local gauge invariance

2.5.3. The Higgs boson

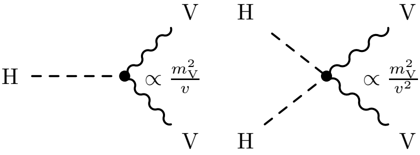
Higgs-boson couplings

Higgs-Boson Couplings

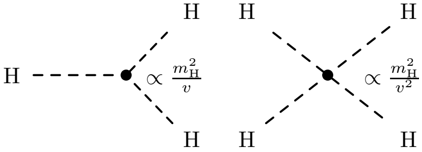
to fermions:



to massive gauge bosons $V = W^{\pm}, Z$:



self coupling:



- Coupling terms can be read-off from Lagrangian
  - H is indistinguishable particle: additional combinatorial factor to all amplitudes with more than 1 H field at vertex
- NB: decay width additionally depends on Higgs-boson mass (later)

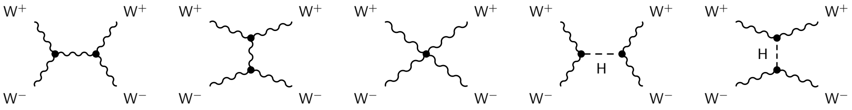
Summary

- **Consequence of the Higgs mechanism**: massive scalar particle "**Higgs boson**"
- Very specific **coupling** to gauge bosons and fermions (and self-interaction), **depending on particle masses**
  - Dominant coupling to heaviest particles
- **Only free parameter in SM Higgs sector: Higgs boson mass m_H**
- **As soon as m_H known: all Higgs-boson interactions determined!**

Theoretical Bounds on the Higgs-Boson Mass

Unitarity

- Several Standard Model scattering cross-sections violate unitarity, i. e. become divergent at large $\sqrt{s}$, e. g. WW $\rightarrow$ WW scattering

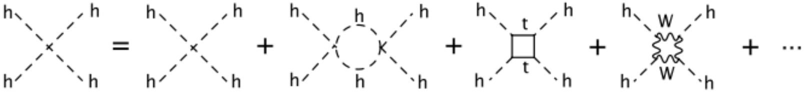


- Adding **contributions from a scalar particle (the Higgs boson) cancels divergencies**, $\sigma \rightarrow$ const for $\sqrt{s} \rightarrow \infty$

Cancellation of divergencies only if $m_{\mathrm{H}} \lesssim 700\,\mathrm{GeV}$
(otherwise perturbation theory not valid)

Extrapolation to Higher Scales

- Like the gauge-coupling constants, also the constants μ^2 and λ of the **Higgs potential** are **subject to higher-order corrections**
- Contributions from all SM particles that couple to the Higgs boson:



$\rightarrow$ **"running" of μ^2 and λ with energy scale Q**

$$V(\phi) = \mu^2(Q)|\phi|^2 + \lambda(Q)|\phi|^4 \,, \qquad m_\mathrm{H}^2 = m_\mathrm{H}^2(Q) = -2\mu^2(Q) = 2\lambda(Q)v^2$$

- **Can the SM (Higgs mechanism) be extrapolated to large scales?**
  - Does $V(\phi)$ behave properly?
  - Does $V(\phi)$ develop a minimum at non-zero $|\phi|$?

Behaviour of $V(\phi)$ at large field-values of $|\phi|$ relevant: **only λ relevant!**

Running of the Higgs Coupling Constant λ

- Running of λ given by renormalisation group equation

$$\frac{d\lambda}{d\ln Q^2} = \beta = \frac{3}{4\pi^2}\Big[\underbrace{\lambda^2}_{\text{Higgs}} + \underbrace{\tfrac{1}{2}\lambda y_t^2 - \tfrac{1}{4}y_t^4}_{\text{top quark}} - \underbrace{\tfrac{1}{8}\lambda(3g^2 + g'^2)}_{W^\pm,\,Z\text{ bosons}} + \ldots\Big]$$

with β function at 1-loop accuracy

- **Dominant non-Higgs contributions from processes involving top quarks due to large mass**
  - Large top-quark mass $\leftrightarrow$ large top-Higgs Yukawa coupling y_t
  - Top-Higgs coupling $\propto y_t/\sqrt{2}$
- Subdominant contributions from massive gauge bosons (neglected in the following)

Triviality Bound

- Running of λ given by renormalisation group equation

$$\frac{d\lambda}{d\ln Q^2} = \beta = \frac{3}{4\pi^2}\Big[\underbrace{\lambda^2}_{\text{Higgs}} + \underbrace{\tfrac{1}{2}\lambda y_t^2 - \tfrac{1}{4}y_t^4}_{\text{top quark}} - \underbrace{\tfrac{1}{8}\lambda(3g^2 + g'^2)}_{\text{W}^\pm,\text{ Z bosons}} + \ldots\Big]$$

- Case: large $\lambda \gg y_t, g, g'$ (= heavy Higgs boson since $m_\text{H}^2 = 2\lambda v^2$)
  - Higgs boson contribution dominates

$$\frac{d\lambda}{d\ln Q^2} \approx \frac{3}{4\pi^2}\lambda^2(Q^2) \quad\longrightarrow\quad \lambda(Q^2) = \frac{\lambda(v^2)}{1 - \frac{3}{4\pi^2}\lambda(v^2)\ln(\frac{Q^2}{v^2})}$$

- Relates value of λ at the EWK scale v to its value at a higher scale Q
- λ increases with Q until it hits pole

Require the SM to remain finite up to cut-off scale Λ
$\lambda(\Lambda^2) < \infty$: **upper limit** on $\lambda(v^2)$ and thus on Higgs-boson mass

Stability Bound

- Running of λ given by renormalisation group equation

$$\frac{d\lambda}{d\ln Q^2} = \beta = \frac{3}{4\pi^2}\Big[\underbrace{\lambda^2}_{\text{Higgs}} + \underbrace{\tfrac{1}{2}\lambda y_t^2 - \tfrac{1}{4}y_t^4}_{\text{top quark}} - \underbrace{\tfrac{1}{8}\lambda(3g^2 + g'^2)}_{W^\pm,\,Z\text{ bosons}} + \ldots\Big]$$

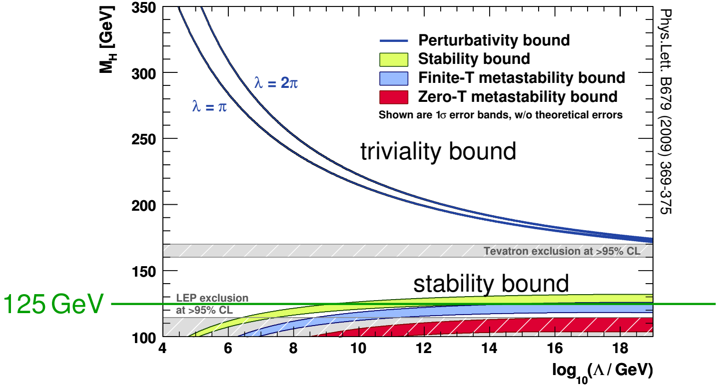
- Case: small $\lambda \ll y_t, g, g'$ (= light Higgs boson since $m_H^2 = 2\lambda v^2$)
  - Top-quark contribution dominates

$$\frac{d\lambda}{d\ln Q^2} \approx -\frac{3}{16\pi^2}y_t^4 \quad \longrightarrow \quad \lambda(Q^2) = \lambda(v^2) - \frac{3}{4\pi^2}\frac{m_t^4}{v^4}\ln\left(\frac{Q^2}{v^2}\right)$$

- Relates value of λ at the EWK scale v to its value at a higher scale Q
- λ decreases with Q until it becomes negative: instable vacuum not bound from below

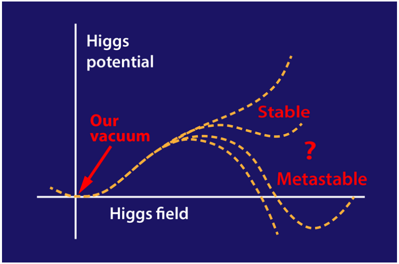
Require $V(\phi)$ to have minimum at finite $|\phi|$ up to cut-off scale Λ
$\lambda(\Lambda^2) > 0$: **lower limit** on $\lambda(v^2)$ and thus on Higgs-boson mass

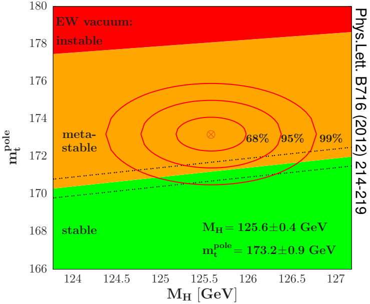
Intrinsic Bounds on Higgs-Boson Mass



- **Cut-off scale** Λ up to which Standard Model should be valid: **bounds on Higgs-boson mass**
- With $m_H = 125\,\text{GeV}$: SM in metastable vacuum up to Planck scale (where validity has to end because gravity becomes strong)

Intrinsic Bounds on Higgs-Boson Mass





- With $m_H = 125\,\text{GeV}$: **SM in metastable vacuum** up to Planck scale
  - Second minimum below SM vacuum due to higher-order contributions to the Higgs potential
  - Current state can tunnel into absolute minimum, but probability such that lifetime larger than age of the universe
- **Standard Model valid up to Planck Scale?**
  - Uncertainties due to **uncertainty on top-quark mass**

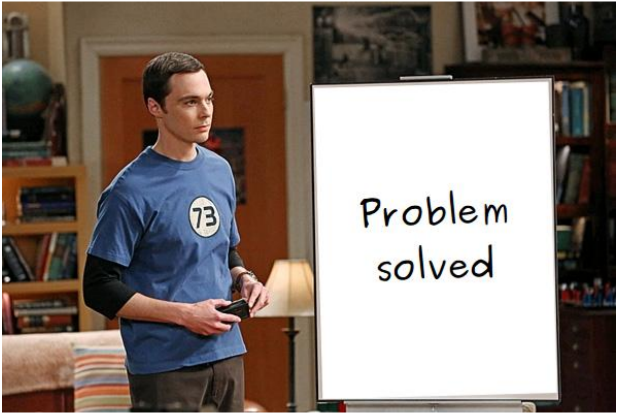
Summary

- **Higg-boson mass m_H not predicted** by the SM Higgs-mechanism
- But **intrinsic upper and lower bounds** from consistency arguments in running of Higgs self-coupling parameter λ with energy scale
  - **Perturbativity (triviality)**: upper bound
  - **Stability of vacuum**: lower bound
  - Bounds depend on energy scale up to which SM is assumed to be valid (appearance of new physics beyond the SM can change the picture)
- With $m_H = 125\,\text{GeV}$ and Standard Model valid up to Planck scale: **metastable vacuum**

Summary: The Higgs Mechanism

- No prediction of but **allows masses of the elementary particles** without breaking local gauge invariance
  - Higgs field ϕ with spontaneously-symmetry-breaking potential $\rightarrow$ non-zero vacuum expectation value v of ϕ
  - Coupling of gauge bosons to ϕ (by covariant derivative) generates boson mass-terms $\propto v$ ('eat up' Goldstone bosons to gain mass)
  - In addition: Yukawa coupling of fermions to ϕ generates fermion mass-terms $\propto v$ (and introduces freedom for mixing between fermion mass- and interaction-eigenstates)
- Predicts a **massive scalar particle (the Higgs boson)**
  - Coupling to fermions and bosons depending on their masses
  - Additional self-interaction
- Higgs sector **determined by Higgs-boson mass** (free parameter)

Summary: The Higgs Mechanism



Programme

Date	Room	Type	Topic
Wed Apr 24.	Kl. HS B	LE 01	1. Organisation and introduction: particle physics at colliders + W/Z/H history
Tue Apr 30.	30.23 11/12	—	*no class*
Wed May 01.	Kl. HS B	—	*no class*
Tue May 07.	30.23 11/12	LE 02	2.1 Gauge theory & 2.2 The electroweak sector of the SM I
Wed May 08.	Kl. HS B	LE 03, EX 01	2.3 Discovery of the W and Z bosons & EX gauge theories
Tue May 14.	30.23 11/12	LE 04	2.4 The Higgs mechanism
Wed May 15.	Kl. HS B	EX 02	Exercise "SM Higgs mechanism"
Tue May 21.	30.23 11/12	—	*no class*
→ Wed May 22.	Kl. HS B	LE 05	2.5 The electroweak sector of the SM II (Higgs mechanism + Yukawa couplings)
Tue May 28.	30.23 11/12	SP 01	Specialisation of 2.4 and 2.5
Wed May 29.	Kl. HS B	LE 06	3.1 From theory to observables & 3.2 Reconstruction + analysis of exp. data
Tue Jun 04.	30.23 11/12	EX 03	Exercise "Trigger efficiency measurement"
Wed Jun 05.	Kl. HS B	LE 07	3.3 Measurements in particle physics (part 1)
Tue Jun 11.	30.23 11/12	EX 04	Exercise on statistical methods
Wed Jun 12.	Kl. HS B	LE 08	3.3 Measurements in particle physics (part 2)
Tue Jun 18.	30.23 11/12	SP 02	Specialisation "Limit setting"
Wed Jun 19.	Kl. HS B	SP 03	Specialisation "Unfolding"
Tue Jun 25.	30.23 11/12	LE 09	4.1 Determination of SM parameters
Wed Jun 26.	Kl. HS B	LE 10	4.2 Measurement and role of W/Z bosons at the LHC
Tue Jul 02.	30.23 11/12	EX 05	Paper seminar "Z pole measurements"
Wed Jul 03.	Kl. HS B	LE 11	4.3 Processes with several W/Z bosons
Tue Jul 09.	30.23 11/12	EX 06	Paper seminar Higgs
Wed Jul 10.	Kl. HS B	LE 12	5.1 Discovery and first measurements of the Higgs boson
Tue Jul 16.	30.23 11/12	EX 07	Exercise "Machine learning in physics analysis"
Wed Jul 17.	Kl. HS B	LE 13	5.2 Measurement of couplings and kinematic properties
Tue Jul 23.	30.23 11/12	EX 08	Presentations: results of ML challenge
Wed Jul 24.	Kl. HS B	LE 14	5.3 Search for Higgs physics beyond the SM & 5.4 Future Higgs physics