Karlsruher Institut für Technologie

Teilchenphysik 2 — W/Z/Higgs an Collidern

Sommersemester 2019

Matthias Schröder und Roger Wolf | Vorlesung 6

INSTITUT FÜR EXPERIMENTELLE TEILCHENPHYSIK (ETP)

Summary SM Lagrangian [1. Generation Leptons]

$$\mathcal{L}_{\text{SM}} = \mathcal{L}_{\text{kin}}^{\text{electron}} + \mathcal{L}_{\text{IA}}^{\text{CC}} + \mathcal{L}_{\text{IA}}^{\text{NC}} + \mathcal{L}_{\text{kin}}^{\text{gauge}} + \mathcal{L}_{\text{kin}}^{\text{Higgs}} + \mathcal{L}_{V(\phi)}^{\text{Higgs}} + \mathcal{L}_{\text{Yukawa}}^{\text{Higgs}}$$

$$\mathcal{L}_{\text{kin}}^{\text{electron}} = i\overline{\nu}\gamma^{\mu}\partial_{\mu}\nu + i\overline{e}\gamma^{\mu}\partial_{\mu}e$$

$$\mathcal{L}_{\text{IA}}^{\text{CC}} = -\frac{q_e}{\sqrt{2}\sin\theta_W}\left[(\overline{\nu}\gamma^{\mu}e_L)\mathrm{W}_{\mu}^{+} + (\overline{e}_L\gamma^{\mu}\nu)\mathrm{W}_{\mu}^{-}\right]$$

$$\mathcal{L}_{\text{IA}}^{\text{NC}} = -\frac{q_e}{2\sin\theta_W\cos\theta_W}\left[(\overline{\nu}\gamma^{\mu}\nu) + (\overline{e}_L\gamma^{\mu}e_L)\right]\mathrm{Z}_{\mu} + e(\overline{e}\gamma^{\mu}e)\left[A_{\mu} + \tan\theta_W \mathrm{Z}_{\mu}\right]$$

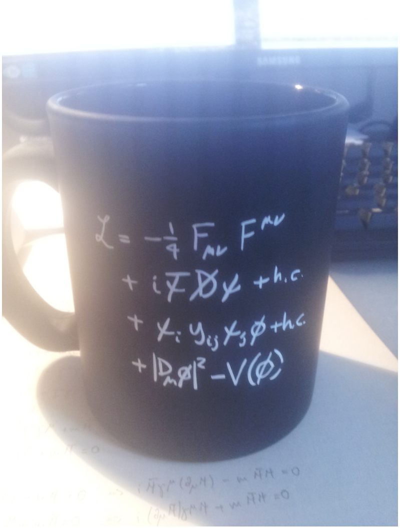
$$\mathcal{L}_{\text{kin}}^{\text{gauge}} = -\frac{1}{2}\mathrm{W}_{\mu\nu}^{a}\mathrm{W}^{a\mu\nu} - \frac{1}{2}\mathrm{B}_{\mu\nu}\mathrm{B}^{\mu\nu}$$

$$\begin{aligned}\mathcal{L}_{\text{kin}}^{\text{Higgs}} = {} & \frac{1}{2}(\partial_{\mu}\mathrm{H})(\partial^{\mu}\mathrm{H}) + m_{\mathrm{W}}^2\mathrm{W}_{\mu}^{+}\mathrm{W}^{-,\mu} + \frac{1}{2}m_{\mathrm{Z}}^2\mathrm{Z}_{\mu}\mathrm{Z}^{\mu} \\ & + 2\frac{m_{\mathrm{W}}^2}{v}\mathrm{H}\mathrm{W}_{\mu}^{+}\mathrm{W}^{-,\mu} + \frac{m_{\mathrm{Z}}^2}{v}\mathrm{H}\mathrm{Z}_{\mu}\mathrm{Z}^{\mu} + \frac{m_{\mathrm{W}}^2}{v^2}\mathrm{H}^2\mathrm{W}_{\mu}^{+}\mathrm{W}^{-,\mu} + \frac{1}{2}\frac{m_{\mathrm{Z}}^2}{v^2}\mathrm{H}^2\mathrm{Z}_{\mu}\mathrm{Z}^{\mu}\end{aligned}$$

$$\mathcal{L}_{V(\phi)}^{\text{Higgs}} = -\frac{1}{2}m_{\mathrm{H}}\mathrm{H}^2 + \frac{m_{\mathrm{H}}^2}{2v}\mathrm{H}^3 - \frac{m_{\mathrm{H}}^2}{8v^2}\mathrm{H}^4$$

$$\mathcal{L}_{\text{Yukawa}}^{\text{Higgs}} = -m_e\overline{e}e - \frac{m_e}{v}\mathrm{H}\overline{e}e$$

Summary SM Lagrangian

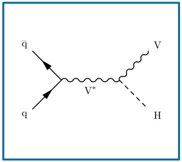


Programme

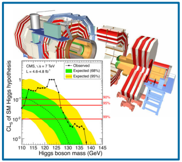
Basics of electroweak theory

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + i\bar{\Psi}\not{D}\psi + D_\mu\Phi^\dagger D^\mu\Phi - V(\Phi) + \bar{\Psi}_L\hat{Y}\Phi\Psi_R + h.c.$$

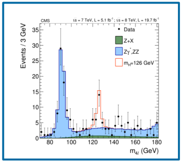
From theory to observables



Experimental techniques



Results and open questions



3. From Theory to Experiment (and Back)

3.1 From theory to observables
- Cross-section calculation: basic picture
- Fermion propagator and perturbation theory
- Scattering matrix and Feynman rules

3.2 Reconstruction of experimental data
- Reminder: accelerators and particle detectors
- Trigger
- Reconstruction of physics objects

3.3 Measurements in particle physics
- Basic tools (PDFs, Histograms, Likelihood)
- Parameter estimation
- Hypothesis testing
- Determination of physics properties (confidence intervals)
- Search for new physics (exclusion limits)

3.4 Experimental techniques
- Efficiency measurements
- Background estimation

3. From Theory to Experiment (and Back)

3.1 From theory to observables
- Cross-section calculation: basic picture
- Fermion propagator and perturbation theory
- Scattering matrix and Feynman rules

3.2 Reconstruction of experimental data
- Reminder: accelerators and particle detectors
- Trigger
- Reconstruction of physics objects

3.3 Measurements in particle physics
- Basic tools (PDFs, Histograms, Likelihood)
- Parameter estimation
- Hypothesis testing
- Determination of physics properties (confidence intervals)
- Search for new physics (exclusion limits)

3.4 Experimental techniques
- Efficiency measurements
- Background estimation

3.1 From theory to observables

From Theory to Observables

Lagrange density ("Lagrangian")

$$\mathcal{L} = \overline{\psi} i\gamma^\mu \partial_\mu \psi \; - m\overline{\psi}\psi$$

$\downarrow$ Euler–Lagrange eq. $\leftarrow$ from $\mathrm{d}\mathcal{S} = 0$

eq. of motion

$$i\gamma^\mu \partial_\mu \psi - m\psi = 0$$

$\downarrow$

quantum-mechanical state ψ (e. g. plane wave)

$\downarrow$

observables

3.1.1. Cross-section calculation: basic picture

Cross Section

- Measure of **transition rate** *initial* $\rightarrow$ *final* state for given process
- Follows from Fermi's golden rule:

$$\sigma = \frac{|\text{matrix element}|^2 \cdot \text{phase space}}{\text{flux of colliding particles}}$$

  - matrix element: probability amplitude, encodes process dynamics
  - phase space: number of available final states

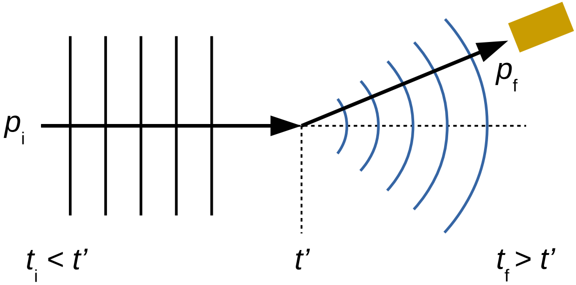
- **Link between**
  - **theory**: compute cross section
  - **experiment**: measure cross section

Model for Particle Scattering

Scattering matrix $\mathcal{S}$ transforms initial state ψ_i into scattering wave

$$\psi_{scat} = \mathcal{S} \cdot \psi_i$$

Observation: projection of plane wave ψ_f out of scattering wave ψ_{scat}



initial particle: plane wave ψ_i

localised potential A

spherical **scattering wave** ψ_{scat}

Transition **probability amplitude** $\mathcal{S}_{fi} = \psi_f^\dagger \cdot \psi_{scat} = \psi_f^\dagger \cdot \mathcal{S} \cdot \psi_i$

Example: QED

- Electron in electromagnetic field

$$\mathcal{L} = \overline{\psi} i\gamma^\mu \partial_\mu \psi - \underbrace{q\overline{\psi}\gamma^\mu A_\mu \psi}_{\text{covariant deriv.}} - \underbrace{m\overline{\psi}\psi}_{\text{Yukawa coupl.}}$$

$$\xrightarrow[\text{eqs.}]{\text{Euler–Lagrange}} \quad \boxed{(i\gamma^\mu \partial_\mu - m)\,\psi = q\gamma^\mu A_\mu \psi}$$

inhomogeneous Dirac eq.

- *Formal* **solution** of inhomogeneous Dirac equation

$$\psi(x) = \psi_0(x) + q \int d^4x' K(x, x')\gamma^\mu A_\mu(x')\psi(x')$$

with

Plane wave (free electron) ψ_0 : $(i\gamma^\mu \partial_\mu - m)\psi_0(x) = 0$

Green's function K : $(i\gamma^\mu \partial_\mu - m)K(x, x') = \delta^4(x - x')$

Solution of Inhomogeneous Dirac Equation

KIT – Karlsruher Institut für Technologie

- The function

$$\boxed{\psi(x) = \psi_0(x) + q \int d^4x' K(x,x') \gamma^\mu A_\mu(x') \psi(x')}$$

is a *formal* solution of the inhomogeneous Dirac equation:

$$\begin{aligned}(i\gamma^\mu \partial_\mu - m)\,\psi(x) &= \underbrace{(i\gamma^\mu \partial_\mu - m)\,\psi_0(x)}_{=0} \\ &\quad + q \int d^4x'\, \underbrace{(i\gamma^\mu \partial_\mu - m)\,K(x,x')}_{=\delta^4(x-x')} \gamma^\mu A_\mu(x') \psi(x') \\ &= q\gamma^\mu A_\mu(x)\psi(x) \quad \checkmark\end{aligned}$$

∂_μ acts on x, not on x'

But not a direct solution: ψ appears on LHS and RHS

Turns differential equation into **integral equation**:
propagates the solution from x' to x

3.1.2. Fermion propagator and perturbation theory

Green's Function K

- Best way to find Green's function K is via its **Fourier transform $\tilde{K}$**

$$K(x,x') \equiv K(x-x') = \frac{1}{(2\pi)^4} \int d^4p\, \tilde{\boldsymbol{K}}(p)\, e^{-ip(x-x')}$$

- Applying Dirac equation:

$$\underbrace{(i\gamma^\mu\partial_\mu - m)K(x-x')}_{} = \frac{1}{(2\pi)^4}\int d^4p\, \underbrace{(\gamma^\mu p_\mu - m)\tilde{K}(p)}_{}\; e^{-ip(x-x')}$$

$\|$ per definition $\qquad\qquad\qquad \|$

$$\delta^4(x-x') = \frac{1}{(2\pi)^4}\int d^4p \quad \mathbb{1}_4 \quad e^{-ip(x-x')}$$

Thus it is: $\quad (\gamma^\mu p_\mu - m)\tilde{K}(p) = \mathbb{1}_4$

Fermion Propagator

- Fourier transform of the Green's function is called **fermion propagator**

$$(\gamma^\mu p_\mu - m)\tilde{K}(p) = \mathbb{1}_4$$

$$(\gamma^\mu p_\mu + m) \cdot (\gamma^\mu p_\mu - m)\tilde{K}(p) = (\gamma^\mu p_\mu + m) \cdot \mathbb{1}_4$$

$$\boxed{\tilde{K}(p) = \frac{(\gamma^\mu p_\mu + m)}{p^2 - m^2}}$$

  - Fermion propagator is a 4×4 matrix, acts in the spinor space
  - Only defined for virtual fermions since $p^2 - m^2 = E^2 - \vec{p}^2 - m^2 \neq 0$

Green's Function $\leftrightarrow$ Fermion Propagator

Karlsruher Institut für Technologie

- The Green's function can be obtained from the propagator by an inverse Fourier transformation

$$K(x-x') = \frac{1}{(2\pi)^4} \int \mathrm{d}^3\vec{p}\, e^{-i\vec{p}(\vec{x}-\vec{x}')} \int_{-\infty}^{+\infty} \mathrm{d}p_0 \quad \frac{(\gamma^\mu p_\mu + m)}{p^2 - m^2} \quad e^{-ip_0(t-t')}$$

Green's Function $\leftrightarrow$ Fermion Propagator

- The Green's function can be obtained from the propagator by an inverse Fourier transformation

$$K(x-x') = \frac{1}{(2\pi)^4} \int d^3\vec{p}\, e^{-i\vec{p}(\vec{x}-\vec{x}')} \int_{-\infty}^{+\infty} dp_0 \, \frac{(\gamma^\mu p_\mu + m)}{(p_0 - E)(p_0 + E)} \, e^{-ip_0(t-t')}$$

$$\downarrow$$

$$E^2 = \vec{p}^2 + m^2$$

- $K(x-x')$ has 2 poles in the integration plane at $p_0 = \pm E$
- Can be solved with methods of *function theory* (see e. g. Schmüser)
- Correct expression for the **fermion propagator** (ϵ infinitesimal):

$$\tilde{K}(p) = \frac{(\gamma^\mu p_\mu + m)}{p^2 - m^2 + i\epsilon} \qquad \epsilon > 0$$

Green's Function

(see e. g. Schmüser)

- For $t > t'$ (*forward evolution*)

$$K(x - x') = \frac{-i}{(2\pi)^3} \int \mathrm{d}^3\vec{p}\, \frac{+\gamma^0 E - \vec{\gamma}\vec{p} + m}{2E}\, e^{-iE(t-t')+i\vec{p}(\vec{x}-\vec{x}')}$$

- For $t < t'$ (*backward evolution*)

$$K(x - x') = \frac{-i}{(2\pi)^3} \int \mathrm{d}^3\vec{p}\, \frac{-\gamma^0 E - \vec{\gamma}\vec{p} + m}{2E}\, e^{+iE(t-t')+i\vec{p}(\vec{x}-\vec{x}')}$$

Propagator and Time Evolution

(see e. g. Schmüser)

- K describes **time evolution** of free fermion
- **General solution** to Dirac equation

$$\psi(t,\vec{x}) = i\int \mathrm{d}^3\vec{x}'\; K(x-x')\gamma^0\psi(t',\vec{x}') \quad \text{for } t > t'$$

particle with $E > 0$ going forward in time

$$\overline{\psi}(t,\vec{x}) = i\int \mathrm{d}^3\vec{x}'\; \overline{\psi}(t',\vec{x}')\gamma^0 K(x-x') \quad \text{for } t > t'$$

particle with $E < 0$ going backward in time

Solution of Inhomogeneous Dirac Equation

- The function

$$\psi(x) = \psi_0(x) + q \int d^4x' K(x, x') \gamma^\mu A_\mu(x') \psi(x')$$

is a *formal* solution of the inhomogeneous Dirac equation

But not a direct solution: ψ appears on LHS and RHS

Turns differential equation into **integral equation**:
propagates the solution from x' to x

The Perturbative Series

- Integral equation can be **solved iteratively** by **expansion in coupling**

$$\boxed{\psi(x) = \psi_0(x) + q \int \mathrm{d}^4 x' K(x, x') \gamma^\mu A_\mu(x') \psi(x')}$$

$$\psi^{(0)}(x) = \psi_0(x)$$

- **0th order**: neglect external field (no scattering)

The Perturbative Series

- Integral equation can be **solved iteratively** by **expansion in coupling**

$$\boxed{\psi(x) = \psi_0(x) + q \int d^4x' K(x, x')\gamma^\mu A_\mu(x')\psi(x')}$$

$$\psi^{(0)}(x) = \psi_0(x)$$

$$\psi^{(1)}(x) = \psi^{(0)}(x) + q \int d^4x' K(x, x')\gamma^\mu A_\mu(x')\psi^{(0)}(x')$$

- **1st order**: assume $\psi^{(0)}(x)$ is close to actual solution

The Perturbative Series

- Integral equation can be **solved iteratively** by **expansion in coupling**

$$\boxed{\psi(x) = \psi_0(x) + q \int \mathrm{d}^4x' K(x,x')\gamma^\mu A_\mu(x')\psi(x')}$$

$$\psi^{(0)}(x) = \psi_0(x)$$

$$\psi^{(1)}(x) = \psi^{(0)}(x) + q \int \mathrm{d}^4x' K(x,x')\gamma^\mu A_\mu(x')\psi^{(0)}(x')$$

$$\psi^{(2)}(x) = \psi^{(0)}(x) + q \int \mathrm{d}^4x' K(x,x')\gamma^\mu A_\mu(x')\psi^{(1)}(x')$$

- **2nd order**: $\psi^{(1)}(x)$ as better approximation at RHS

The Perturbative Series

- Integral equation can be **solved iteratively** by **expansion in coupling**

$$\boxed{\psi(x) = \psi_0(x) + q \int d^4x' K(x,x')\gamma^\mu A_\mu(x')\psi(x')}$$

$$\begin{aligned}
\psi^{(0)}(x) &= \psi_0(x) \\
\psi^{(1)}(x) &= \psi^{(0)}(x) \\
&\quad + q \int d^4x' K(x,x')\gamma^\mu A_\mu(x')\psi^{(0)}(x') \\
\psi^{(2)}(x) &= \psi^{(0)}(x) \\
&\quad + q \int d^4x' K(x,x')\gamma^\mu A_\mu(x')\psi^{(0)}(x') \\
&\quad + q^2 \int\int d^4x' d^4x'' K(x,x')\gamma^\mu A_\mu(x') K(x',x'')\gamma^\nu A_\nu(x'')\psi^{(0)}(x'')
\end{aligned}$$

- **2nd order**: $\psi^{(1)}(x)$ as better approximation at RHS

The Perturbative Series

- Integral equation can be **solved iteratively** by **expansion in coupling**

$$\boxed{\psi(x) = \psi_0(x) + q \int \mathrm{d}^4 x' K(x,x')\gamma^\mu \mathsf{A}_\mu(x')\psi(x')}$$

$$\psi^{(0)}(x) = \psi_0(x)$$

$$\psi^{(1)}(x) = \psi^{(0)}(x) + q \int \mathrm{d}^4 x' K(x,x')\gamma^\mu \mathsf{A}_\mu(x')\psi^{(0)}(x')$$

$$\psi^{(2)}(x) = \psi^{(0)}(x) + q \int \mathrm{d}^4 x' K(x,x')\gamma^\mu \mathsf{A}_\mu(x')\psi^{(0)}(x') + q^2 \int\int \mathrm{d}^4 x' \mathrm{d}^4 x'' K(x,x')\gamma^\mu \mathsf{A}_\mu(x') K(x',x'')\gamma^\nu \mathsf{A}_\nu(x'')\psi^{(0)}(x'')$$

- Terms in $\psi^{(2)}(x)$ correspond to 0, 1, 2 scatterings at potential A_μ

The Perturbative Series

- Integral equation can be **solved iteratively** by **expansion in coupling**

$$\boxed{\psi(x) = \psi_0(x) + q\int \mathrm{d}^4x' K(x,x')\gamma^\mu A_\mu(x')\psi(x')}$$

$$\begin{aligned}
\psi^{(0)}(x) &= \psi_0(x) \\
\psi^{(1)}(x) &= \psi^{(0)}(x) \\
&\quad + q\int \mathrm{d}^4x' K(x,x')\gamma^\mu A_\mu(x')\psi^{(0)}(x') \\
\psi^{(2)}(x) &= \psi^{(0)}(x) \\
&\quad + q\int \mathrm{d}^4x' K(x,x')\gamma^\mu A_\mu(x')\psi^{(0)}(x') \\
&\quad + q^2\int\int \mathrm{d}^4x'\mathrm{d}^4x'' K(x,x')\gamma^\mu A_\mu(x')K(x',x'')\gamma^\nu A_\nu(x'')\psi^{(0)}(x'')
\end{aligned}$$

- RHS in inhomogeneous Dirac equation treated as small “perturbation”

The Perturbative Series

- Integral equation can be **solved iteratively** by **expansion in coupling**

$$\boxed{\psi(x) = \psi_0(x) + q \int \mathrm{d}^4x' K(x,x')\gamma^\mu \mathsf{A}_\mu(x')\psi(x')}$$

$$
\begin{aligned}
\psi^{(0)}(x) &= \psi_0(x) \\
\psi^{(1)}(x) &= \psi^{(0)}(x) \\
&\quad + q \int \mathrm{d}^4x' K(x,x')\gamma^\mu \mathsf{A}_\mu(x')\psi^{(0)}(x') \\
\psi^{(2)}(x) &= \psi^{(0)}(x) \\
&\quad + q \int \mathrm{d}^4x' K(x,x')\gamma^\mu \mathsf{A}_\mu(x')\psi^{(0)}(x') \\
&\quad + q^2 \int\int \mathrm{d}^4x'\mathrm{d}^4x'' K(x,x')\gamma^\mu \mathsf{A}_\mu(x')K(x',x'')\gamma^\nu \mathsf{A}_\nu(x'')\psi^{(0)}(x'')
\end{aligned}
$$

- Expansion in coupling justified since $\boxed{q = e = \sqrt{4\pi\alpha_{\mathrm{em}}} \ll 1}$

3.1.3. Scattering matrix and Feynman rules

The Matrix Element $\mathcal{S}_{\mathbf{fi}}$

- Scattering matrix $\mathcal{S}$ transforms initial state ψ_i into scattered state ψ_scat
- **Matrix element $\mathcal{S}_{\mathbf{fi}}$** ("scattering amplitude") given by projection of final state ψ_f out of ψ_scat

$$\begin{aligned} \mathcal{S}_\mathrm{fi} &= \int \mathrm{d}^3 x_\mathrm{f}\, \psi_\mathrm{f}^\dagger(x_\mathrm{f})\, \psi_\mathrm{scat}(x_\mathrm{f}) \\ &= \int \mathrm{d}^3 x_\mathrm{f}\, \psi_\mathrm{f}^\dagger(x_\mathrm{f}) \left[\psi_\mathrm{i}(x_\mathrm{f}) + q \int \mathrm{d}^4 x' K(x_\mathrm{f}, x')\gamma^\mu A_\mu(x')\psi_\mathrm{i}(x') + \ldots\right] \\ &= \delta_\mathrm{fi} + \mathcal{S}_\mathrm{fi}^{(1)} + \mathcal{S}_\mathrm{fi}^{(2)} + \ldots \end{aligned}$$

  - δ_fi: no scattering (undisturbed initial state)
  - $\mathcal{S}_\mathrm{fi}^{(1)}$: one scattering process, "leading order" (LO)
  - $\mathcal{S}_\mathrm{fi}^{(2)}$: two scattering processes, "next-to-leading order" (NLO)

LO Matrix-Element $\mathcal{S}_{\mathbf{fi}}^{(1)}$

Karlsruher Institut für Technologie

- Matrix element at 1st order perturbation theory

$$\mathcal{S}_{\text{fi}}^{(1)} = q \int \mathrm{d}^3 x_\text{f}\ \psi_\text{f}^\dagger(x_\text{f}) \int \mathrm{d}^4 x' K(x_\text{f}, x') \gamma^\mu A_\mu(x') \psi_\text{i}(x')$$

LO Matrix-Element $\mathcal{S}_{\mathbf{fi}}^{(1)}$

- Matrix element at 1st order perturbation theory

$$\mathcal{S}_{\text{fi}}^{(1)} = q \int \mathrm{d}^4x' \underbrace{\int \mathrm{d}^3x_\text{f}\, \psi_\text{f}^\dagger(x_\text{f}) K(x_\text{f}, x')}_{= -i\overline{\psi}_\text{f}(x')\ \text{(see 3.1.2)}} \gamma^\mu \mathrm{A}_\mu(x') \psi_\text{i}(x')$$

- Propagator $K(x_\text{f}, x')$ extrapolates the state ψ_f measured in the detector at $x_\text{f} = (t_\text{f}, \vec{x}_\text{f})$ back to the scattering target at $x' = (t', \vec{x}')$

$$\boxed{\mathcal{S}_{\text{fi}}^{(1)} = -iq \int \mathrm{d}^4x'\, \overline{\psi}_\text{f}(x') \gamma^\mu \psi_\text{i}(x') \mathrm{A}_\mu(x')}$$

LO Matrix-Element $\mathcal{S}_{\mathbf{fi}}^{(1)}$

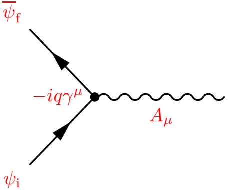
- Matrix element at 1st order perturbation theory

$$\mathcal{S}_{\mathrm{fi}}^{(1)} = q \int \mathrm{d}^4 x' \underbrace{\int \mathrm{d}^3 x_\mathrm{f}\, \psi_\mathrm{f}^\dagger(x_\mathrm{f}) K(x_\mathrm{f}, x')}_{=-i\overline{\psi}_\mathrm{f}(x')} \gamma^\mu A_\mu(x') \psi_\mathrm{i}(x')$$

- Propagator $K(x_\mathrm{f}, x')$ extrapolates the state ψ_f measured in the detector at $x_\mathrm{f} = (t_\mathrm{f}, \vec{x}_\mathrm{f})$ back to the scattering target at $x' = (t', \vec{x}')$

$$\mathcal{S}_{\mathrm{fi}}^{(1)} = -iq \int \mathrm{d}^4 x'\, \overline{\psi}_\mathrm{f}(x') \gamma^\mu \psi_\mathrm{i}(x') A_\mu(x')$$

Corresponds exactly to the IA term in $\mathcal{L}$ (see lecture 2)



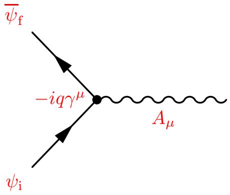
LO Matrix-Element $\mathcal{S}_{\mathbf{fi}}^{(1)}$

- Matrix element at 1st order perturbation theory

$$\mathcal{S}_{\text{fi}}^{(1)} = q \int \mathrm{d}^4 x' \underbrace{\int \mathrm{d}^3 x_{\text{f}}\, \psi_{\text{f}}^{\dagger}(x_{\text{f}}) K(x_{\text{f}}, x')}_{=-i\overline{\psi}_{\text{f}}(x')} \gamma^{\mu} \mathrm{A}_{\mu}(x') \psi_{\text{i}}(x')$$

- Propagator $K(x_{\text{f}}, x')$ extrapolates the state ψ_{f} measured in the detector at $x_{\text{f}} = (t_{\text{f}}, \vec{x}_{\text{f}})$ back to the scattering target at $x' = (t', \vec{x}')$

$$\mathcal{S}_{\text{fi}}^{(1)} = -iq \int \mathrm{d}^4 x'\, \overline{\psi}_{\text{f}}(x') \gamma^{\mu} \psi_{\text{i}}(x') \mathrm{A}_{\mu}(x')$$



- Incoming fermion ψ_{i}
- Outgoing fermion ψ_{f}
- Interaction with potential A_{μ} at x'
- LO matrix-element: sum of contributions at all x'

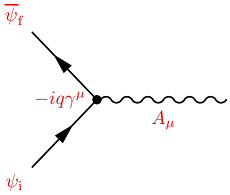
LO Matrix-Element $\mathcal{S}_{\mathbf{fi}}^{(1)}$

- Matrix element at 1st order perturbation theory

$$\mathcal{S}_{\text{fi}}^{(1)} = q \int \mathrm{d}^4x' \underbrace{\int \mathrm{d}^3x_\text{f}\, \psi_\text{f}^\dagger(x_\text{f}) K(x_\text{f}, x')}_{=-i\overline{\psi}_\text{f}(x')} \gamma^\mu \mathrm{A}_\mu(x') \psi_\text{i}(x')$$

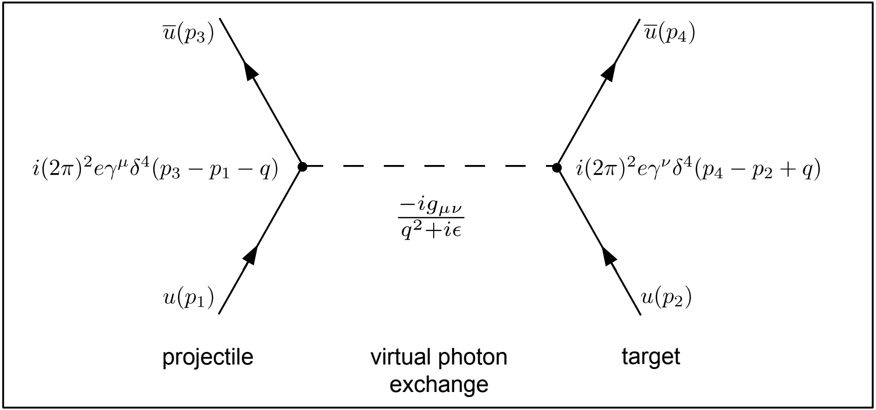
- Propagator $K(x_\text{f}, x')$ extrapolates the state ψ_f measured in the detector at $x_\text{f} = (t_\text{f}, \vec{x}_\text{f})$ back to the scattering target at $x' = (t', \vec{x}')$

$$\boxed{\mathcal{S}_{\text{fi}}^{(1)} = -iq \int \mathrm{d}^4x'\, \overline{\psi}_\text{f}(x') \gamma^\mu \psi_\text{i}(x') \mathrm{A}_\mu(x')}$$



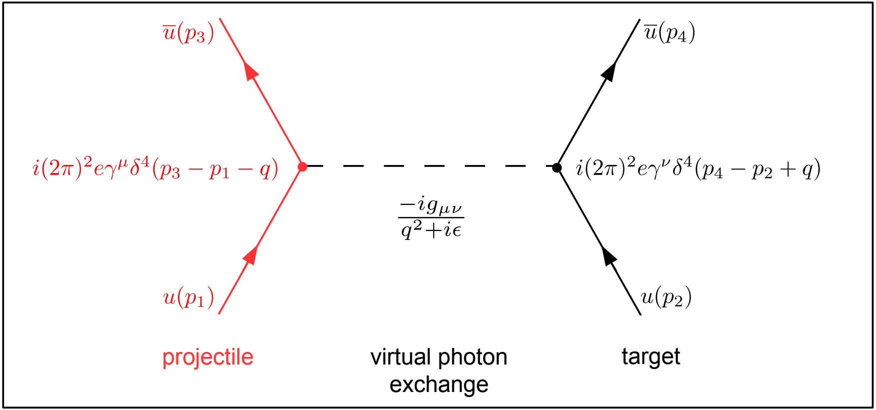
- But **also $\mathbf{A}_\mu$ evolves**: photon is back-scattered
  - Evolution according to inhomogeneous wave equation $\Box \mathrm{A}_\mu = J_\mu$ (Lorentz gauge)
  - Solution via Green's function: **photon propagator**

Fermion-Fermion Scattering (LO)



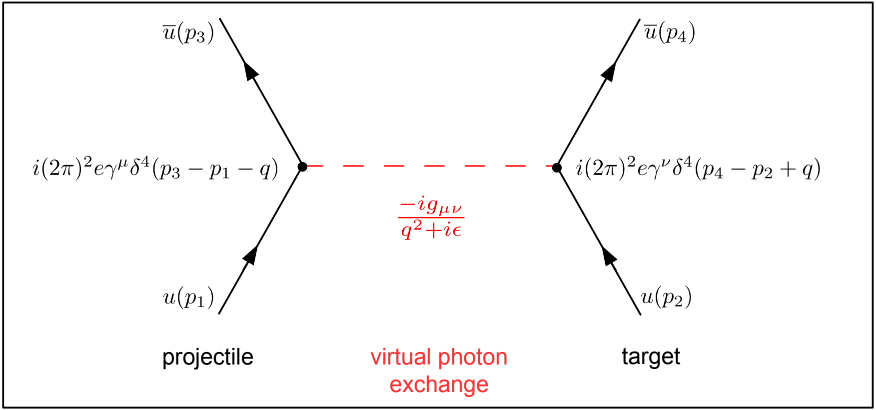
$$\mathcal{S}_{\mathrm{fi}}^{(1)} = i(4\pi^2 q)^2 \int \mathrm{d}^4 q \; \delta^4(p_3 - p_1 - q)\overline{u}(p_3)\gamma^\mu u(p_1) \; \frac{-g_{\mu\nu}}{q^2 + i\epsilon} \; \delta^4(p_4 - p_2 + q)\overline{u}(p_4)\gamma^\nu u(p_2)$$

Fermion-Fermion Scattering (LO)



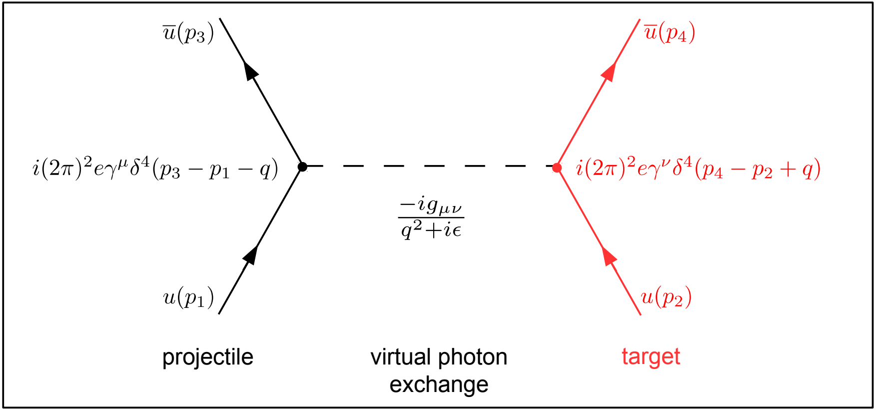
$$\mathcal{S}_{\text{fi}}^{(1)} = i(4\pi^2 q)^2 \int \mathrm{d}^4 q \, \delta^4(p_3 - p_1 - q)\overline{u}(p_3)\gamma^\mu u(p_1) \frac{-g_{\mu\nu}}{q^2 + i\epsilon} \, \delta^4(p_4 - p_2 + q)\overline{u}(p_4)\gamma^\nu u(p_2)$$

Fermion-Fermion Scattering (LO)



$$\mathcal{S}_{\rm fi}^{(1)} = i(4\pi^2 q)^2 \int \mathrm{d}^4 q \; \delta^4(p_3 - p_1 - q)\overline{u}(p_3)\gamma^\mu u(p_1) \frac{-g_{\mu\nu}}{q^2 + i\epsilon} \; \delta^4(p_4 - p_2 + q)\overline{u}(p_4)\gamma^\nu u(p_2)$$

Fermion-Fermion Scattering (LO)



$$\mathcal{S}_{\text{fi}}^{(1)} = i(4\pi^2 q)^2 \int \mathrm{d}^4 q\ \delta^4(p_3 - p_1 - q)\overline{u}(p_3)\gamma^\mu u(p_1);\ \frac{-g_{\mu\nu}}{q^2 + i\epsilon}\delta^4(p_4 - p_2 + q)\overline{u}(p_4)\gamma^\nu u(p_2)$$

Feynman Rules (QED)

Matrix element calculation can be represented with Feynman diagrams

Legs:		
	$u(p) \qquad (\overline{u}(p))$	• Incoming (outgoing) fermion.
	$\epsilon_\mu(k) \qquad (\epsilon^*_\mu(k))$	• Incoming (outgoing) photon.
Vertices:		
	$i(2\pi)^2 e\gamma^\mu \cdot \delta^4(p_f - p_i - q)$	• Lepton-photon vertex.
Propagators:		
	$\frac{i(\gamma^\mu p_\mu + m)}{p^2 - m^2 + i\epsilon}$	• Fermion propagator.
	$\frac{-ig^{\mu\nu}}{q^2 + i\epsilon}$	• Photon propagator.
Four-momenta of all virtual particles have to be integrated out.		

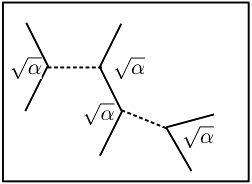
Higher Orders

- Scattering amplitude $\mathcal{S}_{\text{fi}}$ only known in perturbation theory
- Works **the better the smaller the perturbation** is
  - QED: $\alpha_{\text{em}} \approx \frac{1}{137}$
  - EWK: $\alpha_{\text{weak}} = \frac{\alpha_{\text{em}}}{\sin 2\theta_W} \approx 4\alpha_{\text{em}}$
  - QCD: $\alpha_{\text{s}}(m_{\text{Z}}) \approx 0.12$
- If well in perturbative regime, first-order contribution already sufficient to describe main features of scattering process
  - Contribution of order "α"
  - Called "leading order", "tree level", or "Born level"

Higher Orders

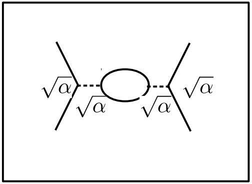
- So far, discussed contributions to $\mathcal{S}_{\mathrm{fi}}$ at order α^1
  - e. g. LO $e^-e^- \rightarrow e^-e^-$ scattering
- Contributions **at order $\boldsymbol{\alpha^2}$**:

Additional legs:



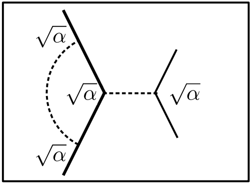
NLO contribution to the $2 \rightarrow 2$ process
LO contribution to a $2 \rightarrow 4$ process
$\rightarrow$ **Opens phase space**

Loops:



(in propagators or legs)

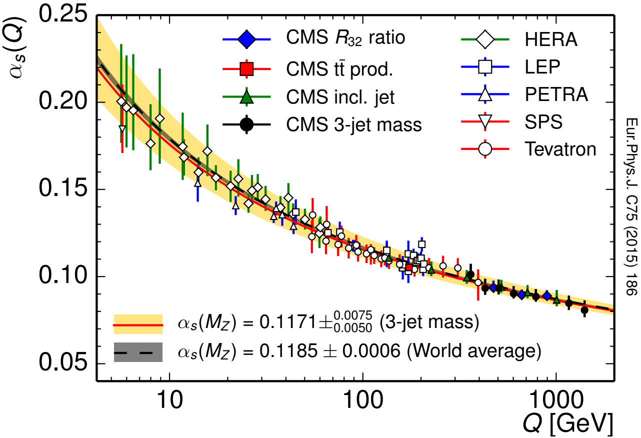
Modifies (effective) masses of particles
$\rightarrow$ **Running masses**



(in vertices)

Modifies (effective) couplings of particles
$\rightarrow$ **Running couplings**

Examples for Running Constants



- Running of the constants can be observed
- Value needs to be measured at one scale
  $\rightarrow$ evolution predicted (DGLAP equations)

Effect of Higher Order Corrections

- **Change of overall normalisation** of cross sections
  - Change of coupling and kinematic opening of phase space
- **Change of kinematic distributions**
  - For example, softer or harder p_T spectrum of final-state particles
- Effects (correction to LO) usually
  - **"small" in QED** $\mathcal{O}(1\,\%)$
  - **"large" in QCD** $\mathcal{O}(10\,\%)$: reliable predictions often require (N)NLO
- Higher order corrections can be mixed, e. g. $\mathcal{O}(\alpha_\mathrm{em}\alpha_\mathrm{s}^2)$
- Challenge: **number of diagrams quickly explodes** for higher-order calculations

Summary

- **Cross section** link between theory and experiment
- Computed from quantum-mechanical **amplitude of scattering process**: obtained from perturbation theory
  - **Propagator** as formal solution of equation of motions
  - Amplitude **expanded in coupling constant**: each term corresponds to distinct process
- **Feynman rules**: compact recipe to compute amplitudes