Teilchenphysik 2 — W/Z/Higgs an Collidern

Sommersemester 2019

Matthias Schröder und Roger Wolf | Vorlesung 9

INSTITUT FÜR EXPERIMENTELLE TEILCHENPHYSIK (ETP)

Termine

Date	Room	Type	Topic
Wed Apr 24.	Kl. HS B	LE 01	1. Organisation and introduction: particle physics at colliders + W/Z/H history
Tue Apr 30.	—	—	*no class*
Wed May 01.	—	—	*no class*
Tue May 07.	30.23 11/12	LE 02	2.1 Gauge theory & 2.2 The electroweak sector of the SM I
Wed May 08.	Kl. HS B	LE 03, EX 01	2.3 Discovery of the W and Z bosons & EX gauge theories
Tue May 14.	30.23 11/12	LE 04	2.4 The Higgs mechanism
Wed May 15.	Kl. HS B	EX 02	Exercise “SM Higgs mechanism”
Tue May 21.	—	—	*no class*
Wed May 22.	Kl. HS B	LE 05	2.5 The electroweak sector of the SM II (Higgs mechanism + Yukawa couplings)
Tue May 28.	30.23 11/12	SP 01	Specialisation of 2.4 and 2.5
Wed May 29.	Kl. HS B	LE 06	3.1 From theory to observables & 3.2 Reconstruction + analysis of exp. data
Tue Jun 04.	30.23 11/12	EX 03	Exercise “Trigger efficiency measurement”
Wed Jun 05.	Kl. HS B	LE 07	3.3 Measurements in particle physics (part 1)
Tue Jun 11.	30.23 11/12	EX 04	Exercise on statistical methods
Wed Jun 12.	Kl. HS B	LE 08	3.3 Measurements in particle physics (part 2)
Tue Jun 18.	30.23 11/12	SP 02	Specialisation “Limit setting”
Wed Jun 19.	Kl. HS B	LE 09	4.1 Determination of SM parameters
Tue Jun 25.	30.23 11/12	SP 03	Specialisation “Unfolding”
Wed Jun 26.	—	—	*no class*
Tue Jul 02.	30.23 11/12	EX 05	Paper seminar “Z pole measurements”
Wed Jul 03.	Kl. HS B	LE 10	4.2 W/Z bosons at the LHC & 4.3 Processes with several W/Z bosons
Tue Jul 09.	30.23 11/12	EX 06	Paper seminar Higgs
Wed Jul 10.	Kl. HS B	LE 11	5.1 Discovery and first measurements of the Higgs boson
Tue Jul 16.	30.23 11/12	EX 07	Exercise “Machine learning in physics analysis”
Wed Jul 17.	Kl. HS B	LE 12	5.2 Measurement of couplings and kinematic properties
Tue Jul 23.	30.23 11/12	EX 08	Presentations: results of ML challenge
Wed Jul 24.	Kl. HS B	LE 13	5.3 Search for Higgs physics beyond the SM & 5.4 Future Higgs physics

Laptops for Exercises/Specialisations

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Laptops for Exercises/Specialisations

The exercise will be a computer exercise, and it will be done during class time ("Präsenzübung"). The exercise runs standalone on a ROOT input file. *Please bring a laptop and make sure* ***beforehand*** *that there is a working installation of a recent ROOT6 and Python 2 version* (the exercise has been tested with **ROOT version 6.1.3.08** and Python version 2.7.6). It is encouraged that you work in small groups of up to three persons, and it is sufficient to have one laptop per group.

3. From Theory to Experiment (and Back)

3.1 From theory to observables
  - ○ Cross-section calculation: basic picture
  - ○ Fermion propagator and perturbation theory
  - ○ Scattering matrix and Feynman rules

3.2 Reconstruction of experimental data
  - ○ Reminder: accelerators and particle detectors
  - ○ Trigger
  - ○ Reconstruction of physics objects

3.3 Measurements in particle physics
  - ○ Parameter estimation
  - ○ Hypothesis testing
  - ○ Search for new physics (exclusion limits)

3.4 Monte Carlo simulation

3.3.3. Search for new physics (exclusion limits)

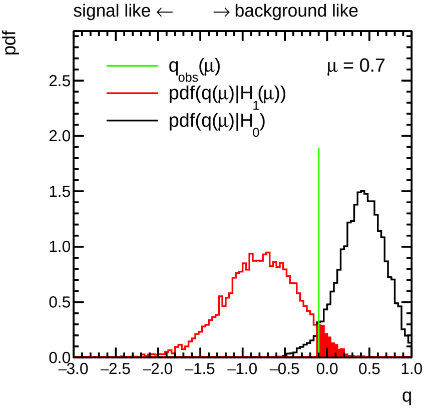
Excluding Parameters

- Assume measurement with a given sensitivity: no signal observed
- How much signal can “hide” in the bkg. fluctuations (+uncertainty)?
- **How large could a signal be at most?**

(Observed) Upper Limit: Interpretation

- “Maximal signal that we would still reject”
- 95 % C.L. upper limit on μ: largest value of μ that would still be rejected in a test with significance 5% given the data
  - NB: limit is a **function of the data** (depends on q_{obs})!
- μ for which $\text{CL}_{\text{s+b}} = 0.05$:

$$\boxed{0.05 = \int_{q_{\text{obs}}}^{\infty} \mathrm{d}q\, \mathcal{P}(q(\mu_{95})|H_1)}$$

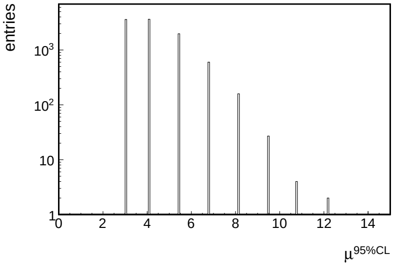


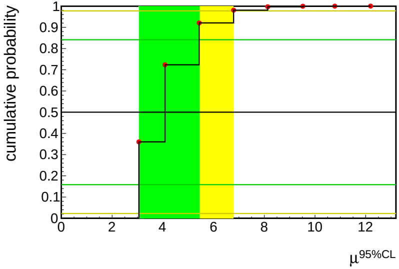
- Upper **limit covers true value ($\mu_{\text{true}} < \mu_{95}$) with probability C.L. $=$ 95 %**
  - If the experiment is repeated many times, μ_{95} would be larger than μ_{true} in 95 % of the cases
- Still **5 % chance of wrong exclusion**, i. e. that $\mu_{\text{true}} > \mu_{95}$

Expected Limit

- **Estimate what observed limit would look like in case of no signal**
- Obtained e. g. from *toy dataset*
  - Sample toy data for q under background-only hypothesis from $\mathcal{P}(q|H_0)$
  - Treat each as observation and compute μ_{95} limit
  - Obtain quantiles from distribution of all μ_{95}
- **Expected limit = median of μ_{95} distribution**
  - 16 and 84% quantiles: 68% confidence interval
  - 2.5 and 97.5% quantiles: 95% confidence interval

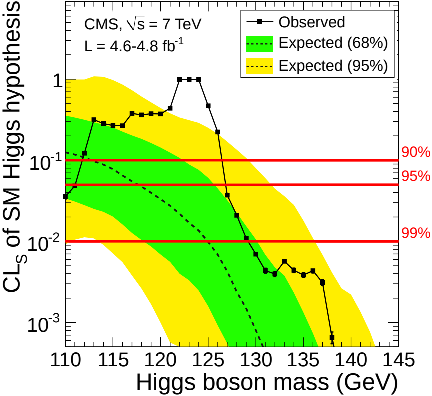
    $\rightarrow$ “Brazilian band” plots





Before the Higgs-Boson Discovery

- Combination of Higgs-boson search results by CMS [Phys.Lett. B710 (2012) 26]



- Tested hypotheses
  - H_0: no Higgs boson
  - H_1: SM Higgs boson
- Test statistic q evaluated for SM Higgs boson of different mass ($\mu = 1$ in each case)

- Excluding a SM Higgs boson at 95% CL with masses
  - $m_H > 118$ GeV expected (from toy data under H_0)
  - $m_H > 127$ GeV observed (from real data)

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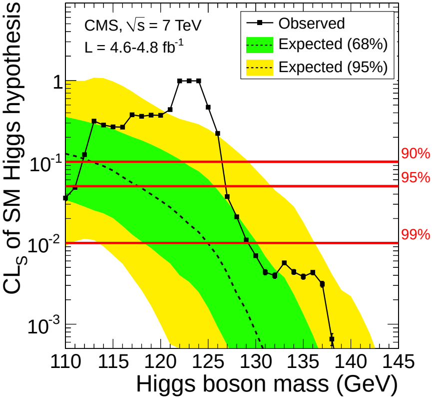

- Tested hypotheses
  - H_0: no Higgs boson
  - H_1: SM Higgs boson
- Test statistic q evaluated for SM Higgs boson of different mass ($\mu = 1$ in each case)

- Observed exclusion weaker than expected (smaller mass range)
- Around $m_H = 125\,\text{GeV}$, q_obs differs significantly from expectation: **indication that H_0 is wrong!**

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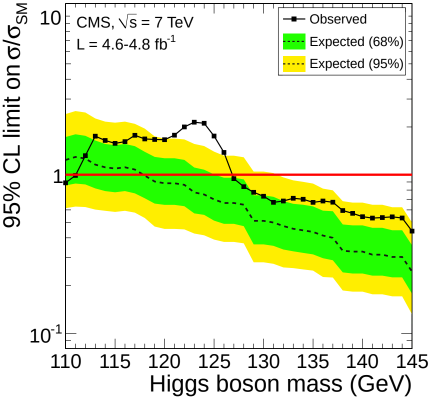

- Tested hypotheses
  - H_0: no Higgs boson
  - H_1: a Higgs boson
- Test statistic evaluated for a Higgs boson of different mass and variable signal strength
  - Signal strength μ can vary in each case (not SM any more!)

- Exclusion limits on μ for different masses at 95 % C.L.

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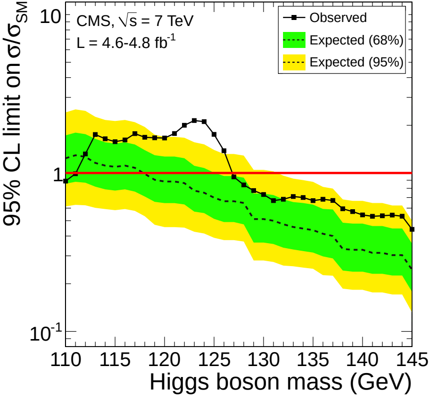

- Tested hypotheses
  - H_0: no Higgs boson
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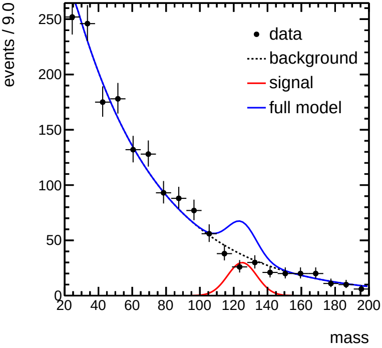
- Test statistic evaluated for a Higgs boson of different mass and variable signal strength
  - Signal strength μ can vary in each case (not SM any more!)

- Striking: observed limit weaker than expected around $m_H = 125\,\text{GeV}$
- Difference (locally) beyond 2σ: **indication that H_0 is wrong!**

What If the Data Trick Us?

- Suppose **data fluctuates low**, sizably below backgr. expectation
- **CL_{s+b}: artificially strong limit** on signal, i. e. $\mu_{1-\alpha}$ is small



- In extreme case, $\mu_{1-\alpha} \to 0$, i. e. exclude signal entirely
- **Not desirable**: just downward fluctuation of the data!
- Often **problem: searches in extreme phase-space regions** with few background events

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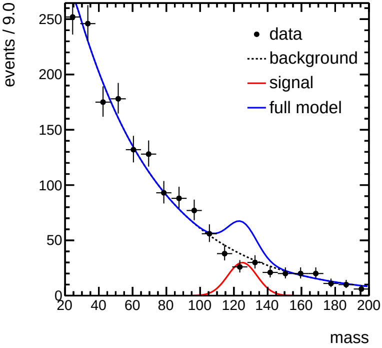

CL_s method

- Compute both

$$\mathrm{CL}_{s+b} = \int_{q_{obs}}^{\infty} \mathrm{d}q\, \mathcal{P}(q(\mu)|H_1)$$

$$\mathrm{CL}_{b} \equiv \int_{q_{obs}}^{\infty} \mathrm{d}q\, \mathcal{P}(q(\mu)|H_0)$$

- Define limit as that μ for which

$$\boxed{\mathrm{CL}_s \equiv \frac{\mathrm{CL}_{s+b}}{\mathrm{CL}_b} = \alpha}$$

normalise CL_{s+b} to "bkg-only p-value"

What If the Data Trick Us?

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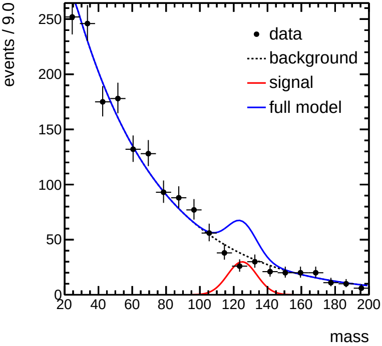

$\mathbf{CL_s}$ method

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- Define limit as that μ for which

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- In case of extreme under fluctuation: $\mathrm{CL_s} \to 1$
- H_1 not excluded by mistake – but also weaker limit in case of no signal

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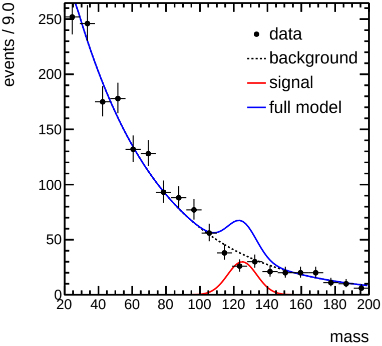

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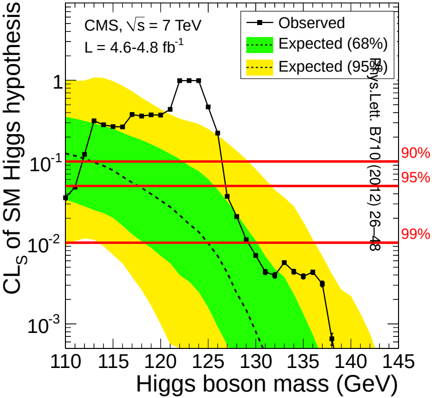
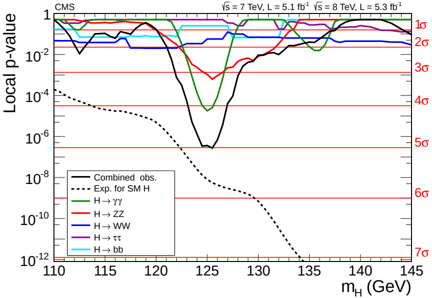
$\mathrm{CL_s}$ protects from fluctuations in the data at cost of lower sensitivity
(procedure used in LHC (Higgs boson) searches)

Et Voilà

December 2011



July 2012

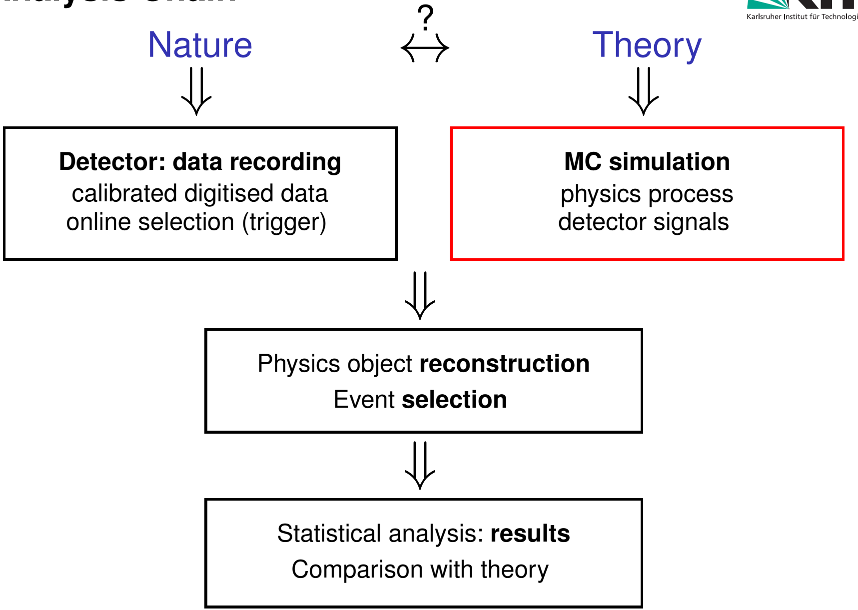


In the following lectures, will frequently see plots like these

Summary

- **Statistical analysis crucial tool** in particle physics
- Does not tell probability of a certain model (at least not without further assumptions) but allows
  - quantifying the compatibility of the data with a tested model, e. g. via p **value or significance**
  - determining the parameter values of a given model that describe the data best (estimators), e. g. **via maximum-likelihood fit**
- In practice often **comparison of two alternative hypotheses** H_0 and H_1, e. g. background-only and signal+background
  - Rules when to reject H_0 in favour of H_1, which allow to quantify type-I and II errors
  - Test statistic combines information of multi-channel data into one single number for application in hypothesis testing
  - **Likelihood ratio is most powerful test statistic**
- Exclusion **limits provide information on model parameter in case no signal found**

Analysis Chain



Nature $\overset{?}{\longleftrightarrow}$ Theory

Detector: data recording
calibrated digitised data
online selection (trigger)

MC simulation
physics process
detector signals

Physics object **reconstruction**
Event **selection**

Statistical analysis: **results**
Comparison with theory

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  - Cross-section calculation: basic picture
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  - Trigger
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  - Parameter estimation
  - Hypothesis testing
  - Search for new physics (exclusion limits)

3.4 Monte Carlo simulation

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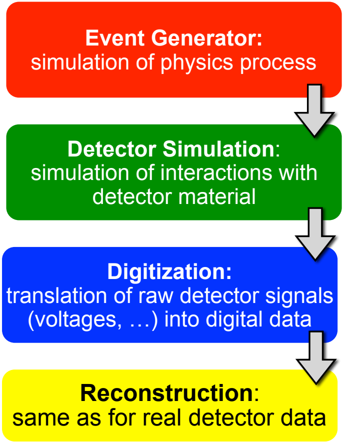
MC Simulations in Particle Physics

- Goal: **comprehensive simulation** of collision events based on best knowledge of all physics processes (collision events and interactions in detector)
- Main tools based on **Monte Carlo (MC) method**
  - In general: **numerical** techniqus to compute **probabilities** using **random numbers**
  - Excellent tool to generate **physics events** (probabilistic theory) and simulate **particle interactions** in detectors



Event Generator:
simulation of physics process

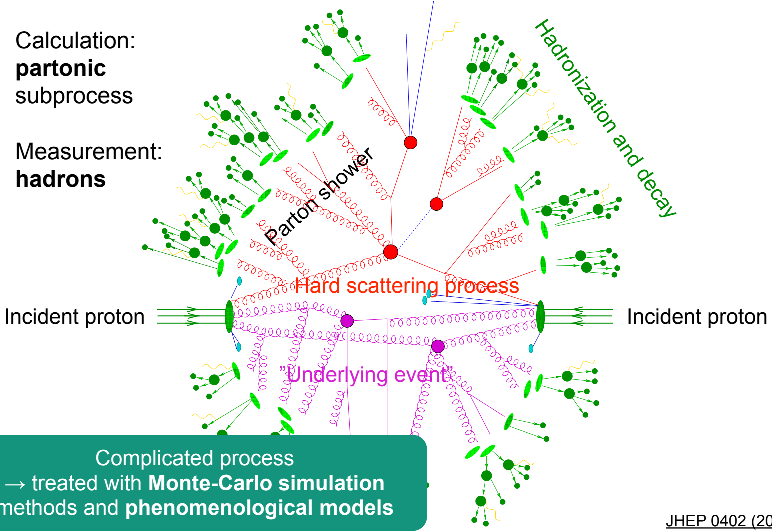
Detector Simulation:
simulation of interactions with detector material

Digitization:
translation of raw detector signals (voltages, …) into digital data

Reconstruction:
same as for real detector data

Simulation of a Collision Event

- Calculation: **partonic** subprocess
- Measurement: **hadrons**



Complicated process
→ treated with **Monte-Carlo simulation** methods and **phenomenological models**

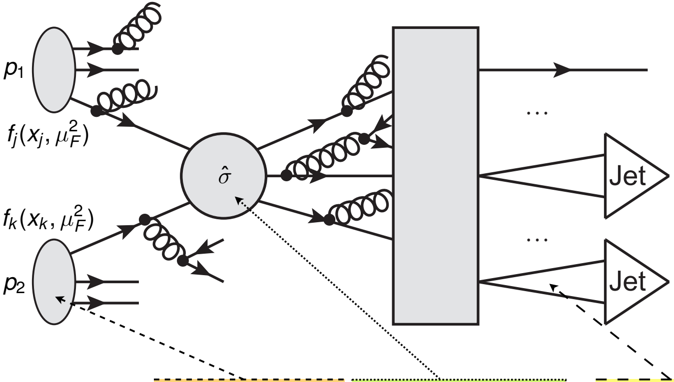
JHEP 0402 (2004) 056

Monte Carlo (MC) Event Generators

- Goal: **realistic simulation** of all relevant physics processes in a particle collision
- Problem: **complexity** of hadron-hadron collisions
  - Initial state: hadrons = compound objects, constituents (quarks and gluons) confined in hadron (running of α_s)
  - Final state: many hadrons and leptons
- Solution: **QCD factorisation**
  - **Separate treatment** of processes at low and high Q^2
  - High Q^2 ("hard scattering process"): **perturbation theory** in leading order or higher orders
  - Low Q^2 ("soft physics"): phenomenological **models**

QCD Factorisation Theorem

Cross section = PDFs ⊗ hard process ⊗ hadronization



$$\sigma_{\text{QCD}} = \sum_{jk} \int \mathrm{d}x_j\, \mathrm{d}x_k\, f_j(x_j, \mu_F^2)\, f_k(x_k, \mu_F^2)\, \hat{\sigma}(x_j x_k s, \mu_F^2, \alpha_S(\mu_R^2)) \otimes \text{Hadronization}$$

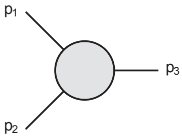
Overview of MC Generators

- Central step in any MC generator: **MC integration** of cross section of hard scattering process in fixed order perturbation theory using PDFs
- **Parton-level** MC generators
  - Simulation stops at level of partons (quarks and gluons)
  - No hadronisation, only events weighted with differential cross-section → no full event simulation (still useful for theoretical studies)
- **Particle-level** MC generators
  - Full event simulation: parton level + parton shower + hadronisation (number of MC events corresponds to theoretical expectation)
  - Provided as single comprehensive package or as combinatin of ME provider and parton shower MC (SMC) programme

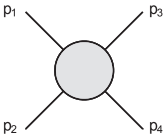
Hard-Scattering Matrix Element

- First generation MC codes: **LO matrix elements (ME)** for $2 \to 1$ and $2 \to 2$ processes
  - Available for all SM and BSM processes
- Improvement 1: LO ME for important $2 \to n$ processes
  - Additional real emission of quarks and gluons (approx. of higher orders)
- Improvement 2: NLO ME (real emission + virtual corrections)
  - 2019: available for all SM processes and many BSM processes

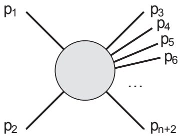
2→1 Process
(e.g. pp → Z)



2→2 Process
(e.g. pp → $t\bar{t}$)



2→*n* Process
(e.g. pp → $t\bar{t}$+(*n*–2) jets)



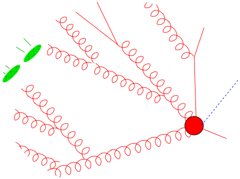
- Often MC cross-section corrected to most accurate calculation (today often NNLO+resummation) via k factor $k = \sigma(\text{NNLO})/\sigma(\text{MC})$
  - Corrects only inclusive cross section, not differential distributions

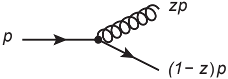
Parton Shower

- **Coherent** emission of soft coloured particles
  - Can be modelled by sequence of **1 → 2 parton splitting** processes
- Parton shower: **probabilistic model** of quark fragmentation
- Description via **Sudakov form factor**
  - Probability for a parton *i* to emit a parton *j*: **splitting function P_{ij}**
  - Solution of **DGLAP equation** for parton shower: Sudakov form factor

$$\Delta_i(t) = \exp\left[-\sum_j \int_{t_0}^{t} \frac{dt'}{t'} \int_0^1 dy\ \frac{\alpha_s}{2\pi} P_{ji}(y)\right]$$

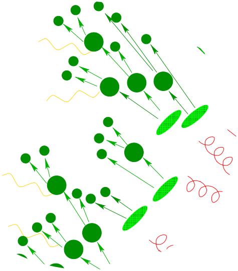
Interpretation: probability for a parton **not to split** during the evolutioin from t_0 to t





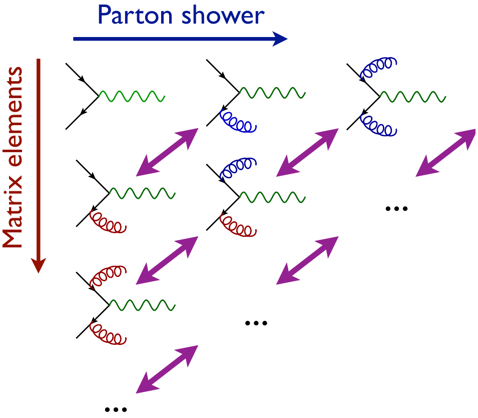
Hadronisation Models

- Parton-hadron transition: **non-perturbative** processes
- Phenomenological MC models very successful
  - Basic assumption: **parton-hadron duality** → very close relation between parton dynamics and properties of final-state hadrons
  - Advantage: **full** event simulation
    → can be used **directly** for experiments
  - Disadvantage: often many **ad-hoc** parameters
    → (rather extensive) **tuning** required
- Most well-known models
  - Lund **string** model (Pythia)
  - **cluster** model (Herwig, Sherpa)



Double Counting and MC Matching

- Real emission from both LO 2 → n ME and PS → **double counting**



F. Maltoni

- Solution: **matching** between ME and PS, **removal of overlap**
  - Different matching algorithms, e. g. MLM, CKKW

Overview of MC Generators

most certainly incomplete …

ME+PS
Alpgen
MG5aMC
Whizard
AcerMC
Grappa
Amegic++
Helac/Phegas
CompHep
Protos
…

Shower MC
Pythia 8
Herwig 7
Sherpa

Parton Level MC
MCFM
FEWZ
NLOJET++
BlackHat
OpenLoops
GoSam
VBFNLO

NLO Generators
MG5aMC
POWHEG BOX

Decays
Tauola
Photos
EvtGen

Summary

- MC generators may be classified by
  - available **physics processes**
  - **highest order** in perturbation theory for hard scattering matrix element
  - **number** of outgoing particles
  - **partonic** or **hadronic** final state
  - **matching**/**merging** between matrix element and parton shower
- Classes of MC generators
  - Pure parton-level MC generator (LO or NLO)
  - General-purpose parton shower MC generator (SMC)
  - LO matrix element provider combined with parton shower (ME+PS)
  - NLO matrix element provider combined with parton shower (NLO+PS)

4. Physics of the W and Z Bosons

4.1 Determination of SM parameters

Z Factories

- Projects to produce **Z bosons in large amounts**
  - e^+e^- collider with $\sqrt{s} = m_Z \approx 91$ GeV ("at the Z pole")
  - Experiments: hermetic 4π detectors

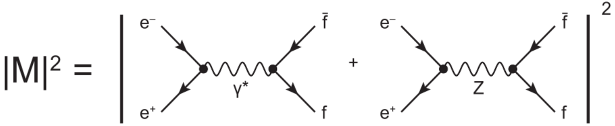
	LEP (1989-2000)	**SLC (1989-1998)**
Data-Taking Periods	LEP 1 (1989-1995): $\sqrt{s} \approx m_Z \approx 91$ GeV	$\sqrt{s} \approx m_Z \approx 91$ GeV Polarized electrons since 1992
	LEP 2 (1996-2000): $\sqrt{s}$ = 160–207 GeV	
Experiments	ALEPH, OPAL, DELPHI, L3	Mark II (until 1991), SLD (1992-1998)
Z Boson Decays Recorded	17,000,000	600,000 (polarized)

Literature

- Results of the LEP Electroweak Working Group
  - `http://lepewwg.web.cern.ch/LEPEWWG/`
  - Comprehensive journal publication by ALEPH, DELPHI, L3, OPAL, SLD collaborations: *Precision electroweak measurements on the Z resonance*, Phys. Rept. 427 (2006) 257

Production Cross-Section

- **Resonant (s-channel) production** of Z bosons in e^+e^- scattering
  - **Photon and Z** boson: same quantum numbers → **interference**
  - LO matrix element



- Cross section: $\sigma(e^+e^- \to \bar{f}f) = \sigma_{\gamma^*} + \sigma_{\gamma^*-Z} + \sigma_Z$
  - $\sqrt{s} \ll m_Z$: **photon exchange** dominates → only QED effects
  - $\sqrt{s} \approx m_Z$: **Z boson exchange** dominates
- Special case: $e^+e^- \to e^+e^-$ (**Bhabha scattering**)
  - **Identical particles** in initial and final state: t-channel process (only photon exchange) in addition
  - Dominant at **small angles**
  - Pure QED, can be calculated very precisely ($1/\sin^4(\theta/2)$ dependence of cross section, see Rutherford scattering)

$\sqrt{s} \ll m_Z$: Photon Exchange

- $e^+e^- \to \bar{f}f$ for $\sqrt{s} \ll m_Z$: essentially pure QED process
- **Inclusive cross section** decreases with $1/(\text{centre-of-mass energy})^2$

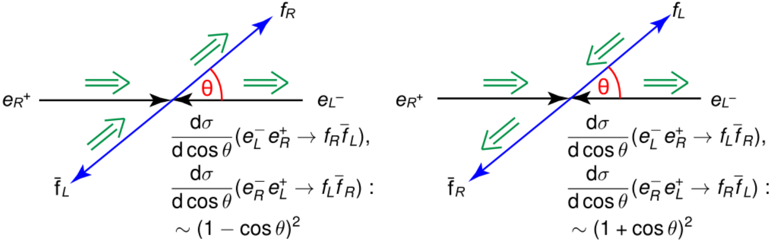
$$\sigma_\gamma = N_{C,f} Q_f^2 \frac{4\pi\alpha^2}{3s}$$

(assumption: all fermion masses can be neglected)

  - $N_{C,f}$: **colour degrees of freedom** (3 for quarks, 1 for leptons)
  - Q_f^2: fermion charge (in units of elementary charge)

$\sqrt{s} \ll m_Z$: Photon Exchange

- **Differential** cross section as a function of scattering angle θ
  - Angular dependence from **particle spins**:

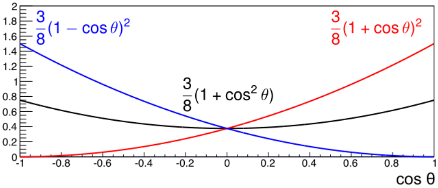


$$\frac{d\sigma}{d\cos\theta}(e_L^- e_R^+ \to f_R \bar{f}_L),$$
$$\frac{d\sigma}{d\cos\theta}(e_R^- e_L^+ \to f_L \bar{f}_R):$$
$$\sim (1-\cos\theta)^2$$

$$\frac{d\sigma}{d\cos\theta}(e_L^- e_R^+ \to f_L \bar{f}_R),$$
$$\frac{d\sigma}{d\cos\theta}(e_R^- e_L^+ \to f_R \bar{f}_L):$$
$$\sim (1+\cos\theta)^2$$

- Superposition of combinations

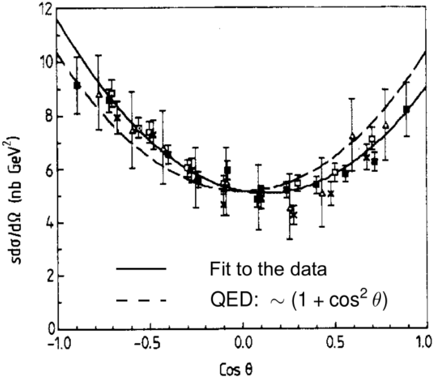
$$\frac{d\sigma_\gamma}{d\cos\theta} = N_{C,f} Q_f^2 \frac{\pi\alpha^2}{2s}(1+\cos^2\theta)$$



Angular Distribution for $\sqrt{s} < m_Z$

- **Interference** between γ^* and Z boson exchange already visible for $\sqrt{s} < m_Z$
- Example: **PETRA** (DESY) $\rightarrow$ first deviations from pure QED

PETRA: √s = 29-35.5 GeV



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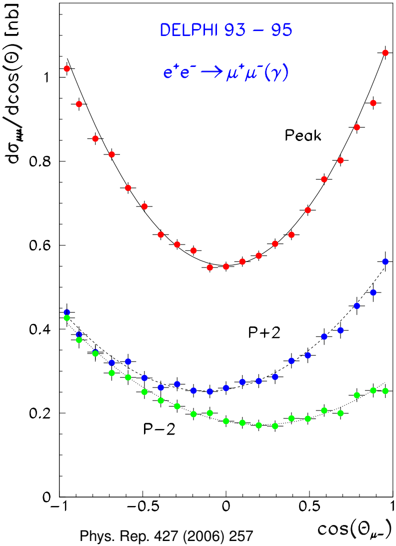
Angular Distribution for $\sqrt{s} \approx m_Z$

- **Interference** between γ^* and Z boson exchange already visible for $\sqrt{s} < m_Z$
- Example: **PETRA** (DESY) → first deviations from pure QED
- LEP: γ^*/Z interference and Z need to be taken into account



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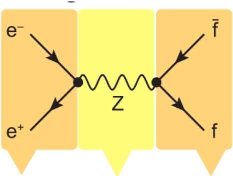
$\sqrt{s} \approx m_Z$: Z Pole

- For $\sqrt{s} \approx m_Z$: Z boson exchange dominates



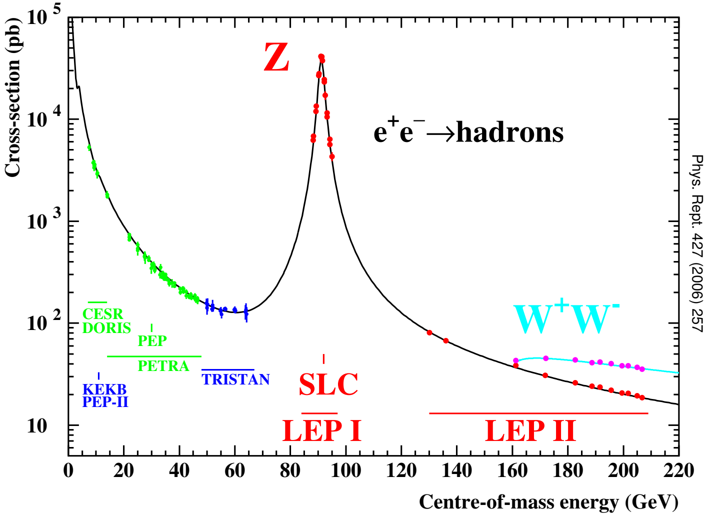
Matrix element $\propto \dfrac{g^2}{(2\cos\theta_W)^2} J^e_{\text{NC},\mu} \dfrac{ig^{\mu\nu}}{s - m_Z^2 + is\frac{\Gamma_Z}{m_Z}} J^f_{\text{NC},\nu}$

- Propagator: Z boson **unstable** → **resonance** in scattering amplitude
  - Wave function of unstable particle

$$\psi \propto \exp[-imt]\exp[-\tfrac{\Gamma t}{2}] \quad \rightarrow \quad \psi^*\psi \propto \exp[-\Gamma t] = \exp[-\tfrac{t}{\tau}]$$

  - **Decay width Γ** = inverse of lifetime τ

$\sqrt{s} \approx m_Z$: Z Pole



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Decay Width Γ

- **Total width Γ_Z** of the Z resonance
  - Sum of **partial (decay) widths**
  - Consider all possible Z-boson decays in the Standard Model:

$$\Gamma_Z = \sum_{\mathrm{f}} \Gamma_{\mathrm{f}} = \sum_{\mathrm{q=u,d,s,c,b}} \Gamma_{\mathrm{q}} + \sum_{l=\mathrm{e},\mu,\tau} \Gamma_l + \sum_{\nu=\nu_{\mathrm{e}},\nu_\mu,\nu_\tau} \Gamma_\nu$$

- **Partial widths Γ_{f}** at LO:

$$\Gamma_{\mathrm{f}} = \Gamma(\mathrm{Z} \to \bar{\mathrm{f}}\mathrm{f}) = N_{C,\mathrm{f}} \frac{G_F m_{\mathrm{Z}}^3}{6\sqrt{2}\pi} \left[(g_V^{\mathrm{f}})^2 + (g_A^{\mathrm{f}})^2\right] \,,\; g_V^{\mathrm{f}} = I_{3,\mathrm{f}} - 2Q_{\mathrm{f}} \sin^2\theta_W,\; g_A^{\mathrm{f}} = I_{3,\mathrm{f}}$$

  Measure **quadratic sum** of vector and axial vector couplings

- **Lepton universality**
  - Same decay width for all charged leptons
  - Same decay width for all neutrinos

Z Boson Decay Channels

Particle	Branching Fraction (PDG 2017)	Detection at Colliders
Left-handed neutrinos	20.00(06)% in total	No direct detection
Left-handed and right-handed charged leptons	3.3658(23)% each	e, μ “simple” τ: depends on decay
Left-handed and right-handed up-type quarks (u,c) in three colors	11.6(6)% each	Jets = collimated bundles of hadrons
Left-handed and right-handed down-type quarks (d,s,b) in three colors	15.6(4)% each	Jets = collimated bundles of hadrons

Cross Section

- Cross section for $e^+e^- \rightarrow Z \rightarrow \bar{f}f$:

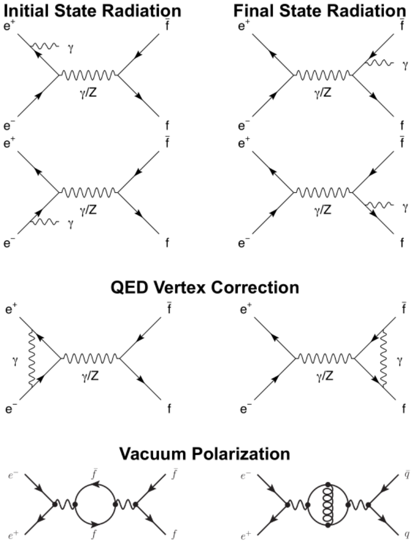
$$\sigma_f = \underbrace{\boxed{\frac{12\pi}{m_Z^2}\frac{\Gamma_e\Gamma_f}{\Gamma_Z^2}}}_{\sigma_f^0} \cdot \underbrace{\boxed{\frac{s\Gamma_Z^2}{(s-m_Z^2)^2 + s^2\frac{\Gamma_Z^2}{m_Z^2}}}}_{\text{Breit–Wigner}}$$

- Resonance peak **height**: $\sigma_f^0 \propto \Gamma_e\Gamma_f$

Radiative Corrections

- Precision of LEP and SLC data: sensitive to **higher-order corrections**
  - **Real emission** of photons and **loop corrections**
- Consequence: running coupling constant

$$\alpha(m_Z^2) \approx \frac{1}{128} > \alpha \approx \frac{1}{137}$$



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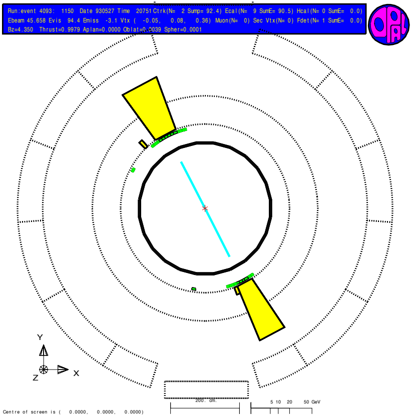
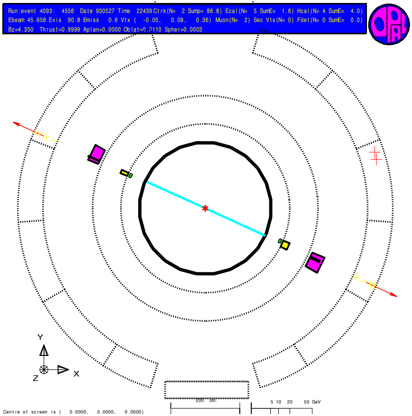
Events at OPAL

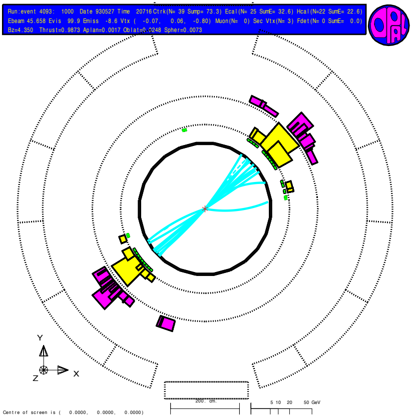


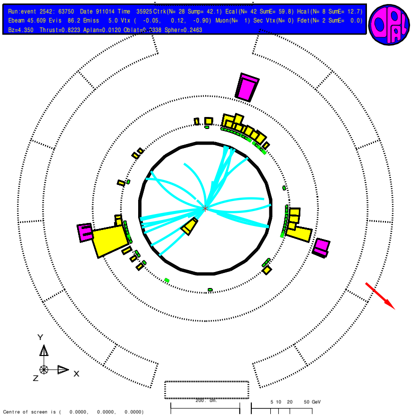
```
Run:event 4093:  1150  Date 930527 Time  20751 Ctrk(N=  2 Sump= 92.4) Ecal(N=  9 SumE= 90.5) Hcal(N= 0 SumE=  0.0)
Ebeam 45.658 Evis  94.4 Emiss  -3.1 Vtx (  -0.05,   0.08,   0.36) Muon(N=  0) Sec Vtx(N= 0) Fdet(N= 1 SumE=  0.0)
Bz=4.350  Thrust=0.9979 Aplan=0.0000 Oblat=0.0039 Spher=0.0001
```



```
Run:event 4093:  4556  Date 930527 Time  22439 Ctrk(N=  2 Sump= 86.8) Ecal(N=  5 SumE=  1.6) Hcal(N= 4 SumE=  4.0)
Ebeam 45.658 Evis  90.8 Emiss   0.6 Vtx (  -0.05,   0.08,   0.36) Muon(N=  2) Sec Vtx(N= 0) Fdet(N= 0 SumE=  0.0)
Bz=4.350  Thrust=0.9999 Aplan=0.0000 Oblat=0.0110 Spher=0.0003
```

Events at OPAL

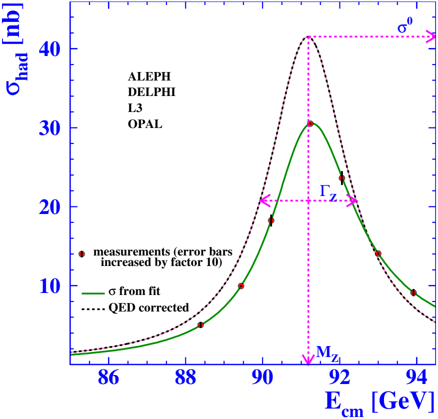




Hadronic Cross Section

- **Shape** of Z resonance
  - **Energy scan**: vary $\sqrt{s}$
  - Selection: $Z \to \bar{q}q$

  $\to$ **hadronic cross section** σ_{had}
- Correct data for QED radiative corrections
- (Pseudo-)Observables
  - **Position** of resonance $\to m_Z$
  - **Height** of resonance $\to \sigma^0_{\text{had}}$
  - **Width** (FWHM) $\to m_Z$



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Hadronic Cross Section: Results

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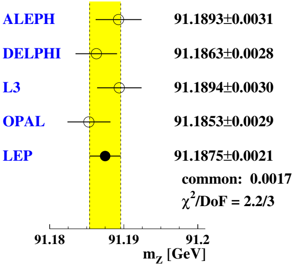
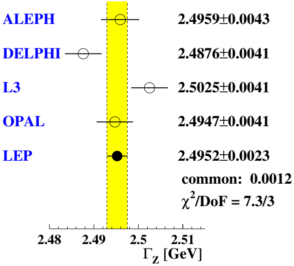
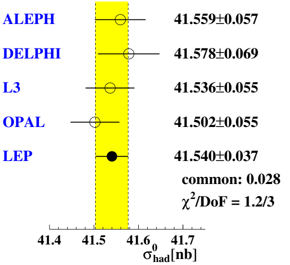






Combination of LEP results

relative uncertainty:

Z boson mass: $2.3 \cdot 10^{-5}$

Z boson width: $9.2 \cdot 10^{-4}$

Cross section: $8.9 \cdot 10^{-4}$

Cross Section and Partial Width

- Cross section for $e^+e^- \to Z \to \bar{f}f$:

$$\sigma_f = \underbrace{\boxed{\frac{12\pi}{m_Z^2}\frac{\Gamma_e\Gamma_f}{\Gamma_Z^2}}}_{\sigma_f^0} \cdot \underbrace{\boxed{\frac{s\Gamma_Z^2}{(s-m_Z^2)^2 + s^2\frac{\Gamma_Z^2}{m_Z^2}}}}_{\text{Breit–Wigner}}$$

- Resonance peak **height**: $\sigma_f^0 \propto \Gamma_e\Gamma_f$
  - Cross section measures **product of partial decay widths**, e. g. $\sigma_{had}^0 \propto \Gamma_e\Gamma_{had}$
  - **Single partial width**: by combining certain ratios of cross sections
- Application: **number of light neutrino flavours**
  - How many (light) invisible particles couple to the Z boson?

Number of Light Neutrino Flavours

- Observable

$$R^0_{\text{inv}} = \sqrt{\frac{12\pi}{m_Z^2}\frac{R^0_l}{\sigma^0_{\text{had}}}} - 3 - R^0_l$$

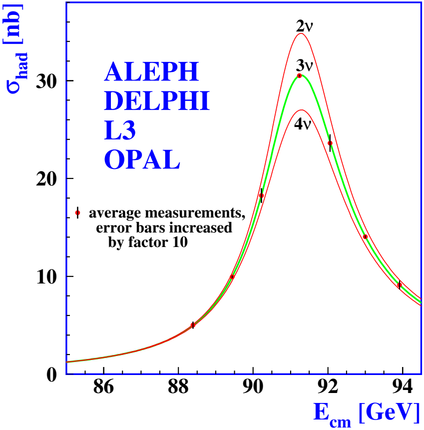
  - Z resonace peak height
  - Ratio hadronic/leptonic width R^0_l
  - Z mass

- Standard Model expectation

$$R^0_{\text{inv}} = N_\nu \left(\frac{\Gamma_\nu}{\Gamma_l}\right)_{\text{SM}} = 1.991(1) N_\nu$$

Result: **number of neutrino flavours** with $m_\nu < \frac{1}{2} m_Z$

$$\boxed{N_\nu = 2.9840(82)}$$



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