Introduction to Particle Physics

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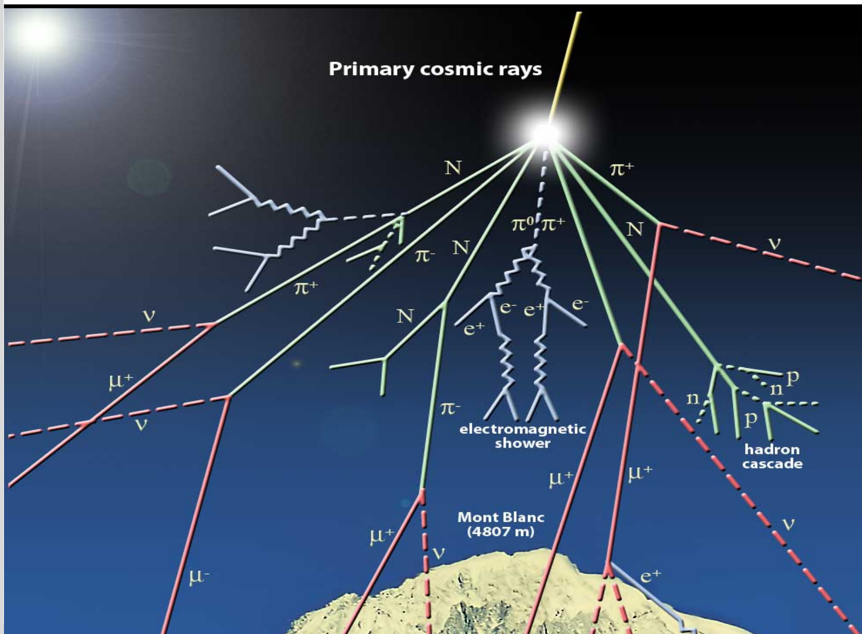
INSTITUTE OF EXPERIMENTAL PARTICLE PHYSICS (IEKP) – PHYSICS FACULTY

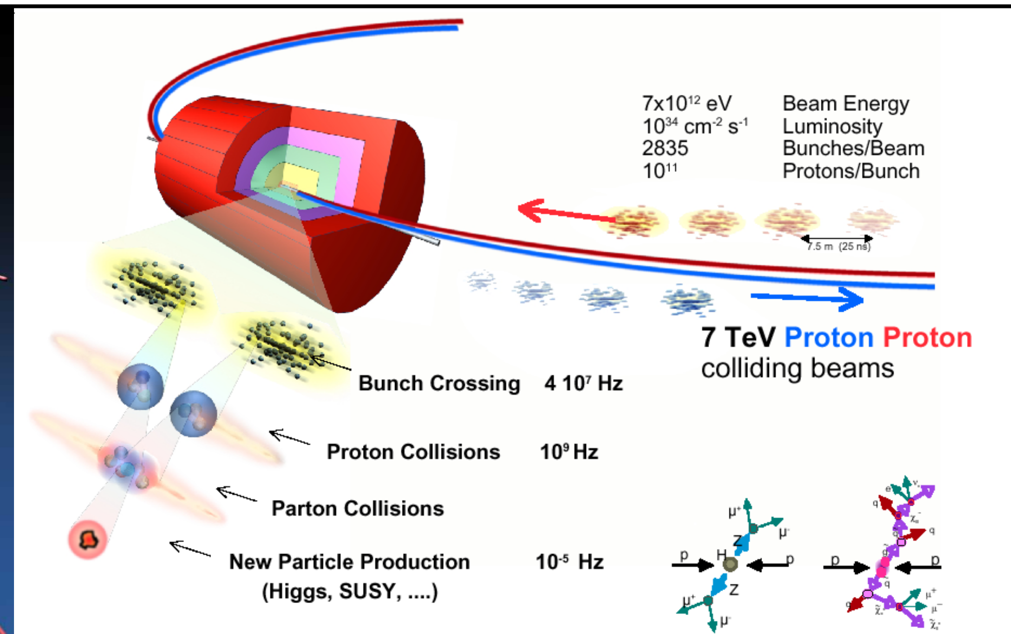
Astroparticle vs. particle physics

Karlsruhe Institute of Technology

- Highest beam energies (up to 10^{21} $\mathrm{eV} \rightarrow$ fixed target).
- Complicated detection medium ($\rightarrow$ atmosphere).
- Large area detectors required.

- Perfect control over initial state under ideal laboratory conditions.
- Compact and tailored detector designs.





Collision kinematics

Center of mass energy of a relativistic two body collision:

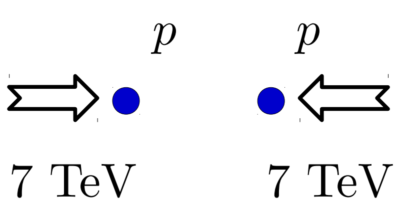
$$
\begin{aligned}
s^2 &= (p_1^\mu + p_2^\mu)^2 \\
&= p_1^2 + p_2^2 + 2p_1^\mu p_{\mu,2} \\
&\approx 2p_1^\mu p_{\mu,2}
\end{aligned}
$$

Boost along z-direction:

$$E' = \gamma\,(E - \beta p_z)$$

$$p_z' = \gamma\,(p_z - \beta E)$$

$$\begin{pmatrix} E \\ 0 \\ 0 \\ E \end{pmatrix} \qquad \begin{pmatrix} E \\ 0 \\ 0 \\ -E \end{pmatrix}$$



$$s^2 = 2 \begin{pmatrix} E \\ 0 \\ 0 \\ E \end{pmatrix} \begin{pmatrix} E \\ 0 \\ 0 \\ -E \end{pmatrix} = 4E^2$$

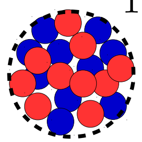
$$E_{cms} = \sqrt{s^2} = \sqrt{4E^2} = 2E = 14\,\mathrm{TeV}$$

Collision kinematics

$$\begin{pmatrix} E \\ 0 \\ 0 \\ E \end{pmatrix}$$

p

10^{19} eV



${}^{16}_{8}O$

$$\begin{pmatrix} M \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

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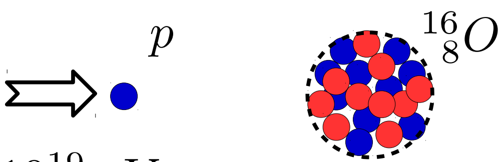

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$$E_{cms} = \sqrt{s^2} = \sqrt{2EM} \approx 567\,\mathrm{TeV}$$

$$\sqrt{\frac{EM}{2}} = \gamma M \to \gamma = \sqrt{\frac{E}{2M}} \approx 17'678$$

$$\gamma = \sqrt{\frac{1}{1-\beta^2}} \to \beta = \sqrt{\frac{\gamma^2 - 1}{\gamma^2}} \approx 0.99999999$$

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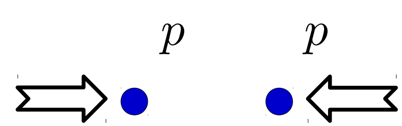

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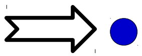
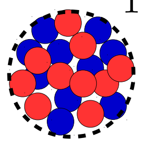


p

10^{19} eV



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expect 1 collision per year detector w/ 1 km^2 surface.

$$\begin{pmatrix} E \\ 0 \\ 0 \\ E \end{pmatrix}$$

p p

7 TeV 7 TeV

$$\begin{pmatrix} E \\ 0 \\ 0 \\ -E \end{pmatrix}$$

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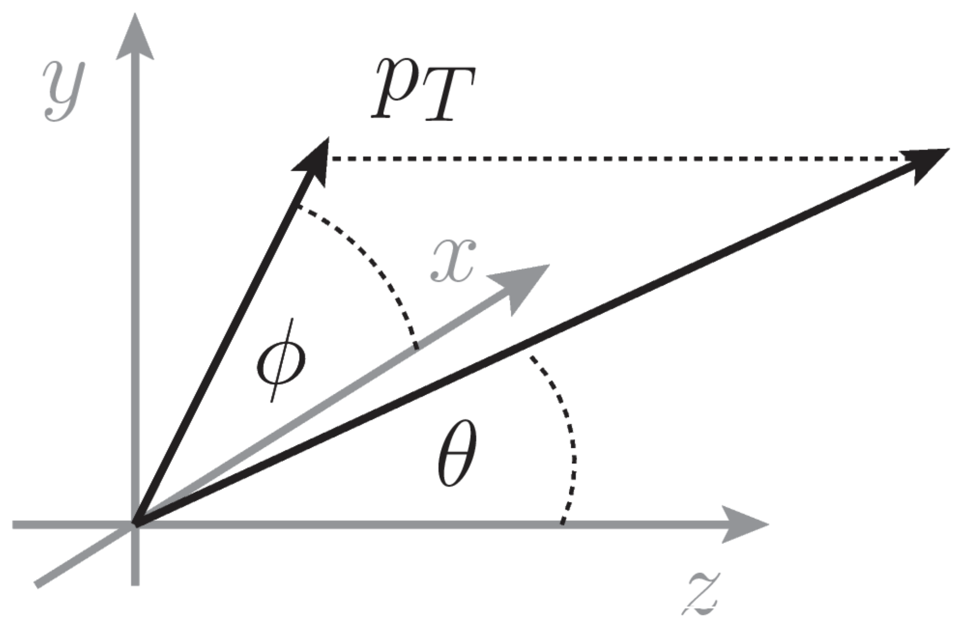
expect 40M collisions per second.

Particle kinematics

- For known mass the kinematics of a single particle are completely described by three variables: $(\ p_x \quad p_y \quad p_z\ )$ or better $(\ p_T \quad \phi \quad \theta\ )$



p_T and ϕ in the plane perpendicular to z are invariant under *boosts* along z, θ not.
Therefore we usually replace θ by:

Rapidity:

$$y = \frac{1}{2}\ln\left(\frac{E+p_z}{E-p_z}\right)$$

which is form invariant under *boosts* along z.

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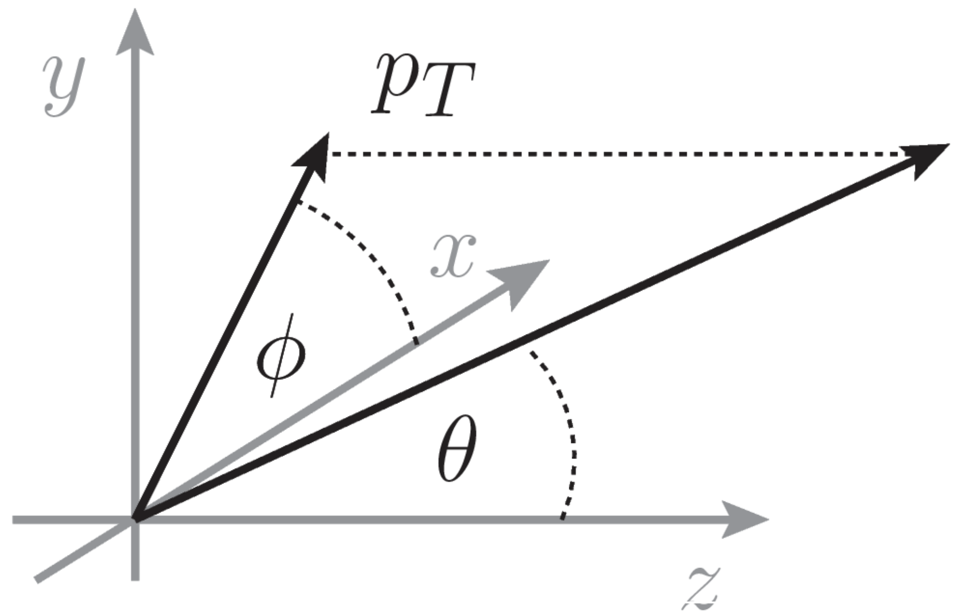

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which is form invariant under

$$y = \frac{1}{2} \ln \left(\frac{E' + p'_z}{E' - p'_z} \right) = \frac{1}{2} \ln \left(\frac{(E - \beta p_z) + (p_z - \beta E)}{(E - \beta p_z) - (p_z - \beta E)} \right) = \frac{1}{2} \ln \left(\frac{(1-\beta)(E+p_z)}{(1+\beta)(E-p_z)} \right)$$
$$= \frac{1}{2} \left(\ln \left(\frac{1-\beta}{1+\beta} \right) + \ln \left(\frac{E+p_z}{E-p_z} \right) \right) = y + \frac{1}{2} \ln \left(\frac{1-\beta}{1+\beta} \right)$$

Pseudorapidity

- For $E \gg m$ the rapidity turns into the pseudorapidity η, which itself only depends on the polar angle θ.

Pseudorapidity:

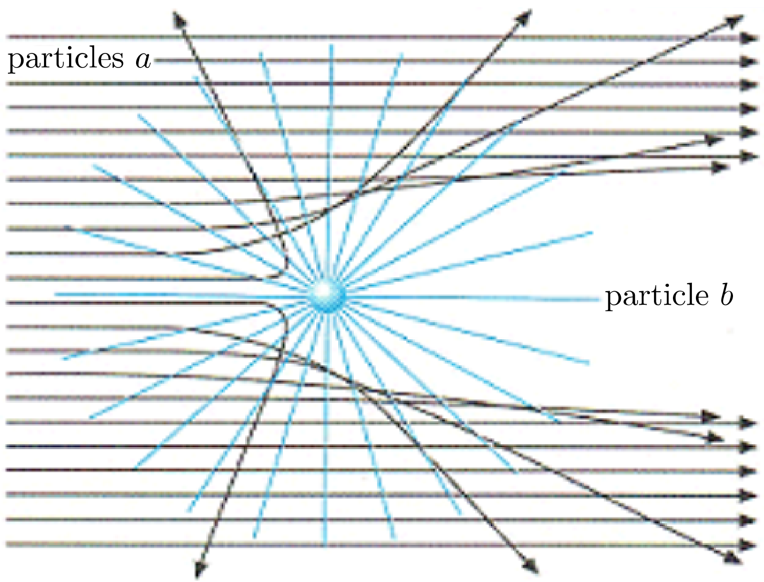
$$\eta = -\ln\left(\tan(\theta/2)\right)$$

$$\begin{aligned} y &= \frac{1}{2}\ln\left(\frac{E(1+\cos\theta)}{E(1-\cos\theta)}\right) \\ &= \frac{1}{2}\ln\left(\frac{(\sin^2 \theta/2 + \cos^2 \theta/2) + (\cos^2 \theta/2 - \sin^2 \theta/2)}{(\sin^2 \theta/2 + \cos^2 \theta/2) - (\cos^2 \theta/2 - \sin^2 \theta/2)}\right) \\ &= \frac{1}{2}\ln\left(\frac{\cos^2 \theta/2}{\sin^2 \theta/2}\right) = -\ln\left(\tan \theta/2\right) = \eta \end{aligned}$$

Imagine in the air shower of slide 4 a particle were scattered at 90° to the axis of its incident direction in the center of mass frame. What is the scattering angle in the **laboratory frame**?

Cross section (classic)

- Imagine a continuous flux of (small) incident particles a impinging on a target particle b at rest and the elastic reaction $a+b \rightarrow a+b$:



$$n_a \ : \ \text{incident particle density} \left[\frac{\text{particles}}{m^3}\right].$$

$$v \ : \ \text{incident particles velocity} \left[\frac{m}{s}\right].$$

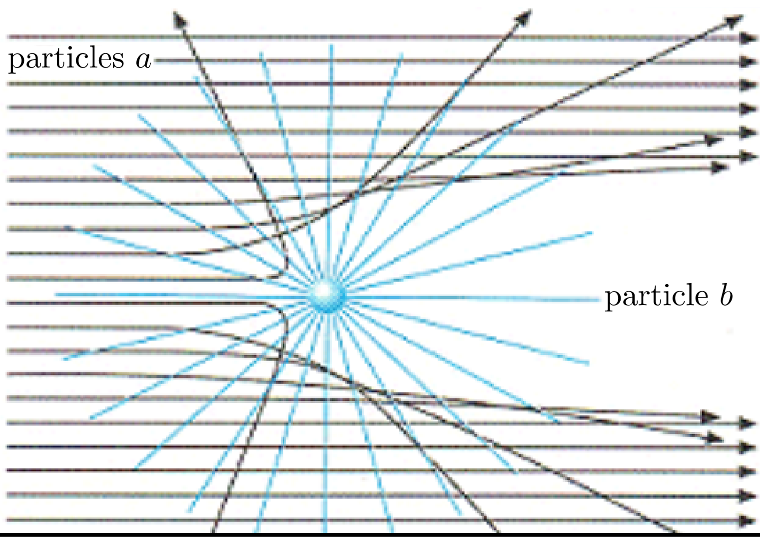
$$\phi = n_a \cdot v \ : \ \text{incident part flux} \left[\frac{\text{particles}}{m^2\, s}\right].$$

$$W = \phi \cdot \sigma \ : \ \text{scattering rate} \left[\frac{1}{s}\right].$$

$$\sigma = \frac{W}{\phi} \ : \ \text{reaction rate/incident part flux.}$$

Cross section (classic)

- Imagine a continuous flux of (small) incident particles a impinging on a target particle b at rest and the elastic reaction $a + b \to a + b$:



n_a : incident particle density $\left[\frac{\text{particles}}{m^3}\right]$.

v : incident particles velocity $\left[\frac{m}{s}\right]$.

$\phi = n_a \cdot v$: incident part flux $\left[\frac{\text{particles}}{m^2\, s}\right]$.

$W = \phi \cdot \sigma$: scattering rate $\left[\frac{1}{s}\right]$.

$\sigma = \frac{W}{\phi}$: reaction rate/incident part flux.

Cross section:

$$\sigma = \frac{N_{obs} - N_{BG}}{T \cdot \epsilon \cdot A} \frac{1}{\phi}$$

N_{obs} : N observed reactions.

N_{BG} : N expected BG reactions.

ϵ : detection efficiency.

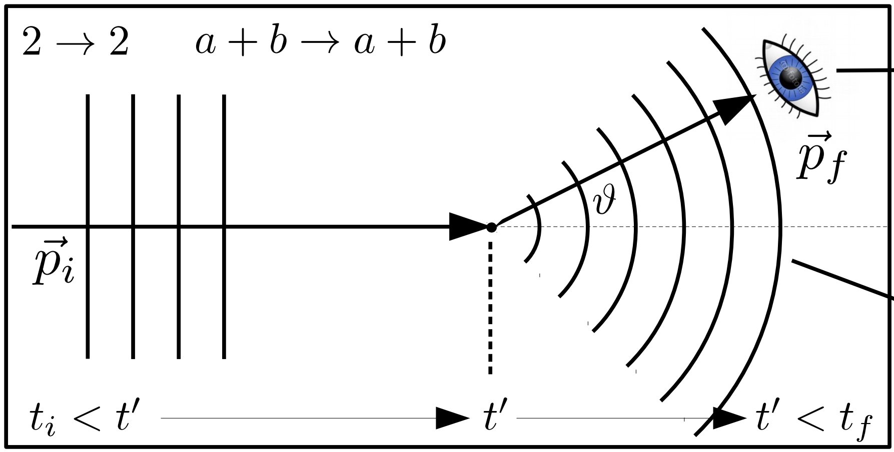
A : detector acceptance.

T : observation time.

In classic elastic scattering the cross section is πr^2.

Cross section (QM)

- Imagine a continuous flux of (small) incident particles a impinging on a target particle b at rest and the elastic reaction $a+b\rightarrow a+b$:



Observation (in $\Delta\Omega$): projection of plain wave ϕ_f out of spherical scattering wave ψ_{scat}.

Spherical scattering wave ψ_{scat}.

Initial particle: described by plain wave ϕ_i.

Localized potential.

Observation probability:

$$\mathcal{S}_{fi} = \phi_f^\dagger \cdot \psi_{\text{scat}} = \phi_f^\dagger \cdot \mathcal{S} \cdot \phi_i$$

Scattering matrix $\mathcal{S}$ transforms initial state wave function ϕ_i into scattering wave ψ_{scat} ($\psi_{\text{scat}} = \mathcal{S}\cdot\phi_i$).

Fermi's golden rule:

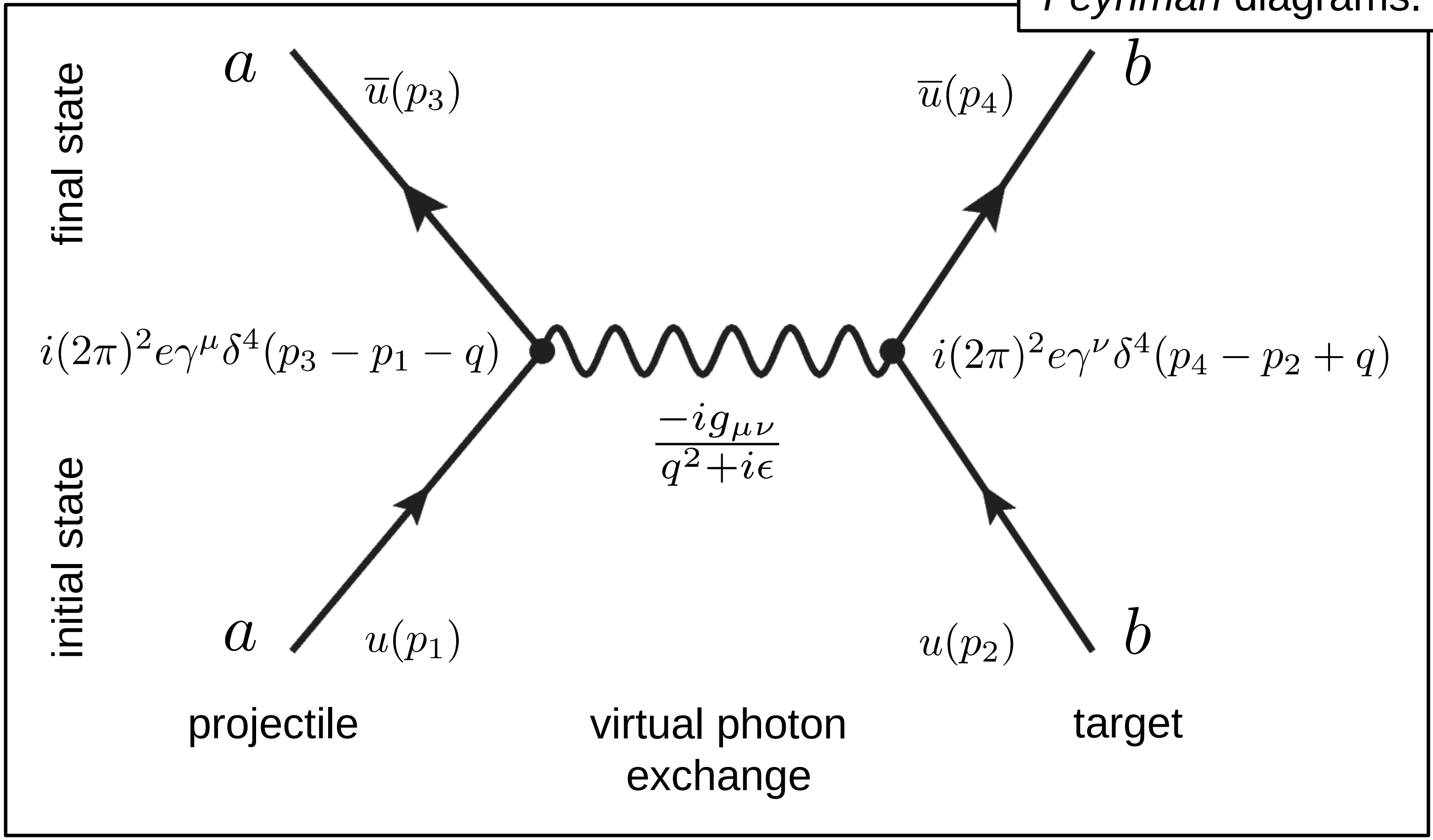
$$W = 2\pi\left|\mathcal{S}_{fi}\right|^2 \rho_f$$

$$\rho_f = \int \prod_{i=a,b} (2\pi)^{-3}\, p_i^2 \mathrm{d}p_i \mathrm{d}\Omega_i$$

phasespace factor for final state products.

The matrix element $\mathcal{S}_{fi}$

Matrix element calculations can be represented pictorially with the help of *Feynman* diagrams.

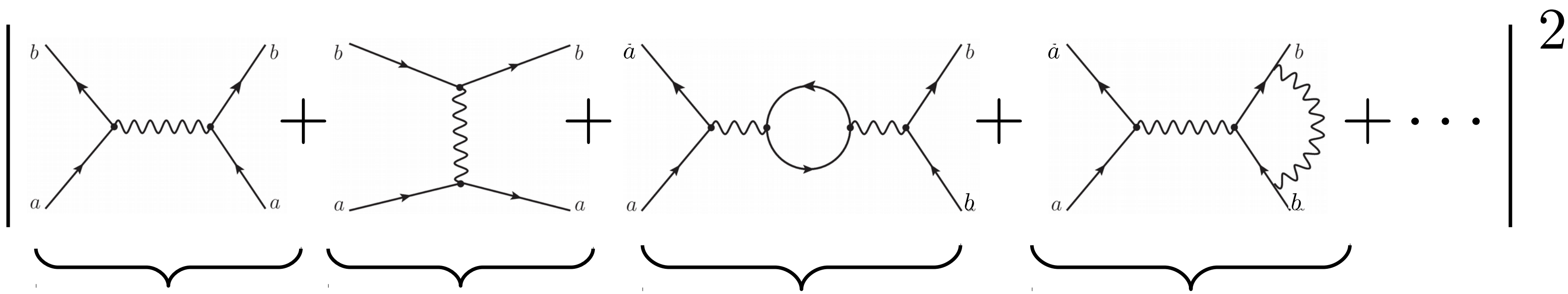


$$\mathcal{S}_{fi}^{(1)} = i\left((2\pi)^2 e\right)^2 \cdot \int \mathrm{d}^4 q\ \delta^4(p_3 - p_1 - q)\overline{u}(p_3)\gamma^\mu u(p_1)\frac{-g_{\mu\nu}}{q^2 + i\epsilon}\delta^4(p_4 - p_2 + q)\overline{u}(p_4)\gamma^\nu u(p_2)$$

The matrix element $\mathcal{S}_{fi}$

- The full calculation (ideally) includes all possible diagrams to all orders in QM perturbation theory:

$$|\mathcal{S}_{fi}|^2 =$$



s-channel, if allowed.

t-channel.

Higher order correction to propagator.

Higher order correction to vertex.

- Coherent sum: includes absolute value squares of individual diagrams and interference terms across different diagrams.

History of particle physics

- Relativistic QM (→ Dirac-Equation 1928)
- Theory of weak IA (→ E. Fermi 1933 – 34)
- Discovery $\mu^{+/-}$ (→ C. D. Anderson 1937) 
- Discovery $\pi^{+/-}$ (→ C. Powel/G. Occhialini 1947) 
- Discovery π^0 (→ R. Bjorklund et al 1950) 
- Discovery $K^{+/-}$ (→ “V”-particles 1947 – 49) 
- Discovery K^0, Λ^0 (→ “V”-particles 1947) 
- Discovery Σ’s, Ξ’s (→ 1950’s) 
- Discovery Δ^{++}, Δ^{+}, Δ^0, Δ^- (→ 1952) 
- Invention of bubble chamber (→ D. Glaser 1952)
- Observation of ν_e (→ C. Cowan, F. Reines 1956)
- Observation P violation of weak IA (→ C. Wu, R. Garwin 1556)
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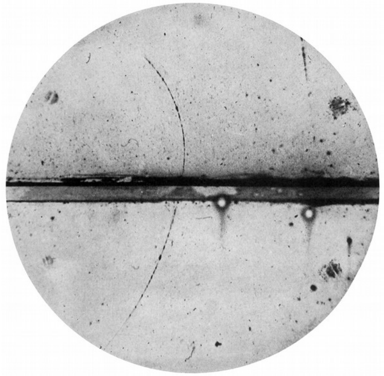
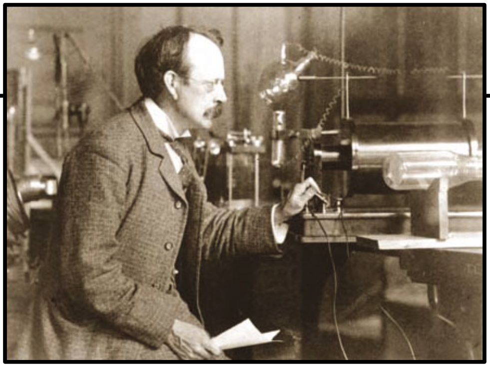
- Discovery J/ψ’s (→ B. Richter, S.Thing, 1974) 
- Discovery Υ’s (→ L. Lederman, E288 collaboration, 1977) 
- Discovery of W, Z (→ UA1 & UA2 collaboration, 1983) 
- Observation of t (→ CDF & D0 collaboration 1995) 
- Observation of ν_τ (→ DONUT collaboration 2000)
- Discovery of H (→ ATLAS & CMS collaboration 2012)

Discovery of the positron (1932)

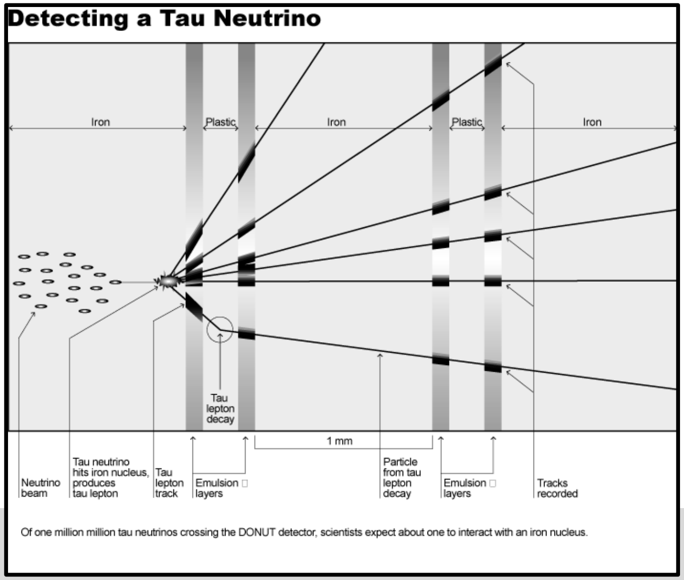


C. D. Anderson (1905 – 1991)

Discovery of the electron (1897)



J. J. Thomson (1856 – 1940)



DONUT collaboration

 discovered in airshower experiments

 discovered in collider experiments

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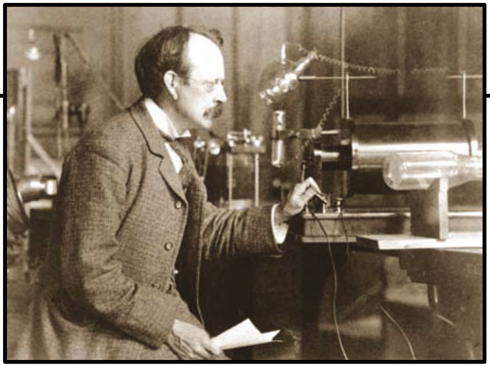

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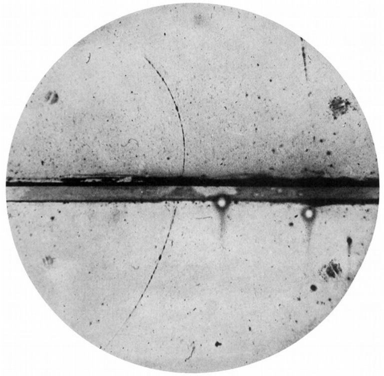

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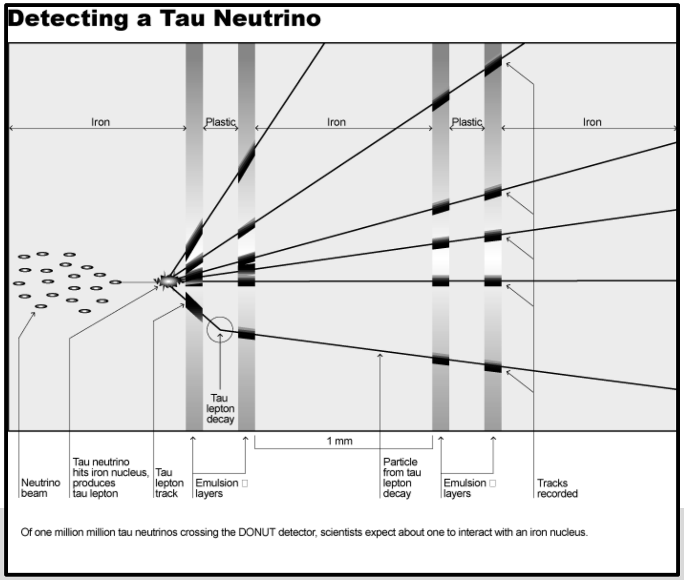
Overall $\mathcal{O}(30)$ Nobel prizes in physics went to directly particle physics related topics.



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discovered in airshower experiments
discovered in collider experiments

Hadrons:

Baryons:

Leptons:

e^-

μ^-

τ^-

ν_τ

ν_e

ν_μ

Mesons:

Hadrons:

Leptons:

e^- μ^- τ^- ν_τ ν_e ν_μ

Mesons:

π^- D^0 B^- η η' η_c B^+ π^+ π^0 η_b D_s^- D_s^+ K^+ K^- K_L^0 K_S^0 D^- D^+ B_c^+ B_s^0 B^0 B_c^-

Baryons:

$J^P = 0^-$

Hadrons:

Leptons:

e^- μ^- τ^- ν_τ ν_e ν_μ

Mesons:

π^- D^0 B^- η J/ψ η' ϕ K^{*-} ρ^0 η_c B^+ ω π^+ ρ^- π^0 ρ^+ η_b D_s^- D_s^+ K^{*+} K^+ Υ K^- K^{*0} D_s^{*-} K_S^0 K_L^0 D^{*+} D^- B_c^{*+} D^{*-} D^+ B_c^{*-} B_c^+ B_s^0 B^{*+} D^{*0} B^{*0} D_s^{*+} B^0 B_c^- B_s^{*0}

Baryons:

$J^P = 0^-$ $J^P = 1^-$

The PARTICLE ZOO

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Hadrons:

Leptons:

e^- μ^- τ^- ν_τ ν_e ν_μ

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π^- D^0 B^- η J/ψ η' ϕ K^{*-} ρ^0 η_c B^+ ω π^+ ρ^- π^0 ρ^+ η_b D_s^- D_s^+ K^{*+} K^+ Υ K^- K^{*0} D_s^{*-} K_S^0 K_L^0 D^{*+} D^- B_c^{*+} D^{*-} D^+ B_c^{*-} B_c^+ B_s^0 B^{*+} D^{*0} B^{*0} D_s^{*+} B^0 B_c^- B_s^{*0}

Baryons:

Ξ_c^+ Σ^0 Ω_{bb}^- p n Σ^+ Ξ^- Λ_b^+ Δ^{++} Ξ^0 $\Xi_{cb}^{\prime 0}$ Σ^- $\Xi_{cb}^{\prime +}$ Λ^0 Λ_c^+ $\Xi_{bb}^{\prime -}$ Σ_c^+ Δ^+ Δ^- Σ_c^{++} Ξ_{cb}^+ Σ_b^- Δ^0 Ξ_c^0 Σ_b^+ Σ_b^0 Ξ_{cc}^{++} $\Xi_c^{\prime +}$ Ξ_{cb}^0 $\Xi_c^{\prime 0}$ Ξ_b^- Ξ_b^0 Ξ_{cc}^+ $\Xi_b^{\prime -}$ $\Xi_{bb}^{\prime 0}$ Σ_c^0 Ω_b^- Ω_c^0 Ω_{cbb}^0 Ω_{cb}^0 Ω_{ccb}^+ Ω_{cc}^+ $\Xi_b^{\prime 0}$ $\Omega_{cb}^{\prime 0}$

$J^P = 0^-$ $J^P = 1^-$ $J^P = 1/2^+$

The PARTICLE ZOO

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Hadrons:

Leptons:

e^- μ^- τ^- ν_τ ν_e ν_μ

Mesons:

π^- D^0 B^- η J/ψ η' ϕ K^{*-} ρ^0 η_c B^+ ω π^+ ρ^- π^0 ρ^+ η_b D_s^- D_s^+ K^{*+} K^+ Υ K^- K^{*0} D_s^{*-} K_S^0 K_L^0 D^{*+} D^- B_c^{*-} B_c^+ B_c^{*+} D^{*-} D^+ B_s^0 B^{*+} D^{*0} B^{*0} D_s^{*+} B^0 B_c^- B_s^{*0}

+150 further known Meson resonances.

Baryons:

Ω_{bbb}^{++} Ξ_c^+ Σ^0 Ω_{bb}^- p n Ω_{ccc}^{++} Ω_{cbb}^{*0} Ω_{ccb}^{*+} Σ^+ Ξ^- Ω_{bb}^{*-} Λ_b^+ Δ^{++} Ξ^0 $\Xi_{cb}^{\prime 0}$ Ω_c^{*0} Δ^{++} $\Xi_{cb}^{\prime +}$ Ω_b^{*-} Σ^- Λ^0 Λ_c^+ $\Xi_{bb}^{\prime -}$ Ξ^{*-} Ω_{cb}^{*0} Ω^- Σ_c^+ Δ^- Σ_b^{*+} Δ^+ Δ^- Ξ_{bb}^{*0} Ω_{cc}^{*+} Σ_c^{++} Ξ_{cb}^+ Ξ_b^{*0} Σ_b^- Δ^0 Ξ_{cb}^{*0} Ξ^{*0} Ξ_c^0 Σ_b^+ Ξ_{cb}^0 $\Xi_c^{\prime 0}$ Ξ_b^{*-} Δ^0 Σ_b^0 Ξ_{cc}^{++} $\Xi_c^{\prime +}$ Ξ_b^- Ξ_b^0 Ξ_{cc}^+ $\Xi_{bb}^{\prime 0}$ Σ_b^{*-} Σ_c^0 Ξ_{bb}^{*-} $\Xi_b^{\prime -}$ Ξ_{cc}^{*++} Ξ_{cb}^{*+} Ω_b^- Ω_c^0 Ω_{cbb}^0 Σ_b^{*0} Ξ_c^{*+} Σ^{*-} Ω_{ccb}^+ Σ_c^{*+} $\Xi_b^{\prime 0}$ Ω_{cb}^0 Ξ_c^{*0} Δ^+ Ω_{cc}^+ Σ^{*0} Σ_c^{*++} $\Omega_{cb}^{\prime 0}$ Σ^{*+} Σ_c^{*0} Ξ_{cc}^{*+}

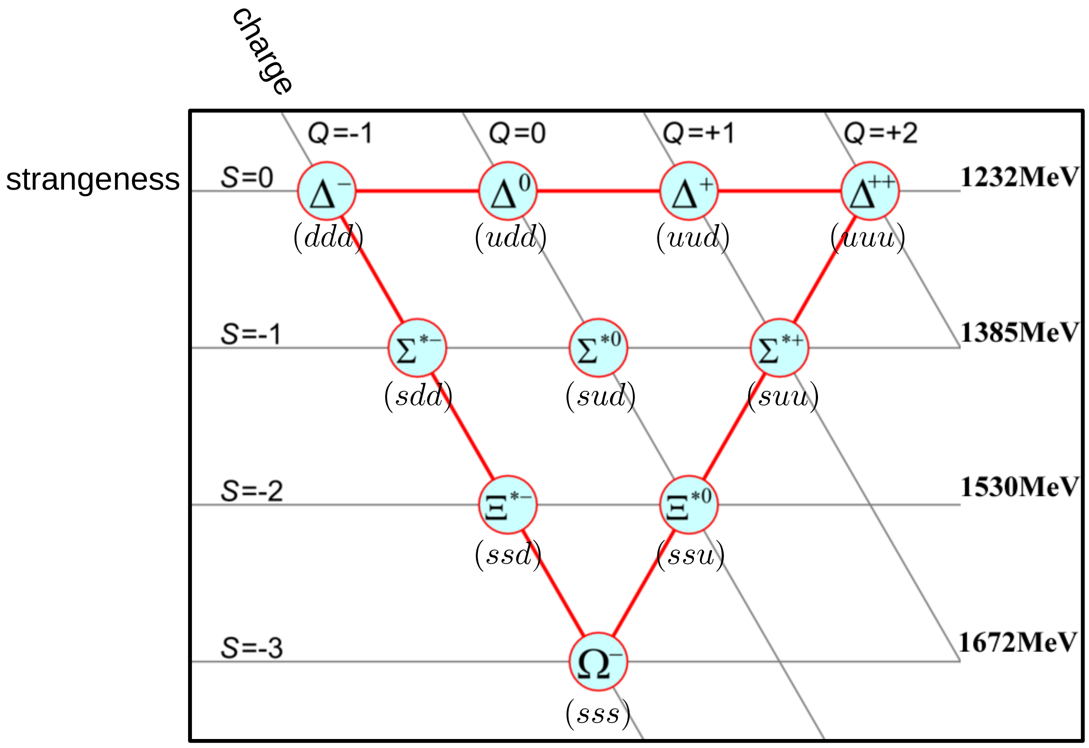
+152 further known Baryon resonances.

$\mathcal{O}(400)$ known elementary particles.

$J^P = 0^-$ $J^P = 1^-$ $J^P = {}^1\!/_2{}^+$ $J^P = {}^3\!/_2{}^+$

More order into the chaos...

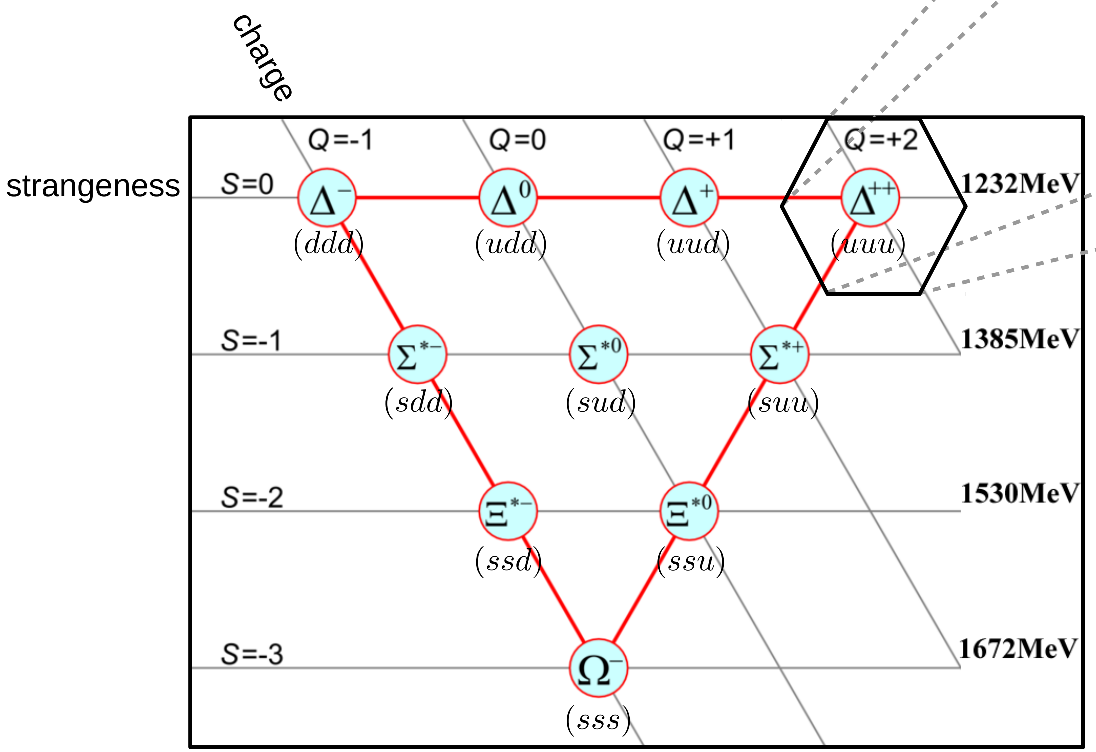
… could be achieved once it was realized that *hadrons* are composed of more fundamental constituents → quarks (first only sorting principle):



$J^P = {}^3\!/_2^+$ *baryon* $SU(3)$ decuplet.

More order into the chaos...

… could be achieved once it was realized that *hadrons* are composed of more fundamental constituents → quarks (first sorting principle only):



$J^P = {}^3\!/_2^+$ *baryon* $SU(3)$ decuplet.

Δ^{++} **requires:**

- all spins up $(\uparrow\uparrow\uparrow)$.
- all same flavors (uuu).
- No orbital momentum $(L = 0)$.

As spin ½ fermion Δ^{++} **needs anti-symmetric wave function:**

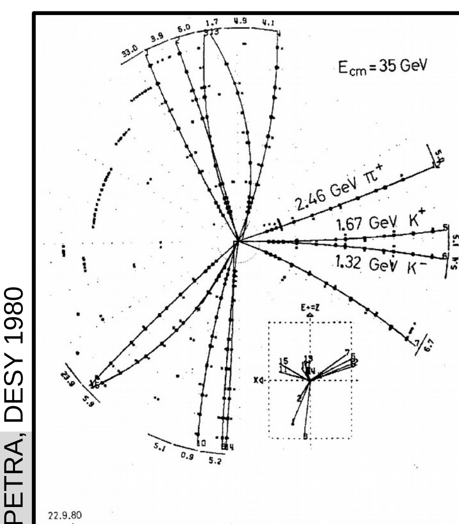
$$\psi = \phi \cdot \chi(uuu) \cdot \eta(\uparrow\uparrow\uparrow)$$

symmetric → Space wave function

symmetric → Flavor wave function

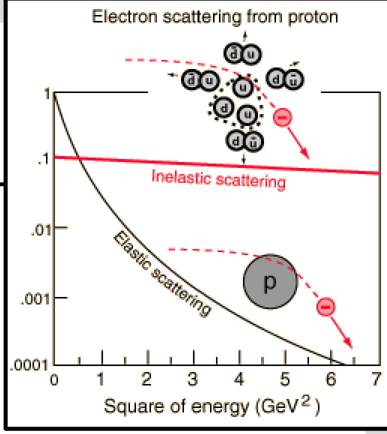
symmetric → Spin wave function

New quantum number required to obtain anti-symmetric wave function (→ first indication for **color**).



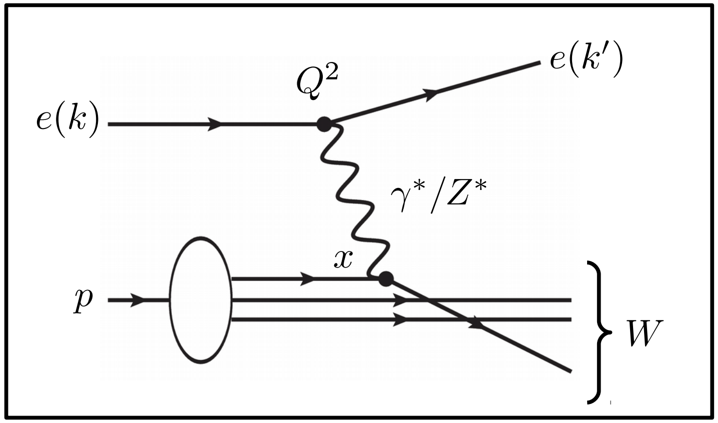
PETRA, DESY 1980

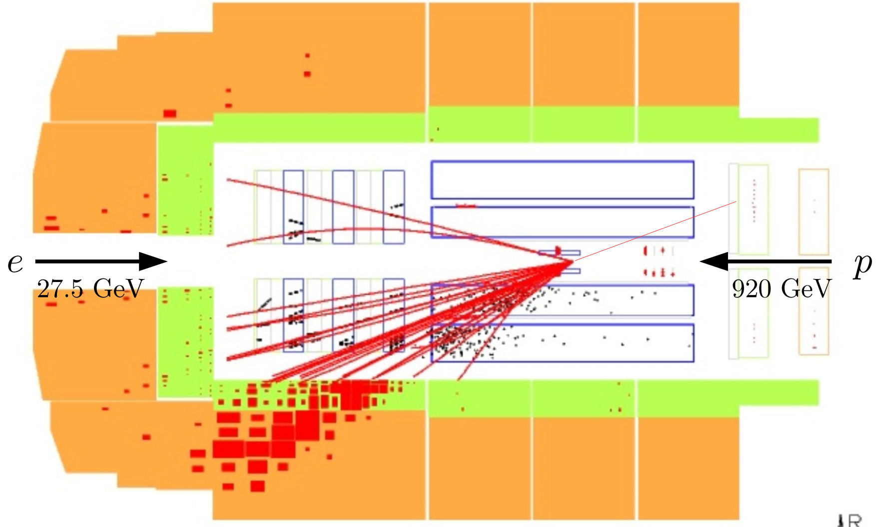
The evidence of quarks...



… emerged from deep inelastic scattering (DIS) experiments (first @SLAC 1969, here shown @HERA ~2000):

$$Q^2 = -q^2 = (k' - k)^2$$
$$s = (p - k)^2 = 4E_p E_e$$
$$x = \frac{Q^2}{2pq}$$
$$y = \frac{pq}{pk}$$
$$Q^2 = xys$$





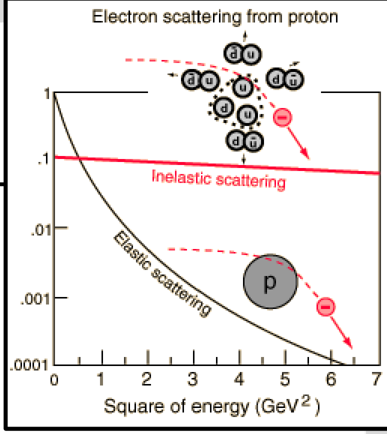
H1 Experiment @ HERA

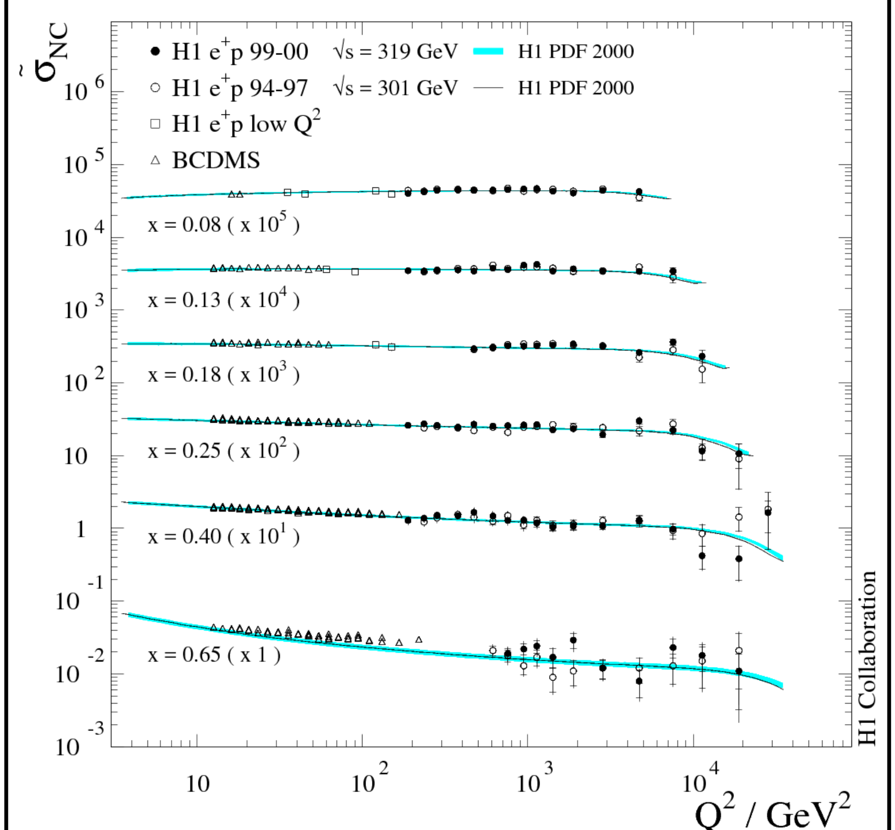
For the DIS process:

$$(xp + q)^2 = m_q^2 + 2xpq - Q^2 = m_q^2$$
$$x = \frac{Q^2}{2pq}$$

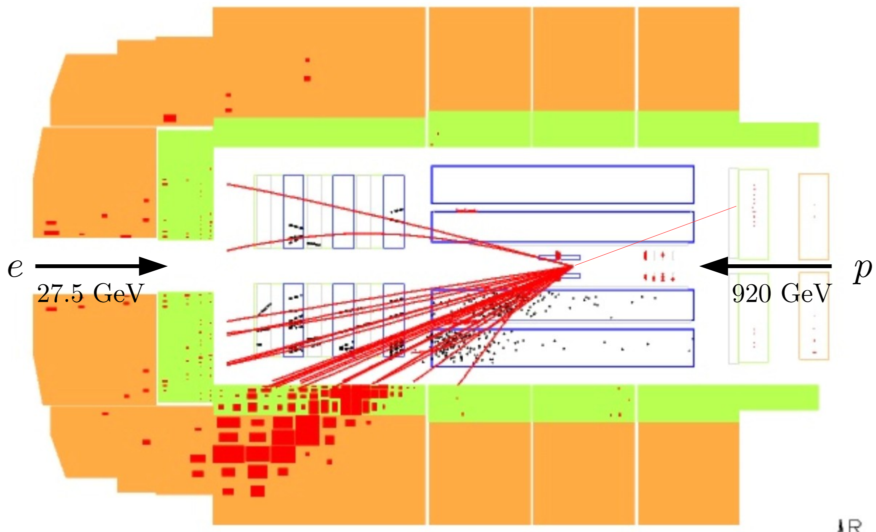
The evidence of quarks...

… emerged from deep inelastic scattering (DIS) experiments (first @SLAC 1969, here shown @HERA ~2000):





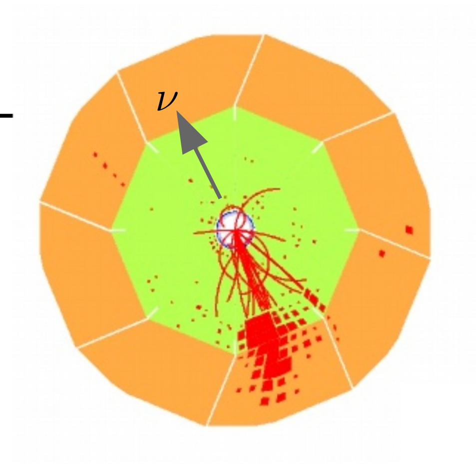
$\tilde{\sigma}_{NC}$: cross section divided by expectation for point-like particle.

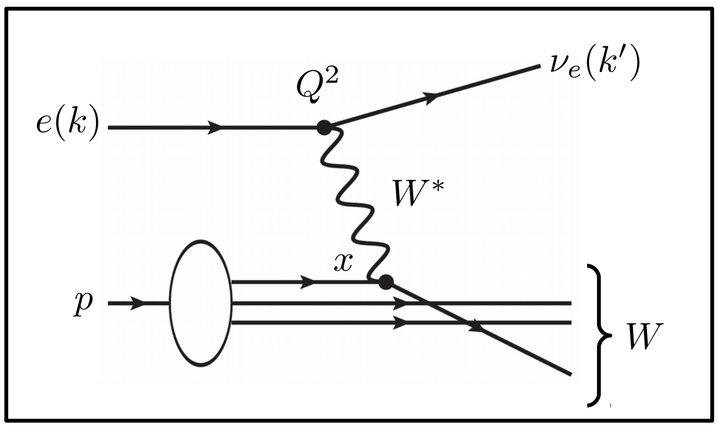


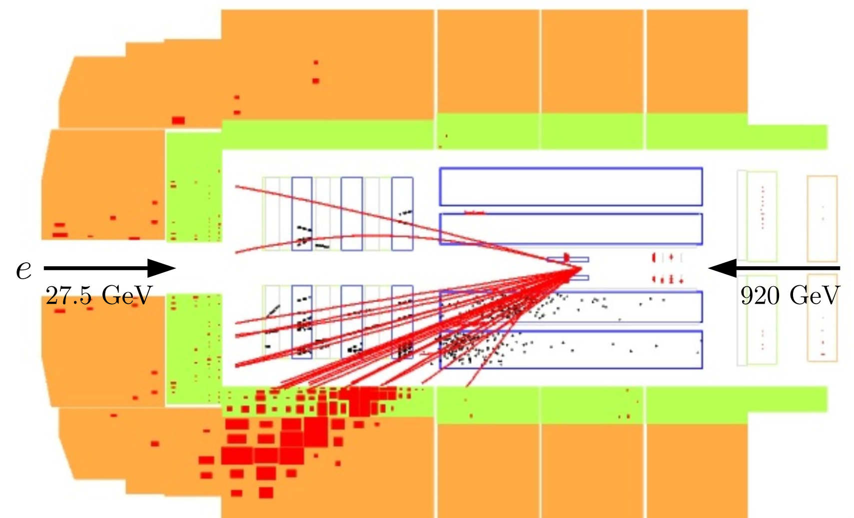
H1 Experiment @ HERA

Change of flavor & charge

- In the scattering vertex the electron can change flavor and charge and leave detector unobserved.
- Opposed to the neutral current (NC) process this is called charged current (CC) process.



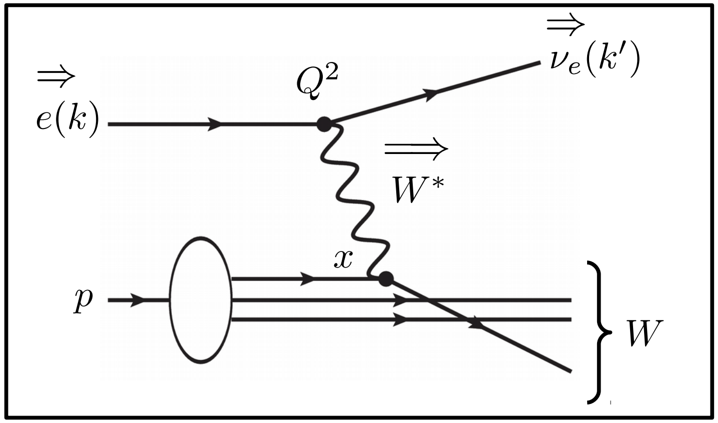


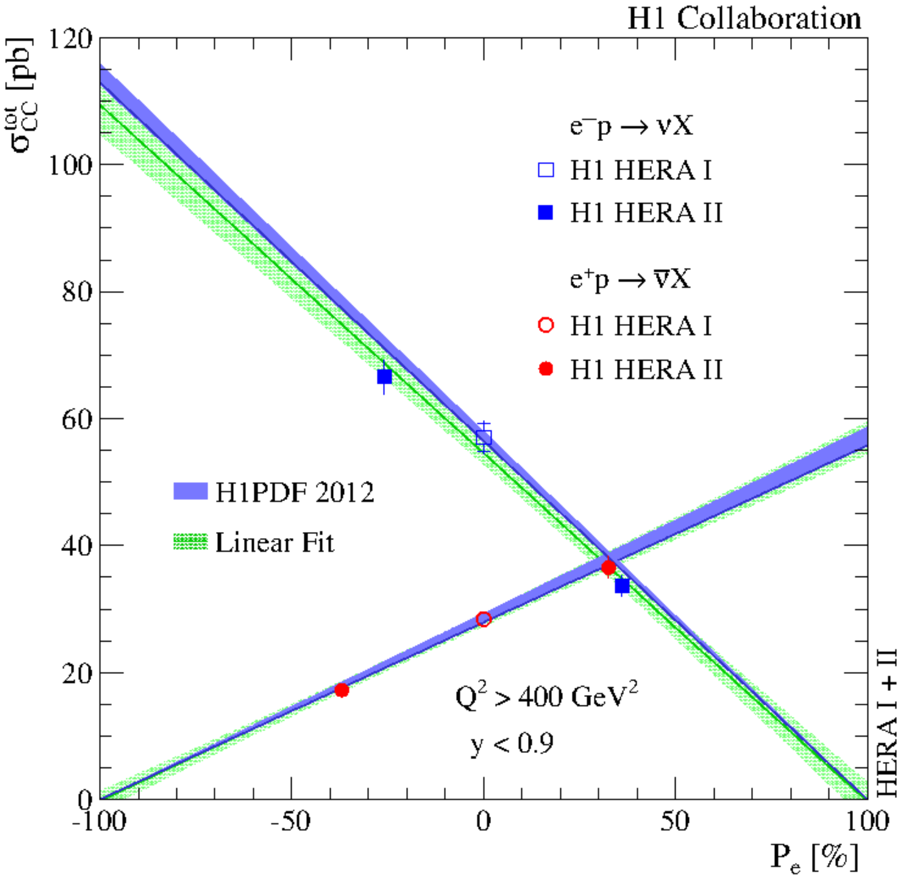


H1 Experiment @ HERA

Parity violation

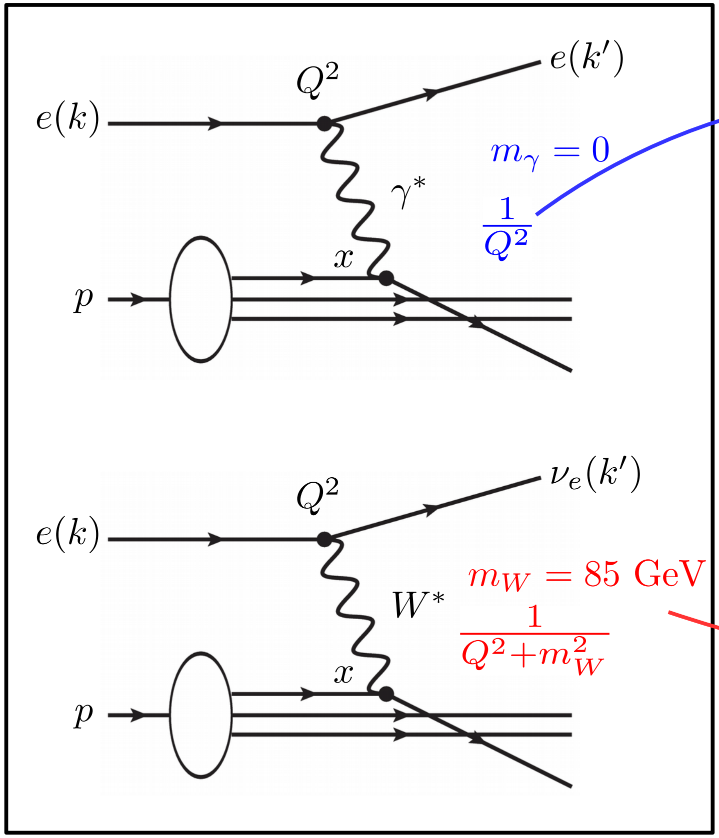
- HERA ran with e-beams of different polarization:
- CC reaction is maximally parity violating!
- W bosons couple only to left-handed particles (right-handed anti-particles).

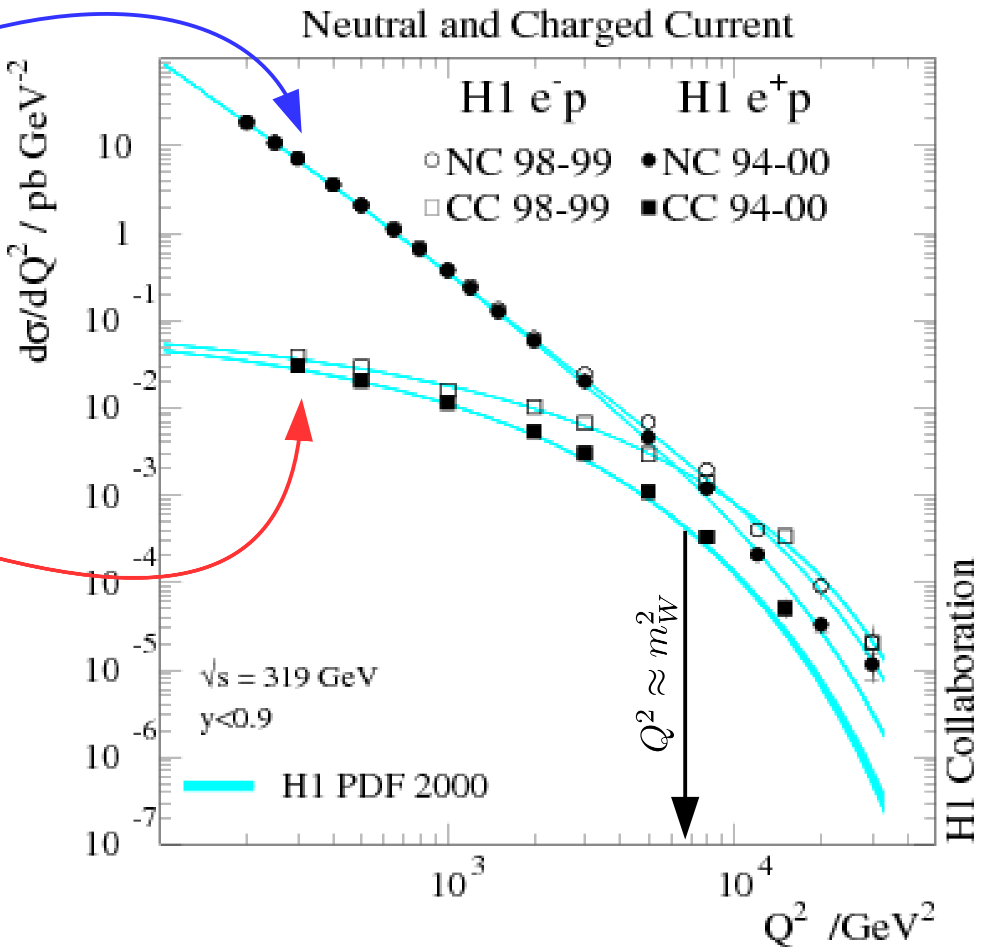




- NB: weak interaction intrinsically also violating CP.

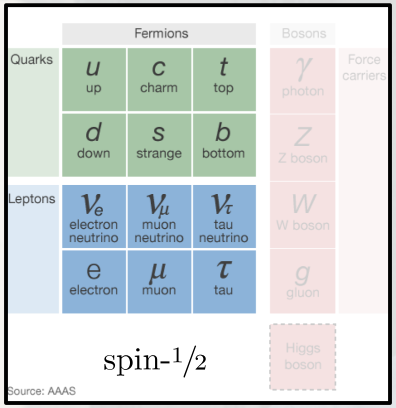
Massive force mediators





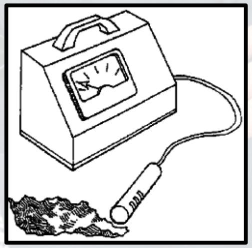
The case of matter

- All matter we know is made up of six quark flavors and six lepton flavors:

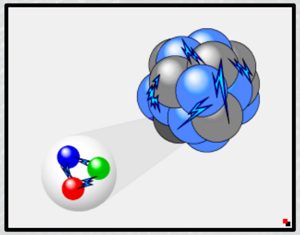


Four fundamental forces act between them (three of importance for particle physics).

Electromagnetism



Weak force



Strong force

The case of matter

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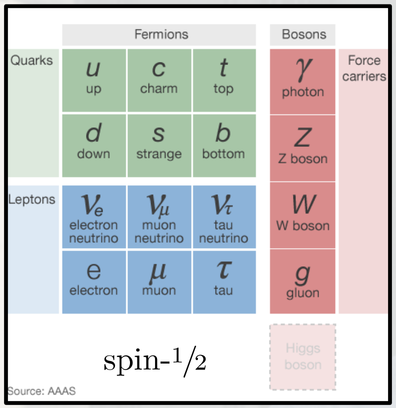
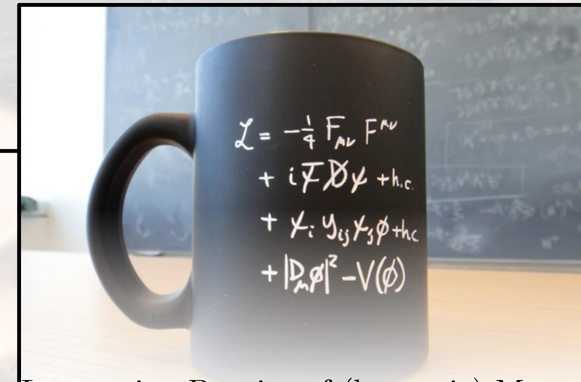




Lagrangian Density of (baryonic) Matter

$$U(1)_Y \times SU(2)_L \times SU(3)_c$$

1d rotations

2d rotations

3d rotations

in a $\mathbb{C}(N)$ hyperspace (w/ $N \geq 5$)

$$\psi \; e^{i\vartheta'}$$

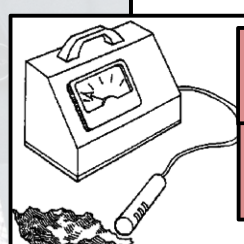
γ photon

Electromagnetism

$$\begin{pmatrix} u \\ d \end{pmatrix}_L e^{i t_a \vartheta_a}$$

$W^\pm$ W boson

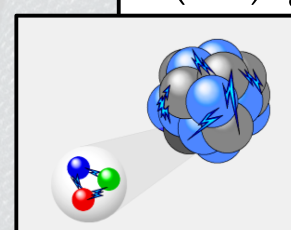
Z Z boson



Weak force

$$\begin{pmatrix} r \\ g \\ b \end{pmatrix}_c e^{i T_a \vartheta_a}$$

g gluon



Strong force

A wealth of structures

$$\mathcal{L}^{\text{SM}} = \mathcal{L}_{\text{kin}}^{\text{Lepton}} + \mathcal{L}_{\text{IA}}^{CC} + \mathcal{L}_{\text{IA}}^{NC} + \mathcal{L}_{\text{kin}}^{\text{Gauge}} + \mathcal{L}_{kin}^{\text{Higgs}} + \mathcal{L}_{V(\phi)}^{\text{Higgs}} + \mathcal{L}_{\text{Yukawa}}^{\text{Higgs}}$$

$$\mathcal{L}_{\text{kin}}^{\text{Lepton}} = i\bar{e}\gamma^{\mu}\partial_{\mu}e + i\bar{\nu}\gamma^{\mu}\partial_{\mu}\nu$$

$$\mathcal{L}_{\text{IA}}^{CC} = -\frac{e}{\sqrt{2}\sin\theta_W}\left[W_{\mu}^{+}\bar{\nu}\gamma_{\mu}e_L + W_{\mu}^{-}\bar{e}_L\gamma_{\mu}\nu\right]$$

$$\mathcal{L}_{\text{IA}}^{NC} = -\frac{e}{2\sin\theta_W\cos\theta_W}Z_{\mu}\left[(\bar{\nu}\gamma_{\mu}\nu) + (\bar{e}_L\gamma_{\mu}e_L)\right] - e\left[A_{\mu} + \tan\theta_W Z_{\mu}\right](\bar{e}\gamma_{\mu}e)$$

$$\mathcal{L}_{\text{kin}}^{\text{Gauge}} = -\frac{1}{2}Tr\left(W_{\mu\nu}^{a}W^{a\mu\nu}\right) - \frac{1}{4}B_{\mu\nu}B^{\mu\nu}\Bigg| \quad \begin{matrix} B_{\mu} \to A_{\mu} \\ W_{\mu}^{3} \to Z_{\mu} \end{matrix}$$

$$\mathcal{L}_{\text{kin}}^{\text{Higgs}} = \frac{1}{2}\partial_{\mu}H\partial^{\mu}H + \left(1 + \frac{1}{v}\frac{H}{\sqrt{2}}\right)^{2} m_W^2 W_{\mu}^{+}W^{\mu -} + \left(1 + \frac{1}{v}\frac{H}{\sqrt{2}}\right)^{2} m_Z^2 Z_{\mu}Z^{\mu}$$

$$\mathcal{L}_{V(\phi)}^{\text{Higgs}} = -\frac{m_H^2 v^2}{4} + \frac{m_H^2}{2}\left(\frac{H}{\sqrt{2}}\right)^{2} + \frac{m_H^2}{v}\left(\frac{H}{\sqrt{2}}\right)^{3} + \frac{m_H^2}{4v^2}\left(\frac{H}{\sqrt{2}}\right)^{4}$$

$$\mathcal{L}_{\text{Yukawa}}^{\text{Higgs}} = -\left(1 + \frac{1}{v}\frac{H}{\sqrt{2}}\right)m_e\bar{e}e$$

Full SM Lagrangian density (first lepton generation)

The power of symmetry

Karlsruhe Institute of Technology

- The SM draws its explaining and predictive power from the level of symmetry of $\mathcal{L}$.
- Each symmetry of $\mathcal{L}$ is related to a conserved quantity. This relation is revealed by the *Noether* theorem:

For illustration assume:

$$\mathcal{L} = \left(\partial_\mu \phi^\dagger \partial^\mu \phi\right) - m^2 \phi^\dagger \phi$$

And the symmetry operation:

$$\phi_j \longrightarrow \phi'_j = \phi_j + \delta\phi_j$$
$$\partial_\mu \phi_j \longrightarrow (\partial_\mu \phi_j)' = \partial_\mu \phi_j + \delta\partial_\mu \phi_j$$

Taylor expansion; symmetry requirement

$$\mathcal{L}(\{\phi_j + \delta\phi_j\}, \{\partial_\mu\phi_j + \delta\partial_\mu\phi_j\}) = \mathcal{L}(\{\phi_j\}, \{\partial_\mu\phi_j\}) + \underbrace{\frac{\delta\mathcal{L}}{\delta(\partial_\mu\phi_j)}\delta\partial_\mu\phi_j + \frac{\delta\mathcal{L}}{\delta\phi_j}\delta\phi_j}_{=0} = \mathcal{L}(\{\phi_j\}, \{\partial_\mu\phi_j\})$$

$$\frac{\delta\mathcal{L}}{\delta(\partial_\mu\phi_j)}\delta\partial_\mu\phi_j + \frac{\delta\mathcal{L}}{\delta\phi_j}\delta\phi_j = \underbrace{\frac{\delta\mathcal{L}}{\delta(\partial_\mu\phi_j)}\delta\partial_\mu\phi_j + \partial_\mu\frac{\delta\mathcal{L}}{\delta(\partial_\mu\phi_j)}\delta\phi_j}_{} = 0$$

$$\partial_\mu\frac{\delta\mathcal{L}}{\delta(\partial_\mu\phi_j)} - \frac{\delta\mathcal{L}}{\delta\phi_j} = 0$$

(on shell requirement)

$$\partial_\mu\left(\frac{\delta\mathcal{L}}{\delta(\partial_\mu\phi_j)}\delta\phi_j\right) = 0$$

$$J^\mu = \frac{\delta\mathcal{L}}{\delta(\partial_\mu\phi_j)}\delta\phi_j$$

(conserved current)

$$\int \mathrm{d}^3x\partial_\mu J^\mu = \int \mathrm{d}^3x\left(\partial_0 J^0 - \partial_i J^i\right) = 0$$

$$\int \mathrm{d}^3x\ \partial_t J^0 = \int \mathrm{d}^3x\ \vec{\nabla}\vec{J} = \int \mathrm{d}\vec{\Omega}\vec{J} = 0$$

J^0 (conserved charge)

The conserved charge is the generator of the symmetry operation that creates it.

Examples of symmetries

- A few examples of symmetry operations and/or conserved quantities on $\mathcal{L}$ are given below (→ try to complete the missing parts on your own):

	internal	external	symmetry	conserved quantity
discrete symmetry		☑	C, P, T, CP, CPT	...
		☑	rotation in $\mathbb{R}^3$	$\vec{L}$
continuous symmetry		☑	tranlsation in $\mathbb{R}^3$	$\vec{p}$
		☑	tranlsation in t	E
symmetry only on fields	☑		$U(1)_Y$, $SU(2)_L$, $SU(3)_c$	...
symmetry only on fields & arguments	☑	☑	Lortentz transformation	...
symmetry only on fields & arguments	☑	☑	Lortentz transformation	...
...	...	...	...	baryon number
...	...	...	...	lepton number

- One last non-trivial symmetry on $\mathcal{L}$ is the symmetry against an operation that transforms bosons into fermions and vice versa.

Remaining lecture program

	Monday (12.03):	Tuesday (13.03.):
13:30 15:00	Introdu[illegible] particle physics[illegible]	Proton structure, QCD jets and flavor (MM).
15:15 16:45	Particle acceleration & detection (RW); data analysis (MM).	Heavy quarks, gauge bosons (MM) & Higgs bosons (RW).

- In case of questions – contact us matthias.mozer@cern.ch (Bld. 30.23 Room 9-8)
  roger.wolf@cern.ch (Bld. 30.23 Room 9-20).

Backup

Transformationen und Gruppen

- Physikalische (Koordinaten-)Transformationen bilden **mathematische Gruppen**:

Gruppe:

Menge ($\mathcal{G}$) + (zweistellige) Verknüpfung ($*$), so dass gilt:

$$* : \mathcal{G} \times \mathcal{G} \to \mathcal{G}, \quad (a,b) \to a * b$$

$$\text{mit}$$

$$(a*b)*c = a*(b*c) \qquad \text{(Assoziativität)}$$

$$\exists e \in \mathcal{G} : e * a = a \ \forall a \in \mathcal{G} \quad \text{(Neutrales Element)}$$

$$\exists a^{-1} \in \mathcal{G} : a * a^{-1} = e \ \forall a \in \mathcal{G} \quad \text{(Inverses Element)}$$

- Wichtig ist, dass die Gruppe “schließt”, d.h. $a * b \in \mathcal{G}$

Transformationen und Gruppen

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mit

$(a * b) * c = a$

$\exists e \quad \in \mathcal{G} : e *$

$\exists a^{-1} \in \mathcal{G} : a *$

- Wichtig ist, da

Beispiel: Drehungen im $\mathbb{R}^2$

Menge ($\mathcal{G} = SO(2)$), Verknüpfung ($*$, Matrixmultiplikation)

$$*: \mathcal{G} \times \mathcal{G} \to \mathcal{G}, \quad (R(\alpha), R(\beta)) \to R(\alpha) * R(\beta) = R(\alpha + \beta)$$

$$\text{Neutrales Element} : 1_2 = R(0)$$

$$\text{Inverses Element} \quad : R^{-1}(\alpha) = R^{\tau}(\alpha) = R(-\alpha)$$

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$$\text{Inverses Element} \quad : R^{-1}(\alpha) = R^{\tau}(\alpha) = R(-\alpha)$$

$$R(\alpha) = \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix}$$ (Darstellung in 2d)

$$R(\alpha) = \begin{pmatrix} \cos\alpha & 0 & \sin\alpha \\ 0 & 1 & 0 \\ -\sin\alpha & 0 & \cos\alpha \end{pmatrix}$$ (Darstellung in 3d)

Beispiele von Transformationsgruppen

- Alle Drehungen im $\mathbb{R}^n$: spezielle orthogonale Gruppe $SO(n)$
- Alle Drehungen im $\mathbb{R}^n$ inklusive Spiegelungen: orthogonale Gruppe $O(n)$ (→ winkeltreue Abbildungen)
- Spiegelungen am Ursprung (→ Parität): Z_2
- Anmerkung: $O(n) = SO(n) \times Z_2$
- Alle Translationen im Raum
- Alle Gallileitransformationen
- Alle Lorentztransformationen, Drehungen und Translationen im $\mathbb{R}^{3+1}$ (→ Poicaré-Gruppe)
- …

Unitäre Transformationen

$U(1)$ Phasentransformation.

$$\psi(\vec{x},t) \rightarrow \psi'(\vec{x},t) = e^{i\vartheta}\psi(\vec{x},t)$$

- $U(n)$: Gruppe der **unitären Transformationen** im $\mathbb{C}^n$ mit den folgenden Eigenschaften: $\mathbf{G} \in U(n)$, $\mathbf{G}^\dagger\mathbf{G} = 1_n$, $|\det \mathbf{G}| = 1$

- Spaltet man eine weitere Phase von $\mathbf{G}$ ab kann man erreichen, dass: $\det \mathbf{G} = +1$

$$U(n) = U(1) \times SU(n)$$

$|\det \mathbf{G}| = 1$

(Unitäre Transformationen)

$\det \mathbf{G} = +1$

(Spezielle unitäre Transformationen)

- Die $SU(n)$ spielen in der Teilchenphysik eine besondere Rolle. Wir werden sie daher im folgenden als Beispiel verwenden, um einige Begriffe einzuführen

Kontinuierliche Gruppentransformationen

- Kontinuierliche Gruppentransformationen → **zusammengesetzt** aus vielen infnitesimalen Transformationen mit einem kontinuierlichen Parameter $\vartheta \in \mathbb{R}$:

$$\vartheta \in \mathbb{R} \qquad \mathbf{t} \in \mathcal{M}(\mathrm{n} \times \mathrm{n})$$

$$\mathbf{G}|_{\text{finite}} = \left(1_n + i\frac{\vartheta}{m}\mathbf{t}\right)^m \xrightarrow{m \to \infty} e^{i\vartheta \cdot \mathbf{t}}$$

- $\mathbf{t}$ Generatoren von $\mathbf{G}$.
- Definieren Struktur von $\mathbf{G}$.

- Die Menge der $\mathbf{G}$ (mit entsprechender Verknüpfung) bildet eine Lie-Gruppe
- Die Menge der $\mathbf{t}$ bildet die Lie-Algebra

Eigenschaften der t

- **Hermitesch**:

$$\mathbf{G}^\dagger \mathbf{G} = 1_n$$

$$= \left(1_n - i\vartheta \mathbf{t}^\dagger\right)\left(1_n + i\vartheta \mathbf{t}\right) = 1_n + i\vartheta \underbrace{\left(\mathbf{t} - \mathbf{t}^\dagger\right)} + O(\vartheta^2)$$

$$\boxed{\mathbf{t} = \mathbf{t}^\dagger}$$

- **Spurfrei**:

$$\det \mathbf{G} = \det\left(1_n + i\vartheta \mathbf{t}\right)$$

$$= 1 + i\vartheta \mathrm{Tr}(\mathbf{t}) + O(\vartheta^2) \overset{!}{=} 1$$

$$\boxed{\mathrm{Tr}(\mathbf{t}) = 0}$$

- **Dimension** des Tangentialraums:

$$\begin{pmatrix} * & * & * & * & * \\ * & * & * & * & \\ * & * & * & & \\ * & * & & & \\ * & & & & \end{pmatrix}$$

- n reelle Einträge auf Diagonale.
- $1/2 \cdot n(n-1)$ komplexe Einträge auf off-Diagonale.
- -1 für $SU(n)$ wegen $\det \mathbf{G} = 1$

- $U(n)$ hat n^2 Generatoren.
- $SU(n)$ Hat $(n^2 - 1)$ Generatoren.

Examples that appear in the SM ($U(1)$)

- $U(1)$ transformations (equivalent to $SO(2)$):
  - Number of generators: $1^2 \qquad = 1$

Was ist der Generator der $U(1)$

Examples that appear in the SM ($U(1)$)

- $U(1)$ transformations (equivalent to $SO(2)$):
  - Number of generators: $1^2 \quad = 1$

Was ist der Generator der $U(1)$ → 1

Examples that appear in the SM ($SU(2)$)

- $SU(2)$ transformations (equivalent to $SO(3)$):
  - Number of generators: $(2^2 - 1) = 3$
  - i.e. there are 3 matrices $\{\mathbf{t}_j\}$, which form a basis of traceless hermitian matrices, for which the following relation holds:

$$\mathbf{G} = e^{i\sum \vartheta_j \mathbf{t}_j} \qquad 1 \leq j \leq 3$$

  - Explicit representation:

$$\mathbf{t}_j = \frac{1}{2}\sigma_j \qquad (j = 1 \ldots 3)$$

(3 Pauli matrices)

$$[\mathbf{t}_i, \mathbf{t}_j] = i\epsilon_{ijk}\mathbf{t}_k$$

- algebra closes.
- structure constants of $SU(2)$.

  - In der schwachen Wechselwirkung im SM: $W^+,\ W^-,\ Z^0$

Examples that appear in the SM ($SU(3)$)

- $SU(3)$ transformations:
  - Number of generators: $(3^2 - 1) = 8$
  - i.e. there are 8 matrices $\{\mathbf{T}_j\}$, which form a basis of traceless hermitian matrices, for which the following relation holds:

$$\mathbf{G} = e^{i \sum \vartheta_j \mathbf{T}_j} \qquad 1 \leq j \leq 8$$

  - Explicit representation:

$$\mathbf{T}_j = \tfrac{1}{2}\lambda_j \quad (j = 1 \ldots 8)$$

(8 Gell-Mann matrices)

$$[\mathbf{T}_i, \mathbf{T}_j] = i f_{ijk} \mathbf{T}_k$$

- algebra closes.
- structure constants of $SU(3)$.

  - In der starken Wechselwirkung im SM: $|r\bar{g}\rangle,\ |r\bar{b}\rangle,\ |g\bar{r}\rangle,\ |g\bar{b}\rangle,\ |b\bar{r}\rangle,\ |b\bar{g}\rangle$

$$\frac{1}{\sqrt{2}}\left(|r\bar{r}\rangle - |g\bar{g}\rangle\right),\ \frac{1}{\sqrt{6}}\left(|r\bar{r}\rangle + |g\bar{g}\rangle - 2\,|b\bar{b}\rangle\right)$$

Abelsche und nicht-abelsche Gruppen

- $U(1)$ ist eine **abelsche Gruppe** → Reihenfolge in der Transformationen ausgeführt werden egal

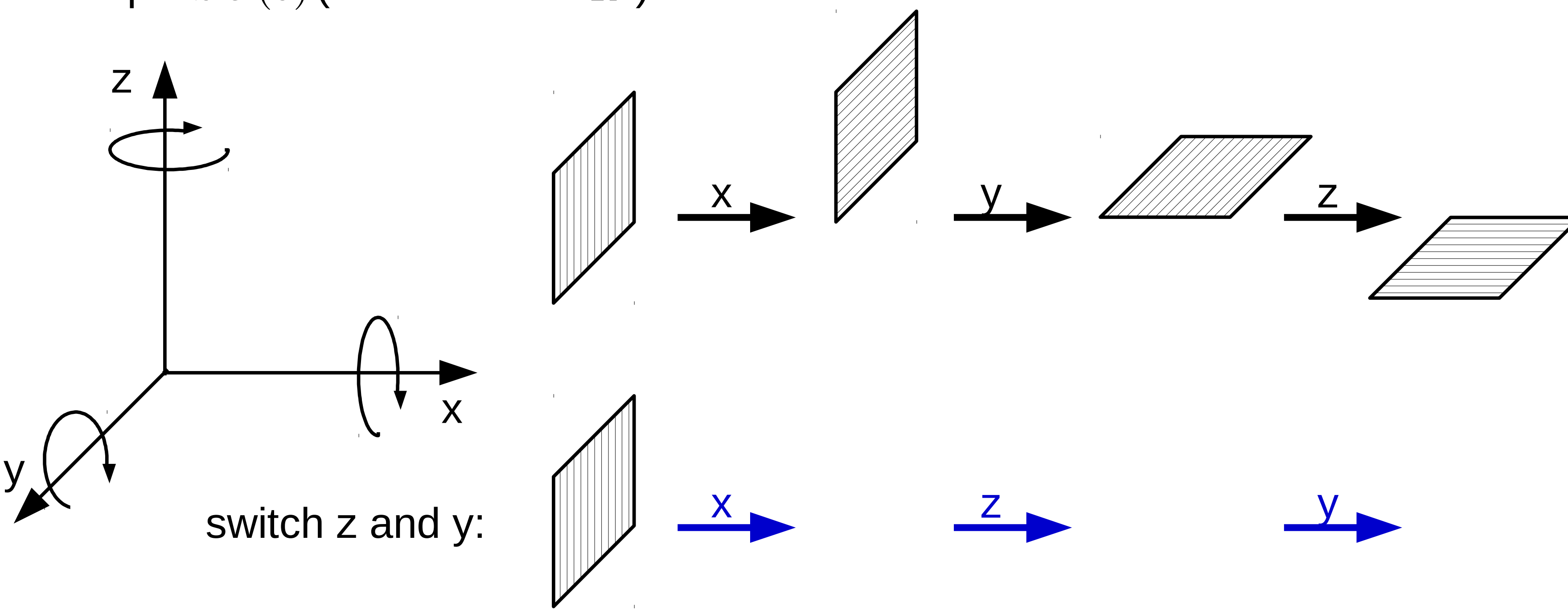
- $SU(2)$ und $SU(3)$ sind nicht-abelsche Gruppen (siehe Kommutator-Relationen) → Reihenfolge in der Transformationen ausgeführt werden spielt eine Rolle!

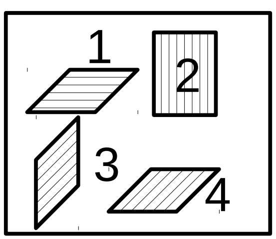
- Für die folgende Übung beachte:

$$U(1) \sim SO(2)$$
$$SU(2) \sim SO(3)$$

(Non-)Abelian symmetry transformations

- Example $SO(3)$ (90º rotations in $\mathbb{R}^3$):





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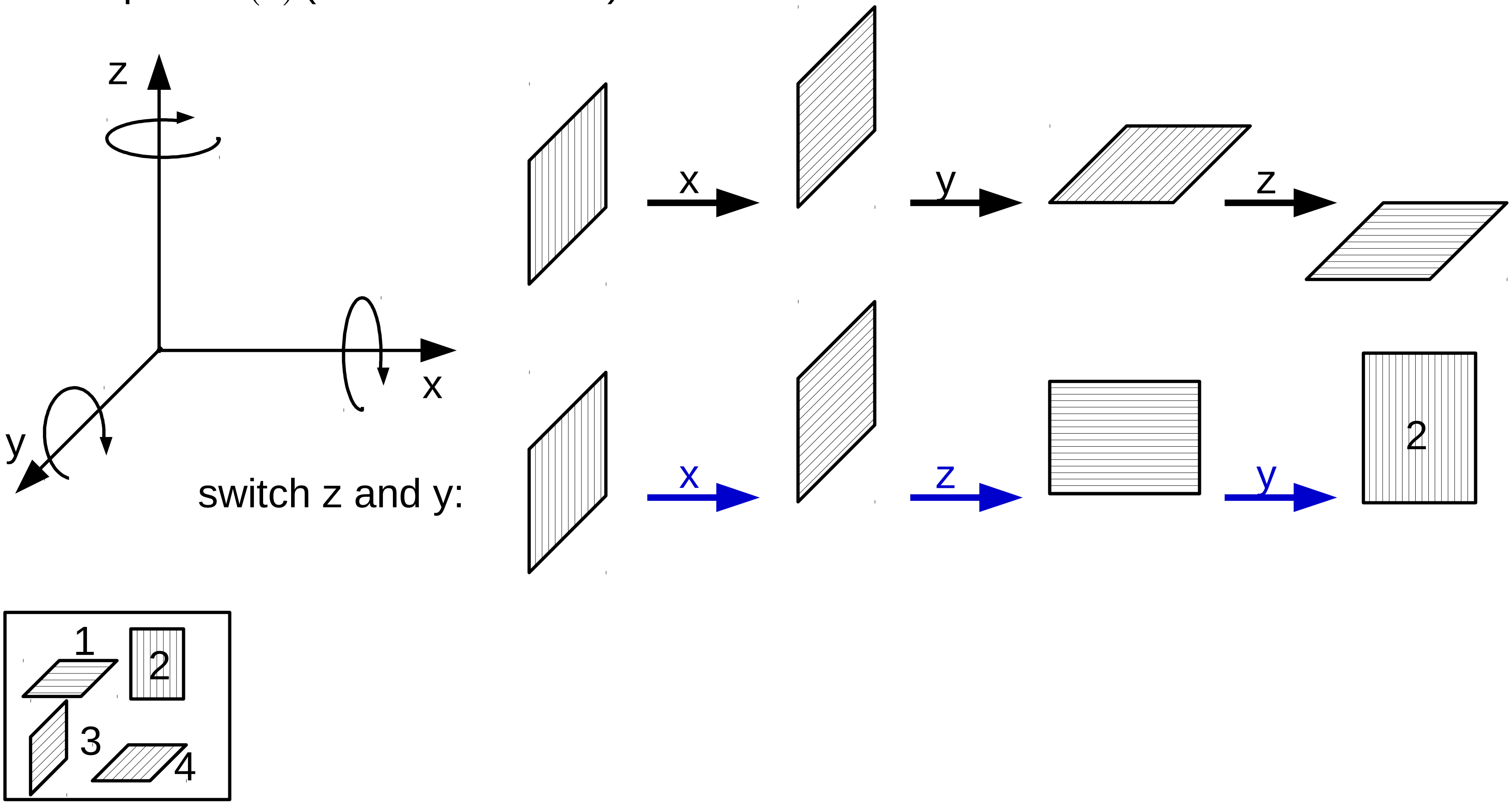

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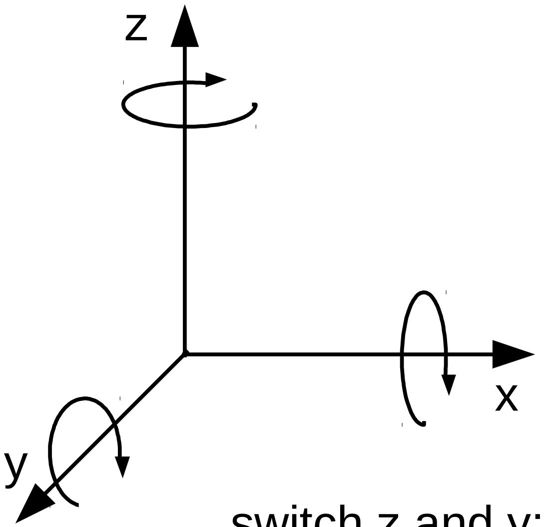
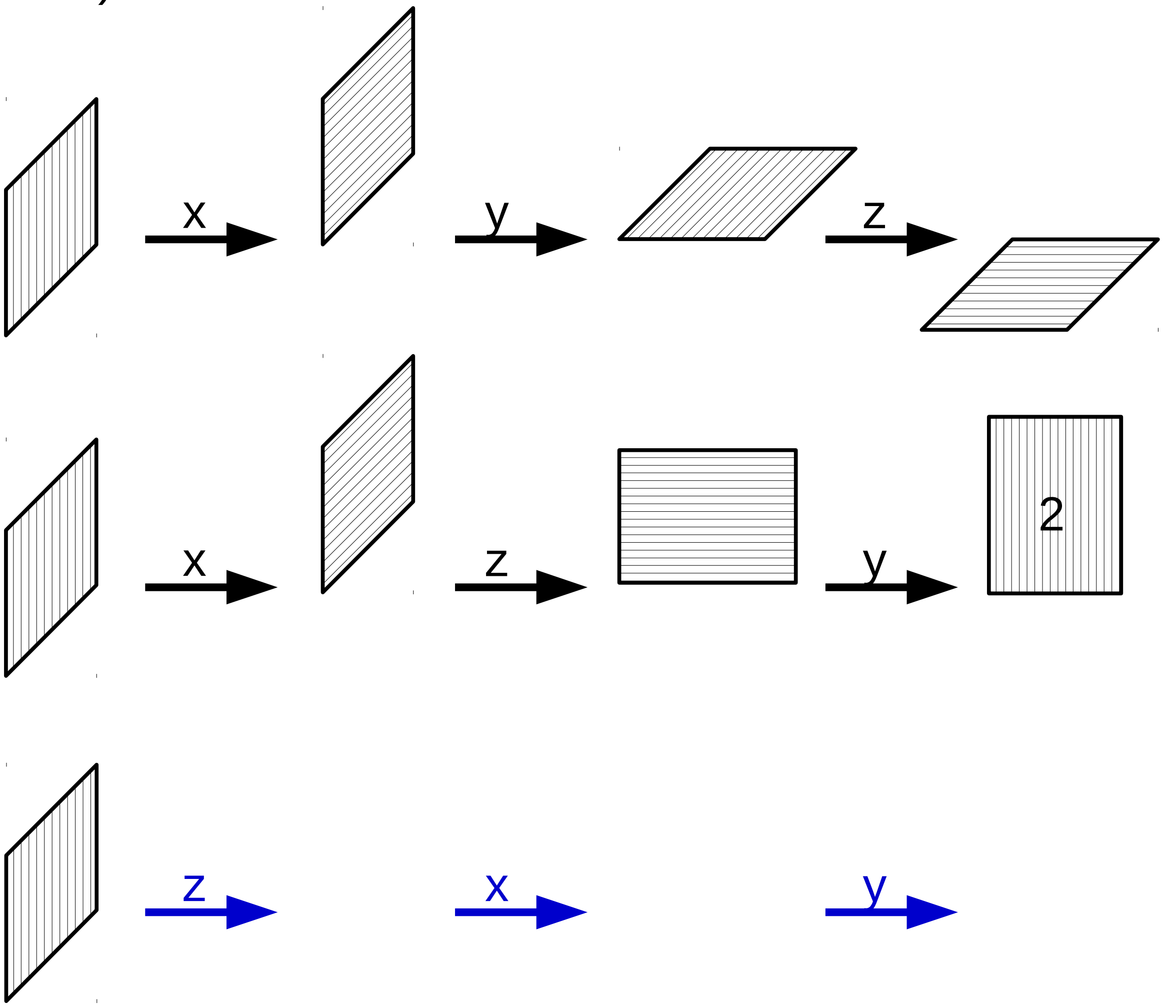
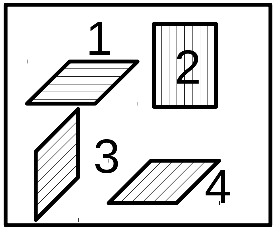


switch z and y:

cyclic permutation:





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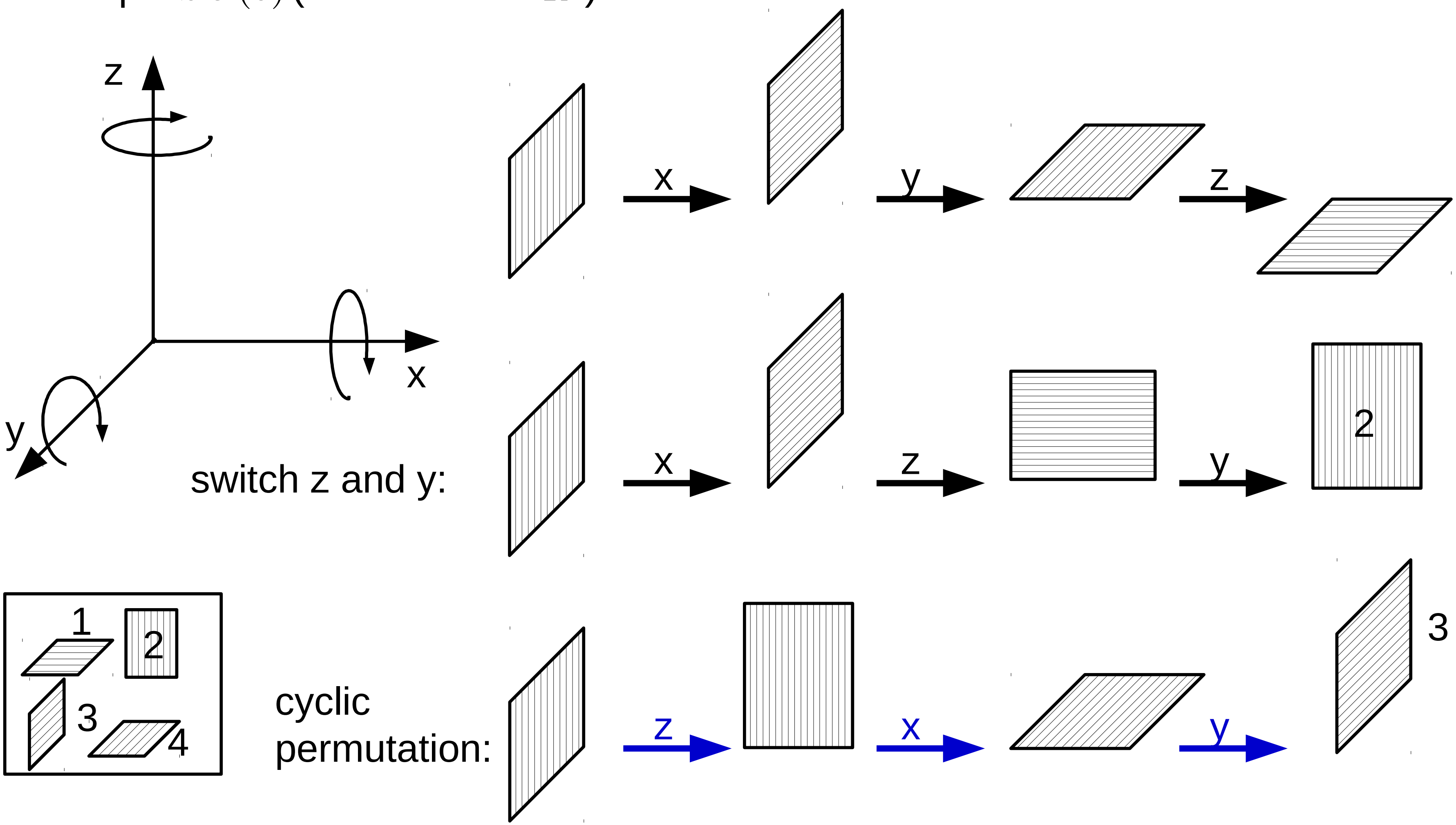

Beispiel: interne Erhaltungsgröße

- Betrachte Lagrangedichte eines komplexen skalaren Feldes:

$$\mathcal{L}(\partial_\mu \phi, \partial_\mu \phi^*, \phi, \phi^*) = \partial_\mu \phi \partial^\mu \phi^* - m^2 \phi \phi^*$$

- $\mathcal{L}$ offensichtlich invariant unter Phasentransformationen auf ϕ:

$$\begin{aligned} \phi &\to \phi' = e^{i\vartheta} \phi = \phi + \delta\phi \\ \phi^* &\to \phi^{*\prime} = e^{-i\vartheta} \phi^* = \phi^* - \delta\phi^* \end{aligned}$$

$mit:$

$$\begin{aligned} \delta\phi &= i\vartheta\phi \\ \delta\phi^* &= -i\vartheta\phi^* \end{aligned}$$

$$\begin{aligned} J^\mu &= \frac{\delta\mathcal{L}}{\delta\partial_\mu \phi_j} \delta\phi_j \\ &= \partial^\mu \phi^* \, (i\vartheta\phi) + \partial^\mu \phi \, (-i\vartheta\phi^*) \propto (\partial^\mu \phi^* \phi - \partial^\mu \phi \;\, \phi^*) \end{aligned}$$

(Noetherstrom = elektr. Strom)

$$J^0 = Q \propto \int_V \dot{\phi}^* \phi - \dot{\phi} \;\, \phi^* = q \cdot 1$$

(Noetherladung = elektr. Ladung)

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Anm.: Generator der Symmetrietransformation ist 1

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Beispiel: interne Erhaltungsgröße

- Beziehungen zwischen Symmetrie und Erhaltungsgröße in der Teilchenphysik:

$U(1)_Y$ ⟶ Elektrische Ladung (im SM Hyperladung Y)

$SU(2)_L$ ⟶ Schwacher Isospin (für linkshändige Teilchen)

$SU(3)_c$ ⟶ Farbladung (rot, grün, blau)